1. Show that the total cross section for the \( e^+e^- \rightarrow \mu^+\mu^- \) reaction at lower energies when \( m_\mu \) can no longer be neglected, but \( m_e \) still can, becomes:

\[
\sigma(e^+e^- \rightarrow \mu^+\mu^-) = \frac{4\pi\alpha^2\hbar^2c^2}{3s} \left(1 + \frac{2m_\mu^2}{s}\right) \sqrt{1 - \frac{4m_\mu^2}{s}}.
\]

Start from the most general form of the \( |\mathcal{M}|^2(e^+e^- \rightarrow \mu^+\mu^-) \) derived in class.

2. Show that for massless particle scattering \( 1 + 2 \rightarrow 3 + 4 \) conservation of 4-momentum implies that \( p_1 \cdot p_2 = p_3 \cdot p_4 \) and \( p_1 \cdot p_4 = p_2 \cdot p_3 \). We have used these relations in several derivations in class.

3. Show that vector and axial vector interactions of the form \( \bar{u}\gamma^\mu u \) and \( \bar{u}\gamma^\mu\gamma^5 u \), respectively, conserve helicity, i.e., that

\[
\bar{u}\gamma^\mu u = \bar{u}_L\gamma^\mu u_L + \bar{u}_R\gamma^\mu u_R \quad \text{and} \quad \bar{u}\gamma^\mu\gamma^5 u = \bar{u}_L\gamma^\mu\gamma^5 u_L + \bar{u}_R\gamma^\mu\gamma^5 u_R.
\]

In other words, the interactions do not couple particle states with opposite helicities.

4. Use the same explicit form of the Dirac spinors \( u^{(1)}, u^{(2)}, v^{(1)}, \) and \( v^{(2)} \) derived in class and used in problem 4 of homework set 5, to prove the completeness relations

\[
\sum_{s=1}^2 u^{(s)}\bar{u}^{(s)} = \not{p} + m \quad \text{and} \quad \sum_{s=1}^2 v^{(s)}\bar{v}^{(s)} = \not{p} - m.
\]

Recall that these relations provide a key step enabling us to calculate spin averaged squared matrix elements, \( |\mathcal{M}|^2 \), by evaluating traces instead of lengthy sums of matrix products. Be mindful that multiplying a \( 4 \times 1 \) column matrix with a \( 1 \times 4 \) row matrix results in a \( 4 \times 4 \) matrix, as indicated by the presence of Dirac \( \gamma \) matrices on the RHS of the expressions.

5. We have seen that the basic elastic \( e-\mu \) or \( e-p \) scattering process, e.g., \( e(k) + \mu(p) \rightarrow e(k') + \mu(p') \), results in the “current-current” interaction

\[
|M|^2 = \frac{g_e^4}{4t^2} L_{\mu\nu}^{(\text{electron})} K_{(\text{proton or muon})}^{\mu\nu},
\]

where the electron current has the form

\[
L_{\mu\nu} = 2[k'_\mu k_\nu + k'_\nu k_\mu - (k'_\nu \cdot k - m^2)g_{\mu\nu}].
\]

(a) Show that \( L_{\mu\nu} = L_{\nu\mu} \).

(b) Show that \( q^\mu L_{\mu\nu} = q^\nu L_{\mu\nu} = 0 \).

(c) Recalling that \( K_{\mu\nu} \) has the same structure, explain why relations (a) and (b) also apply to the muon (proton) vertex, i.e., to \( K_{\mu\nu} \). [Note: this property is retained even in inelastic e-p scattering due to the charge conservation at the \( p\gamma X \) vertex.]