Microgravity Isolation System Design: 
A Modern Control Analysis Framework

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Many acceleration-sensitive, microgravity science experiments will require active vibration isolation from the manned orbiters on which they will be mounted. The isolation problem, especially in the case of a tethered payload, is a complex three-dimensional one that is best suited to modern control design methods. These methods, although more powerful than their classical counterparts, can nonetheless go only so far in meeting the design requirements for practical systems. Once a tentative controller design is available, it must still be evaluated to determine whether or not it is fully acceptable, and to compare it with other possible design candidates. Realistically, such evaluation will be an inherent part of a necessarily iterative design process. In this paper, an approach is presented for applying complex $\mu$-analysis methods to a closed-loop vibration isolation system (experiment plus controller). An analysis framework is presented for evaluating nominal stability, nominal performance, robust stability, and robust performance of active microgravity isolation systems, with emphasis on the effective use of $\mu$-analysis methods. A new technique is also included for deriving guarantees on allowable umbilical and payload mass variations, using complex $\mu$ analysis.

Nomenclature

\begin{align*}
A &= \text{system dynamic matrix} \\
\alpha &= \text{algebraic constant} \\
B &= \text{acceptable region} \\
\beta &= \text{algebraic constant} \\
C &= \text{system control input matrix} \\
\Gamma &= \text{boundary between acceptable and unacceptable regions} \\
c &= \text{umbilical damping, N s/m} \\
\gamma &= \text{algebraic constant} \\
D &= \text{system state output matrix} \\
\Delta &= \text{complex uncertainty matrix} \\
c &= \text{control transmission matrix} \\
\delta &= \text{variation} \\
e &= \text{system disturbance input matrix} \\
\theta &= \text{phase rotation angle, rad} \\
f &= \text{stochastic disturbance} \\
\nu &= \text{structured singular value} \\
G &= \text{transfer-function matrix} \\
\xi &= \text{positive imaginary direction in complex space} \\
g &= \text{direction in complex space} \\
\Omega &= \text{disturbance-accommodation pseudostate} \\
H &= \text{transfer-function matrix} \\
\rho &= \text{algebraic constant} \\
I &= \text{identity matrix} \\
\sigma &= \text{singular value; positive real direction in complex space} \\
j &= \text{square root of \(-1\)} \\
\Omega &= \text{complex gain space corresponding to values of a} \\
K &= \text{regulator feedback gain matrix} \\
\omega &= \text{particular complex uncertainty gain} \\
k &= \text{umbilical stiffness, N/m} \\
\nu &= \text{payload mass, kg} \\
L &= \text{observer gain matrix} \\
\alpha &= \text{nominal} \\
M &= \text{mass or mass moment of inertia, kg m} \\
\beta &= \text{nominal} \\
m &= \text{system mass} \\
\gamma &= \text{system mass} \\
\min &= \text{minimum} \\
n &= \text{payload mass, kg} \\
\nom &= \text{output stochastic disturbance (sensor noise)} \\
\rho &= \text{performance} \\
\st &= \text{nominal} \\
\pf &= \text{real \(\mu\) (with \(\mu\))} \\
\Real &= \text{real uncertainty} \\
\r &= \text{input stochastic disturbance (process noise); lack of} \\
\sigma &= \text{input stochastic disturbance (process noise); lack of} \\
\st &= \text{stability} \\
\tau &= \text{input stochastic disturbance (process noise); lack of} \\
\xi &= \text{input stochastic disturbance (process noise); lack of} \\
\eta &= \text{system states} \\
\zeta &= \text{output stochastic disturbance (sensor noise)} \\
\Delta &= \text{system states} \\
\Delta &= \text{radius of complex uncertainty matrix \(\Delta\) (with \(r\))} \\
\kappa &= \text{stability} \\
\kappa &= \text{unit direction of complex uncertainty matrix \(\Delta\) (with \(g\))} \\
\xi &= \text{disturbance-accommodation pseudostates} \\
\xi &= \text{disturbance-accommodation pseudostates (with \(\xi\))}
\end{align*}

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\[ 2 \quad \text{output disturbance-accommodation pseudostates (with } \xi \text{); synthesis approach using quadratic cost function (with } H; \text{ particular value (with other symbols))} \\
3 \quad \text{particular value} \\
4 \quad \text{particular value} \\

\textit{Superscripts} \\
\text{CL} \quad \text{closed-loop system} \\
\text{T} \quad \text{transpose} \\
1 \quad \text{system augmented by frequency weighting and disturbance accommodation} \\
2 \quad \text{omission of the frequency-weighted pseudostates in the augmented plant model used for determining the observer gains (with } L) \\
-1 \quad \text{inverse} \\

\section*{Introduction}

The microgravity vibration isolation problem has received considerable attention in recent years. A number of materials-processes and fluid-physics experiments, planned for study in a weightless space environment, have run into unacceptably high back-flow and acceleration levels.\textsuperscript{1} The low-frequency disturbances of most concern are a natural accompaniment of space flight, with its large, flexible, unloaded structures and random, human-induced excitations. The combined need, with many experiments, for human interaction and for umbilicals connecting orbiter with payload has resulted in a very difficult, three-dimensional, active-isolation design problem.

Recent work by the authors has produced an extended $H_2$ synthesis framework, along with an associated general design philosophy, for developing a robust microgravity vibration-isolation controller.\textsuperscript{2} Other approaches (e.g., classical design or $\mu$ synthesis) are, of course, also theoretically capable of producing acceptable designs. But no approach can guarantee a satisfactory controller, short of an iterative design-and-analysis procedure. Analysis of any controller candidate is necessary to ensure that it, in fact, meets stability and performance requirements for a reasonable degree of model uncertainty. Methods of $\mu$ analysis can provide conservative guarantees of such acceptability.\textsuperscript{3} They also allow the engineer to assign a relative merit to each competing controller design.

This paper seeks to provide an analysis framework and philosophy for evaluating a given isolation controller candidate, with emphasis on the effective use of $\mu$-analysis methods. Particular stress is placed on evaluating the nominal stability, nominal performance, robust stability, and robust performance of the associated closed-loop isolation system. The work builds on the microgravity control synthesis framework proposed by the authors in Ref. 4.

\section*{Basic Isolation System}

A generic microgravity vibration isolation system is depicted in Fig. 1. A payload, such as a microgravity science experiment, is actuated upon by actuators (typically noncontacting, e.g., Lorentz or electromagnetic) that are commanded by a control system. This control system uses measurements, such as payload positions and accelerations, to develop the control signals, typically currents or voltages. The objective of controller synthesis is to develop a stable, robust controller that meets or exceeds the performance requirements. A framework for developing such a controller, by extended $H_2$ synthesis, has been presented in a previous paper.\textsuperscript{4} For the purposes of the present work, it is assumed that a tentative controller has been synthesized and is in need of evaluation.

Figure 2 shows a transfer-function block diagram of the closed-loop system in state-space form. This is the basic model used for closed-loop system analysis. The plant \([A, B, C, D]\) (i.e., payload plus umbilical plus actuator) is subject to direct and indirect disturbances; both kinds are included in input disturbance vector \(f_i\). The direct disturbances are those that act directly upon the payload; for example, they could be caused by air currents, astronaut contact, the flow of fluids for lubrication or cooling, or rotating machinery mounted on the experiment platform. The indirect disturbances act upon the payload through the umbilical. Because of the umbilical’s stiffness, any motion of the space platform well relative to the experiment will cause such a disturbance.

Some of the plant states \(x\) (not shown) are inaccessible; a subset \(y\) represents the ideal measurements, uncontaminated by sensor noise. The measurement vector \(z\) represents the actual measurements, which are contaminated by the output sensor-noise vector \(n\). The synthesized controller, to be evaluated, has state-space form \([A_{FB}, B_{FB}, C_{FB}, D_{FB}]\). Finally, \(E_z\) and \(E_n\) are simply selection matrices, and the control vector is \(u\).

\section*{Controller Evaluation}

Once a microgravity vibration-isolation controller has been synthesized, it must be evaluated for acceptability and relative merit. One major design consideration is simplicity: the controller’s state-space form should have no more states than necessary. For an analog implementation, controller complexity translates directly into hardware complexity; for a digital controller, the cost is largely in computational power and speed. A controller that is robust in stability and performance but is too complicated is of no use. Modern control methods, especially with the augmentation accompanying frequency weighting and disturbance accommodation, can result in controllers that have an unnecessarily large number of states. The controller dimensionality can usually be reduced substantially by employing modal reduction and/or the balance-and-truncate technique.\textsuperscript{5} Even if controller reduction is not deemed necessary, it is advisable for the sake of design simplicity.

Another important consideration is controller stability. Although unstable controllers are used on occasion (provided the closed-loop system is stable) and need not be rejected out of hand, they cannot easily be bench-tested. An ideal controller designed by extended $H_2$ or $\mu$ synthesis is guaranteed to stabilize the nominal plant; but it is not guaranteed to be open-loop stable, since some plants can only be stabilized by unstable controllers. Application of reduction techniques can also lead to controller destabilization. It is best to conduct an eigenvalue check of any controller candidate’s $A$ matrix (i.e., its dynamic matrix) to ensure controller stability.
Once the controller model has been checked for simplicity and stability, it is ready for attachment to the model of the plant. There are four crucial checks that must now be made of the closed-loop system; viz., nominal stability, nominal performance, robust stability, and robust performance. The nominal analysis model (Fig. 2) is the conceptual starting point for conducting these checks, each of which is treated individually below.

Nominal Stability
An unstable closed-loop system cannot, of course, provide the isolation desired. Stability is the most basic system requirement for any microgravity vibration-isolation system. The extended $H_2$ synthesis method, when used for controller design, provides an inherent guarantee of stability for the nominal plant with full-state feedback. However, $H_2$ synthesis occasionally produces an unstable closed-loop system because of numerical anomalies. So the closed-loop system eigenvalues should still be checked. The same is true with a controller designed by $\mu$ synthesis.

The well-known separation principle guarantees that for a perfectly known plant, a stable asymptotic (i.e., Luenberger) observer will not destabilize the system. Thus, with the full-order observer (i.e., observing all states and pseudostates), nominal closed-loop stability is assured. However, models of dynamic systems are never perfect, so nominal stability is no guarantee of actual isolation-system stability; and the use of an umbilical for the present problem makes precise modeling especially difficult. Still, a check of nominal stability certainly provides a reasonable starting point.

If the order of the feedback controller is reduced, the guarantee of nominal closed-loop stability provided by extended $H_2$ synthesis is lost. But simple checks of the closed-loop-system eigenvalues (i.e., the eigenvalues of the closed-loop-system $A$ matrix) can readily verify (or contraindicate) stability, regardless of the design methodology used. For the system shown in Fig. 2, the closed-loop-system $A$ matrix (from input $u$ to output $z$, for simplicity and without loss of generality) is

$$A_{CL} =
\begin{bmatrix}
A + BD_{FB}(I - D_{FB}D)^{-1}D_{FB}C & B(I - D_{FB}D)^{-1}D_{FB}C \\
B_{FB}(I - DD_{FB})^{-1}C & A_{FB} + B_{FB}(I - DD_{FB})^{-1}DC_{FB}
\end{bmatrix}
$$

where

$$\begin{bmatrix}
A_{FB} & B_{FB} \\
C_{FB} & D_{FB}
\end{bmatrix}
$$

represents the transfer-function matrix in the feedback path from $z$ to $u$.

Nominal Performance
A potential controller design for a microgravity vibration-isolation system must do more than prove stability; it must also give acceptable system performance when its model is attached to the nominal plant model. Orbiter motions and sensor noise must both be rejected to specified levels, and the necessary control signals must be small enough to prevent actuator or amplifier saturation. Using the various transfer functions represented by the nominal analysis model, the designer can verify that the nominal controlled plant meets such design specifications. Since system uncertainties (e.g., umbilical or actuator model inaccuracies) will degrade the actual closed-loop system performance, it is generally desirable to exceed the performance specifications during controller design, so that as large a set of off-nominal plants as possible (or desirable) will still meet the design specifications. Robust performance will be checked at a later stage; but it should be kept in mind here that a nominal performance that is only marginal will probably be unacceptable when model inaccuracies are taken into account. The goal, of course, is for the isolation system to meet the performance requirements in hardware.

The uncontrolled plant for the microgravity vibration-isolation problem has a strictly proper transfer-function matrix (i.e., $D = 0$), the zero matrix). Accordingly, the closed-loop transfer functions from process noise inputs $f_i$ to measurements $z$ are

$$H_{CL}^{CL} = \begin{bmatrix}
A + BD_{FB}C & BC_{FB} & I \\
B_{FB}C & A_{FB} & 0 \\
E, C & O & 0
\end{bmatrix}
$$

and from sensor noise inputs $f_s$ to measurements $z$, the closed-loop transfer functions are

$$H_{CL}^{CL} = \begin{bmatrix}
A + BD_{FB}C & BC_{FB} & BD_{FB} \\
B_{FB}C & A_{FB} & BD_{FB} \\
C & O & I
\end{bmatrix}
$$

The closed-loop system must achieve acceptable isolation, but it must do so without unacceptable sensitivity to sensor noise. These two tasks involve an inherent tradeoff. High loop gains are necessary to reject input disturbances at a given frequency. These correspond to small transmissibility gains, i.e., to small magnitudes of the complementary sensitivity-function matrix $T$. On the other hand, low loop gains are necessary to reject sensor noise. These correspond to small magnitudes of the sensitivity-function matrix $S$. Since the sum of $T$ and $S$ is the identity matrix $I$, the designer cannot achieve arbitrarily low sensitivity to noise in the same frequency band where good disturbance rejection is to be accomplished. This unavoidable tradeoff can be measured by use of the above two transfer-function matrices.

The system must also employ a control signal that does not become excessive for realistic disturbance levels. If amplifiers or actuators are allowed to saturate, the resulting nonlinearities will greatly complicate both the synthesis and the analysis problems. The following transfer-function matrix provides a measure of this aspect of nominal system performance:

$$H_{CL}^{CL} = \begin{bmatrix}
A + BD_{FB}C & BC_{FB} & ES \\
B_{FB}C & A_{FB} & 0 \\
D_{FB}C & E & O
\end{bmatrix}
$$

Simple Form for Nominal $H_2$ Controller
For a controller designed by extended $H_2$ synthesis, the controller transfer-function matrix

$$\begin{bmatrix}
A_{FB} & B_{FB} \\
C_{FB} & D_{FB}
\end{bmatrix}
$$

has a particularly convenient form. First, define $n := \dim(x)$, the number of states in the plant model; and let the disturbance-accommodation pseudostate vector be $\xi := [\xi^T, \xi^T]^T$, with $\xi$ and $\xi$, corresponding to input and output disturbances, respectively. Assume the use of an asymptotic observer for state reconstruction. When all states and pseudostates are reconstructed in the observer the full observer synthesis model is used (Fig. 3a). In this case,

$$\begin{bmatrix}
A_{FB} & B_{FB} \\
C_{FB} & D_{FB}
\end{bmatrix}
$$

where $K$ is the optimal feedback gain matrix, and $L$ is the observer gain matrix. The left-hand superscripts $1$ reflect the appropriate augmentation.

The frequency-weighting pseudostates need not be reconstructed in the observer, since they can be produced from the state observations and control signals, by simple filtering. One can then (more typically) use an observer-synthesis model of reduced order (Fig. 3b), in which case

$$\begin{bmatrix}
A_{FB} & B_{FB} \\
C_{FB} & D_{FB}
\end{bmatrix}
$$

(4a)
convection, or encapsulated crystal growth) can cause actual or apparent mass and/or moment-of-inertia variations, and unembilical and actuator nonlinearities will typically cause the models to be quite inaccurate. A controller design must take into account such effects, so that system stability can be ensured for at least the anticipated range of model uncertainties. A closed-loop system of this character is said to have robust stability.

In classical control theory, the gain margin (GM) and phase margin (PM) of a single-input, single-output (SISO) system are the familiar measures of stability robustness. They measure, respectively, the amount of gain or phase that can be inserted into the feedback loop of a transfer-function block diagram without leading to system instability. This may be conceptualized as the turning of a gain knob or phase knob, respectively, until instability is encountered.) Accordingly, they provide the analyst with separate measures of allowable uncertainty in loop gain or phase.

If the loop gain and phase are allowed to vary simultaneously, this uncertainty can be represented correspondingly by a complex gain $e^{j	heta}$ = re$^{j	heta}$. Let $\Delta(s)$ be a complex ball representing all possible complex gains of some magnitude $r$, and let $M(s)$ be the transfer function of the (stable closed-loop) system seen by these gains. (Refer to Fig. 4.) The magnitude of the scalar product $\Delta(j\omega)M(j\omega)$ must be less than one to guarantee that the uncertainty does not destabilize the system. Consequently, the magnitude of the inverse of $M(j\omega)$ (i.e., $|M^{-1}(j\omega)|$) provides a measure of the magnitude of complex uncertainty allowable in the loop, as a function of frequency $\omega$. The minimum of the plot of $|M^{-1}(j\omega)|$ provides a guarantee of allowable uncertainty size for all frequencies.

For multiple-input, multiple-output (MIMO) systems, a robustness analysis method very similar to the $\Delta$-complex-gain method described above can be used. In this analysis, $M(s)$ is a transfer-function matrix and $\Delta(s)$ is a transfer-function matrix of complex gains representing uncertainty in the system. The maximum singular value of the matrix product $\Delta(j\omega)M(j\omega)$ must be less than one to ensure a stable system. Consequently, the reciprocal of the maximum singular value of $M(j\omega)$ (i.e., $\bar{\sigma}^{-1}[M(j\omega)]$) provides a measure (induced 2-norm) of the complex uncertainty allowable in the loop as a function of frequency $\omega$. The minimum of the plot of $\bar{\sigma}^{-1}[M(j\omega)]$ provides a guarantee of the allowable uncertainty size for all $\omega$.

One problem with the singular-value method is that it may model uncertainties in the system that are not physically meaningful, since it allows any input (to the $\Delta$ block) to be coupled to any $\Delta$-block output. The structured singular value $\mu$ provides an alternative measure of stability robustness that permits the analyst to remove unrealistic couplings. This is accomplished by prescribing a structure for the complex gain matrix $\Delta(s)$. To measure system stability robustness using $\mu$ analysis, the engineer first models system uncertainties as a complex gain matrix $\Delta(s)$ with a block-diagonal structure, appropriately located in a flow path of a transfer-function-matrix block diagram. This structured uncertainty matrix is composed of smaller complex uncertainty blocks ($\Delta_i$ blocks) along its main diagonal. To permit $\mu$ analysis it is required simply that the resulting system, with $\Delta$ block inserted, be capable of arrangement as shown in Fig. 4. Once this has been done, the structured singular value $\mu$ of $M(j\omega)$ can be found. In actuality, $\mu$ can be determined exactly only if the number of $\Delta_i$ blocks is three or fewer; for a larger number of blocks, upper and lower bounds must be employed. The quantity $\mu^{-1}[M(j\omega)]$ provides a measure of the largest magnitude (induced 2-norm) that the $\Delta_i$ blocks can all possess simultaneously without any combination of the $\Delta_i$ blocks leading to system instability. The structured singular value $\mu[M(j\omega)]$, used with a structured uncertainty, is analogous to the maximum singular value $\bar{\sigma}[M(j\omega)]$ that was used in the unstructured case.

\[ 2L_s := \begin{bmatrix} L_s & 0 \\ 0 & L_t \end{bmatrix} \]

(4b)

\[ 2L := \begin{bmatrix} L_s \\ L_t \end{bmatrix} \]

(4c)

with $L_s$ consisting of the first $n$ rows of $2L$. The left-hand superscript 2 indicates that the frequency-weighting pseudostates were omitted in the augmented plant model used for determining the observer gains. Equations (3) and (4) can be used with Eqs. (1) and (2) to evaluate the system nominal stability and performance with relative ease.

Robust Stability

To be practical, a vibration-isolation system must be stable not only for the nominal plant but for a range of off-nominal plants as well. For the microgravity isolation problem, the payload mass will in most cases be known quite accurately. But fluid motion (e.g., for cooling or in studies of liquid bridges, surface-tension-driven
Example: Robust Stability Analysis for Actuator Uncertainties

To illustrate, consider a robust stability analysis for a microgravity isolation system with noncontacting magnetic actuators. Two types of actuators are under development: magnetic bearings and Lorentz actuators. Each type is likely to experience gain variation as a function of payload position in the rattle space. Position-dependent cross coupling is likely as well, between each actuator's nominal line of force and the associated orthogonal axes. Finally, anomalous phase lags can be expected, because of unmodeled physical phenomena (e.g., eddy currents and hysteresis). Such actuator uncertainties can be modeled by use of a multiplicative input block, inserted into the nominal analysis model (closed-loop portion) as shown in Fig. 5. The block has a direct physical correspondence to phase-plus-gain uncertainties in the controller output(s) or actuator(s). The uncertainty can be either unstructured or structured. Assume that each actuator has only one line of action. If a control input to one actuator affects only the output of that actuator, a structured uncertainty model will give more realistic (i.e., less conservative) results. If there is appreciable cross coupling from one actuator input to the output of another actuator (e.g., via the cocking of the payload), then the unstructured form is more appropriate.

Consider the unstructured case first. In this case, $\Delta$ is a full complex matrix, with uncertainties between all of its inputs and outputs. Re-express the system in the form shown in Fig. 6, where $M$ is the transfer-function matrix seen by $\Delta$. For this example (i.e., with a multiplicative input $\Delta$ block),

$$ M = G_2 G_1 (I - G_2 G_1)^{-1} $$  (5)

$W$ is a scalar weighting function, used to normalize the $\Delta$ block so that its maximum singular value is less than one. To have a stable system, it is necessary that the product $\Delta(\omega) W(\omega) M(\omega)$ have a maximum singular value $\sigma$ less than one. Accordingly, $\sigma^{-1}[W M(j\omega)]$ gives the maximum allowable size of $\Delta$ as a function of $\omega$. Specifically, if the maximum singular value of $\Delta$ is less than $\sigma^{-1}[W M(j\omega)]$, there is no uncertainty represented by $\Delta$ that can destabilize a stable system. (For example, for a SISO system one could choose $W$ to reflect the minimum acceptable phase or gain variation from controller output to plant input, as a function of frequency. With this normalization of $\Delta$, if the plot of $\sigma^{-1}[W M(j\omega)]$ is below unity for all $\omega$, the nominal system is guaranteed to have the required stability margin.)

Next consider a structured-uncertainty case, where $\Delta$ has a block-diagonal form. For a system with two control inputs there would be two $\Delta_i$-blocks:

$$ \Delta = \begin{bmatrix} \Delta_1 & 0 \\ 0 & \Delta_2 \end{bmatrix} $$

Physically this means that uncertainties are considered to exist only in-channel between input(s) and output(s) of $\Delta$. Again, re-express the system in the form shown in Fig. 6; $M$ is defined as before. $W$ in this case is a diagonal weighting matrix, with one scalar weighting function per channel of input to $\Delta$. This weighting matrix is used to normalize each $\Delta_i$ block so that its maximum singular value $\sigma_i(\omega)$ is less than one. To have a stable system it is necessary that the product $\Delta_i(\omega) W(\omega) M(\omega)$ have maximum singular value $\sigma_i$ less than one, as before. Accordingly, $\sigma_i^{-1}[W M(j\omega)]$ gives the maximum allowable size of the uncertainty block $\Delta_i$, as a function of $\omega$. Specifically, if the maximum singular value of each $\Delta_i$ block is less than $\sigma_i^{-1}[W M(j\omega)]$, there is no uncertainty represented by $\Delta_i$ that can destabilize a stable system. (For example, for a system with two control inputs one could choose $W$ to reflect the minimum acceptable in-channel phase or gain variation from each controller output to the associated plant input, as a function of frequency.)

With this normalization of $\Delta$, if the plot of $\sigma_i^{-1}[W M(j\omega)]$ lies below unity for all $\omega$, the nominal system is guaranteed to have the required stability margins.) Note that since the uncertainty block(s) $\Delta_i$ can represent complex gains in any direction in the gain plane, even those directions not corresponding to realistic variations, $\mu$-analysis results are conservative in nature.

In the case of an unstructured block the plot of $\sigma^{-1}(M)$ can be used to calculate the MIMO GM and PM at the plant input. In the structured-uncertainty case either $\sigma^{-1}(M)$ or, less conservatively, $\mu^{-1}(M)$ can be used for this purpose. See the Appendix for details.

Other Stability-Robustness Checks for the Microgravity Isolation Problem

As shown above, multiplicative-input $\Delta$ blocks are useful in finding guarantees of allowable variation for magnetic or Lorentz actuators. $\Delta$ blocks can be placed elsewhere in the system block diagram to provide other useful stability-robustness guarantees, in similar fashion. The controller inputs for the microgravity isolation problem typically come from relative-position sensors and accelerometers. Each sensor will filter its input in a manner that is only approximated by its transfer-function model. The analyst can determine guarantees of allowable sensor variations with the use of multiplicative-output $\Delta$ blocks (Fig. 7a). As before, a MIMO GM and PM can be derived. In addition to the above variations, there will be higher system modes for which the payload and umbilical models do not account. An additive-uncertainty $\Delta$ block can be placed in a forward direction around the plant (Fig. 7b) to find a measure of the modal uncertainties allowable. One can also use a feedback uncertainty block (Fig. 7c) to model uncertainties in payload mass or in umbilical stiffness or damping, as will be shown later.

Robust Performance

It is necessary also that the closed-loop system performance not degrade unacceptably if the isolation system model is inaccurate or if the system changes over time. For example, umbilical-model uncertainties must not reduce microgravity isolation below acceptable levels. The level of performance robustness can be measured by using $\mu$ analysis, in a manner similar to that employed for stability-robustness analysis. By posing the closed-loop frequency-domain specifications in the form of a $\Delta$ block with an appropriately selected weighting matrix ($\Delta_{\text{ref}}$ and $W_{\text{ref}}$ respectively), and at the same time using stability $\Delta$-blocks ($\Delta_\text{st}$) as described previously, the designer can determine how much complex uncertainty is allowable at various places in the closed-loop system, without exceeding the performance specifications or causing system instability.

Typically the nonzero elements of $W_{\text{ref}}$ are the reciprocals of the appropriate closed-loop transfer functions. Again, the results are conservative.

Figures 8a and 8b give examples of robust-performance analysis models that are useful for the microgravity isolation problem. The structured singular-value plot for the multiplicative-input (or-output) case can be used to determine a MIMO gain variation (GV) or phase variation (PV). These are conservative measures of the GV or PV allowable in each channel (entirely analogous to MIMO PM and GM) without violating performance specifications or losing stability guarantees. (See the Appendix for details.) These measures are found for a realistic one-dimensional microgravity isolation problem in Ref. 2.
Guarantees of Real Parametric Uncertainty

The multiplicative-input, multiplicative-output, and additive-uncertainty blocks used in the above checks are inadequate by themselves to verify system stability or performance robustness for the microgravity vibration isolation problem. In particular, these checks cannot verify system robustness to uncertainties in umbilical properties or in payload mass or moments of inertia. These kinds of uncertainties are referred to as real parametric uncertainties. Robustness in the face of real parametric uncertainties is of particular concern for the microgravity vibration-isolation problem, since umbilicals are quite difficult to model accurately. Some possible approaches to this problem are surveyed below, along with a new approach using complex feedback uncertainty.

Real \( \mu \)

One possible approach to determining real-parametric-uncertainty guarantees is the use of real \( \mu \) \((\mu_R)\). To use \( \mu_R \) the problem is first expressed in the form of Fig. 6, where the diagonal elements \( \Delta_i \) of the \( \Delta \) block are restricted to being real and uncorrelated. (The initial procedure for producing this form is generally somewhat involved, but straightforward. See Ref. 8.) The interpretation of \( \mu_R \) is analogous to that of complex \( \mu \), with the exception that now only real gains are considered. Unfortunately, exact calculations of \( \mu_R \) are computationally very intensive; accordingly, its practical use is severely limited, to about eight or nine parameters.\(^9\) For some problems, though, this is an acceptable limitation. If the three-dimensional microgravity problem has umbilicals that can be modeled jointly by one three-dimensional spring-and-damper system, and a payload that is isolated by one three-dimensional magnetic actuator, the real-\( \mu \) problem can have one mass, three uncoupled inertias, three uncoupled stiffnesses, and three uncorrelated dampings. If the uncertainties in damping can be considered to be of negligible effect (as indicated by certain studies of the one-dimensional problem\(^5\)), then real \( \mu \) could be a practical analysis option. Note, however, that real \( \mu \) cannot be used for performance-robustness analysis, since only real uncertainties can be represented in the corresponding \( \Delta \) block.

Mixed \( \mu \)

A second approach to obtaining real-parametric-uncertainty guarantees is to treat the problem using mixed \( \mu \) \((\mu_M)\), where both real and complex uncertainties (including repeated blocks) are allowed in the \( \Delta \)-block substructure.\(^10\) In general, \( \mu_M \) is more conservative than \( \mu_R \), but the inclusion of complex gains permits performance-robustness checks while still considering real parametric uncertainties. Again, the starting point is shown in Fig. 6. As with real \( \mu \), exact mixed-\( \mu \) calculation appears to be fundamentally limited to very restricted cases. However, there have been promising advances in finding useful upper and lower bounds on \( \mu_M \) for many engineering problems of practical size (with as many as 100 real parameters).\(^10\)

Complex \( \mu \)

A third approach would be to set up the problem as with real \( \mu \), but to compute complex \( \mu \) instead. This method can be used for both stability- and performance-robustness checks. However, \( \mu \) is more conservative than \( \mu_M \) or \( \mu_R \), and there is still no exact computational method for more than three \( \Delta \) blocks. Again, one must generally settle for upper and lower bounds.
Complex $\mu$ with Feedback Uncertainty

Alternatively, complex $\mu$ can be used in a fundamentally different way for conducting real-parametric-uncertainty analysis. The previous three methods all require the use of separate $\Delta$, blocks for each parameter under consideration, so that the respective parametric uncertainties are considered in mutual isolation. For the one-dimensional microgravity isolation problem, this means that $d_k$, $d_t$, and $d_m$ must each have its own $\Delta$, block in the overall uncertainty-block structure. However, for this problem the uncontrolled plant has a characteristic equation (viz., $ms^3 + cs + k$) that permits exploitation for real-parametric-uncertainty analysis. If the plant is rearranged appropriately for the insertion of a complex feedback $\Delta$, block, it is possible to use the particular structure of the associated complex $\mu$ to determine combinations of payload mass, umbilical stiffness, and umbilical damping for which robust stability can be assured.

(As with the preceding methods, the results are conservative.)

Consider the block diagram shown in Fig. 9a, where $d_m$, $d_t$, and $d_k$ represent real variations in payload mass, umbilical damping, and umbilical stiffness, respectively. This block diagram reduces to the equivalent block diagram shown in Fig. 9b. If a complex $\Delta$, block, $\Delta_{FB}$, is placed in a negative-feedback path around $1/(ms^2 + cs + k)$ (i.e., in place of the feedback transfer function in Fig. 9a), then real parametric-uncertainty bounds on $k$, $c$, and $m$ can be obtained from the structured singular-value plot of the system seen by $\Delta_{FB}$. The method is as follows, for a one-dimensional isolation problem.

First, arrange the closed-loop system so that it contains a transfer-function block having the structure $1/(ms^2 + cs + k)$. For a one-dimensional microgravity isolation system such a block diagram is shown in Fig. 10. To evaluate the stability robustness of this system, one can look at a rearranged (and abbreviated) block diagram, shown in Fig. 11, where a feedback uncertainty block $\Delta_{FB}$ has been inserted into the analysis model. This feedback stability-robustness model can be reduced readily to the form of Fig. 4, with the plant seen by $\Delta_{FB}$ designated $M_{FB}$. For a given value of $\omega$ (say, $\omega_1$), $\Delta_{FB}(\omega)$ can be conceptualized as a ball in the complex gain plane, of radius $r_1(\omega_1)$, with radial unit vector (variable) $\vec{g}$. The value $r_1(\omega_1) = \mu^{-1}(|M_{FB}(j\omega)|)$ gives the maximum allowable size (2-norm) of the complex uncertainty $\Delta_{FB}(\omega)$. In particular, if $r_1(\omega_1)$ is smaller than $\mu_1(\omega_1)$, there exists no complex unit vector $\vec{g}$ at which an uncertainty (gain) represented in $\Delta_{FB}(\omega_1)$ will drive a stable system unstable. However, a $\Delta_{FB}(\omega_1)$ of size $r_1(\omega_1) = \mu_1(\omega_1)$ will drive a system pole onto the $j\omega$ axis for some complex unit vector $\vec{g} = \vec{g}_1$. The result will be system instability for the complex uncertainty $r_1(\omega_1)\vec{g}_1$. (This complex uncertainty might not, of course, actually correspond to a physically possible variation. Accordingly, the term allowable may be correct only in a conservative sense.)

Note that the above discussion is for a particular (arbitrary) frequency, $\omega_1$. When the plotted magnitude of $\Delta_{FB}(\omega)$ remains below the curve $r_1(\omega)$ for all values of $\omega$, this means that there is no complex uncertainty contained in the uncertainty ball that can drive the system unstable at any frequency $\omega$.

Let $\Delta k$ represent a complex ball of potential variations in stiffness $k$. Then the plot $r_1(\omega)$ displays the allowable (in the sense just described) size of $\Delta k$, as a function of $\omega$, given no uncertainty in $c$ or $m$. In other words, at any frequency $\omega$, $r_1(\omega)$ indicates how large $\Delta k$ can be without placing a system pole on the $j\omega$ axis at $s = j\omega$. [Since only real variations in stiffness are physically possible, the plot of $r_1(\omega)$ provides a conservative guarantee of permissible stiffness variations as a function of frequency $\omega$. In this sense, it provides guarantees but not limits on permissible stiffness variations.] If one defines $r_1, \Delta c, r_m, \Delta m$ analogously, the following can be obtained:

$$r_1(\omega) = \mu^{-1}(|M_{FB}(j\omega)|)$$  \hspace{1cm} (6a)$$

$$r_1(\omega) = \mu^{-1}(\beta M_{FB}(j\omega)) = \frac{r_1(\omega)}{\omega}$$  \hspace{1cm} (6b)$$

$$r_m(\omega) = \frac{\mu^{-1}(|M_{FB}(j\omega)|)}{\omega^2}$$  \hspace{1cm} (6c)$$

The minimum of each curve (call these minima $\rho_1, \rho_2$, and $\rho_m$, respectively) tightly bounds the amount of complex variation allowable (again, in the conservative sense described above) in $\Delta k$. 

Fig. 9a Feedback variation block diagram.

Fig. 9b Reduced feedback variation block diagram.

Fig. 10 Closed-loop system block diagram, frequency domain.
$\Delta c$, and $\Delta m$, respectively, without any other feedback variations. If, for example, the size of $\Delta k$ equals the minimum $\rho_1$, instability will occur at some frequency $\omega_1$ for the complex variation $\Delta k$ of that magnitude, in some complex direction (with corresponding unit vector $\hat{g}_k$). Analogous situations exist in the cases of $\Delta c$ and $\Delta m$. Thus $\rho_1$, $\rho_2$, and $\rho_m$ provide the designer with conservative guarantees of allowable real parametric plant uncertainties $\delta k$, $\delta c$, and $\delta m$, taken one at a time.

It is possible also to use the structured singular-value information to obtain guarantees on combinations of real parametric uncertainties. Let $\Gamma_r$ be defined as an acceptable region in the real parameter space made up of points with coordinates $(\delta k, \delta c, \delta m)$. The origin $(0, 0, 0)$ corresponds to a system with nominal plant parameters $k = k_{\text{nom}}$, $c = c_{\text{nom}}$, and $m = m_{\text{nom}}$. (Refer to Fig. 12a.) Let $\Gamma_r$ be the boundary between $\tilde{\sigma}_r$ and the unacceptable region, $\Omega_r$. Used in this way, acceptable and unacceptable refer to system behavior, in terms of stability (for the present consideration) or performance. For a given frequency $\omega$, there may be some combinations $(\delta k, \delta c, \delta m)$ for which the control results in acceptable (e.g., stable) behavior; such points will be in $\Gamma_r$. Other values $(\delta k, \delta c, \delta m)$, which yield unacceptable behavior at that frequency, will be in $\Omega_r$. The boundary $\Gamma_r$ is found, in theory, by varying $s$ from $j0$ to $j\infty$ and plotting each point $(\delta k, \delta c, \delta m)$ that leads to a closed-loop system pole on the $j\omega$ axis. Only purely imaginary values of $s$ are used.

The designer would like to bound $\delta k$, $\delta c$, and $\delta m$ such that he remains in $\Gamma_r$. One possible approach would be to use a brute-force (i.e., point-by-point) method. Such an approach, while having the advantage of remaining entirely inside a real parameter space, can only make guarantees about the specific sample points considered. The approach of $\mu$ analysis, in contrast, considers all values in a region of a complex gain plane, so that the $\Delta$-block is a ball that leads to conservative guarantees for complex uncertainties less than some size. This information, although conservative, is still of great value; and since it is much easier to obtain, it has been used in this study. These complex uncertainties can be used to obtain guarantees on combinations of real parametric uncertainties, as follows.

Consider Fig. 12b. The region $\Omega(\omega_1)$ consists of all points in complex gain space corresponding to values of uncertainty gain $g$ (i.e., of the structured singular value of $\Delta_{FB}$) that place a system pole on the $j\omega$ axis at frequency $\omega_1$. (The symbol $\Omega$ is set in boldface to indicate that it represents a vector quantity with real and imaginary components.) The acceptable region $\tilde{\Omega}(\omega_1)$ is a ball in the gain space, centered at $g = 0$ with radius $r_1(\omega_1) = \mu^{-1}(\Delta_{FB}(j\omega_1))$, such that it just touches the nearest point of $\Omega$. A separate figure would exist, theoretically, for each value of $\omega$.

An uncertainty block $\Delta_{FB}$ whose magnitude (i.e., radius in the gain plane) is less than $r_1(\omega_1)$ is guaranteed not to represent any complex gains that could place a pole on the $j\omega$ axis at $\omega_1$. The maximum allowable size of $\Delta_{FB}$ (i.e., of $r_1(\omega_1)$ varies with frequency; define $\rho_k$ as the size of the largest uncertainty ball $\Delta_{FB}$ (or alternatively, $\Delta k$) that will not lead to instability at any frequency.
Let \( \omega_0 \) be the limiting frequency. Likewise, \( \rho_{\omega} \) and \( \rho_m \) (with \( \omega \) and \( \omega_m \), respectively) provide analogous conceptual pictures. Referring to Fig. 12b again, let \( r_x(\omega) \) represent the vector of \( \Delta x_{\omega} \) with complex gain \( g = g_0 \), expressed in terms of its real and imaginary components. Then \( \hat{g}_x(\omega) \) is the limiting value of \( g(\omega) \) at frequency \( \omega_0 \). Let \( \hat{v}_x \) and \( \hat{v}_y \) be unit vectors in the real and imaginary g-plane directions, respectively; let \( \hat{g}_x(\omega) \) be the unit vector in the direction of \( r_x(\omega) \); and let \( \beta_1(\omega) \) and \( \beta_2(\omega) \) be the scalar components of \( \hat{g}_x(\omega) \), so that

\[
\begin{align*}
    r_x(\omega) &= r_x(\omega)[\beta_1(\omega) \hat{v}_x + \beta_2(\omega) \hat{v}_y] \\
    r_x(\omega) &= r_x(\omega)[\beta(\omega) \hat{v}_x(\omega)]
\end{align*}
\]

(7a)

(7b)

and

\[
\beta_1^2(\omega) + \beta_2^2(\omega) = 1
\]

(7c)

Analogous vectors and equations exist for each frequency \( \omega \). If, for all \( \omega \leq 1 \),

\[
(\delta k - \omega^2 \delta m)^2 < \beta_1^2(\omega) r_1^2(\omega)
\]

(8a)

and

\[
(\delta c)^2 < \beta_1^2(\omega) r_1^2(\omega)
\]

(8b)

where \( \delta k, \delta c, \) and \( \delta m \) are real variations in \( k, c, \) and \( m \) from the nominal, then one can be assured that no instability will occur for those values of \( \omega \). Since \( \rho_\omega \) is \( \min \rho_x(\omega) \), one can replace \( r_x(\omega) \) with \( \rho_\omega \) in (8a, b) and the assertion will still hold. Finally, since the limiting case of \( \beta_1 \) and \( \beta_2 \) (i.e., corresponding to \( \rho_\omega \)) is at a point of tangency to \( \Omega(\omega) \), any other values \( \gamma_1 \) and \( \gamma_2 \) may be substituted for \( \beta_1 \) and \( \beta_2 \), provided that \( \gamma_1^2 + \gamma_2^2 \leq 1 \), without exceeding the boundaries of \( \Omega(\omega) \). Then the following assertion will hold:

**Assertion 1**: If, for all \( \omega \leq 1 \),

\[
(\delta k - \omega^2 \delta m)^2 < \gamma_1^2 \rho_1^2
\]

(9a)

and

\[
(\delta c)^2 < \gamma_1^2 \rho_2^2
\]

(9b)

where

\[
\gamma_1^2 + \gamma_2^2 \leq 1
\]

(9c)

then no instability will occur for those values of \( \omega \).

Defining \( \beta_3(\omega) \) and \( \beta_4(\omega) \) in an analogous way to \( \beta_1(\omega) \) and \( \beta_2(\omega) \), where \( \delta^2 \delta M_{\omega} \) now replaces \( M \) in Fig. 4, one can also arrive at the following: If, for all \( \omega \geq 1 \),

\[
[\delta m - (\delta k/\omega^2)]^2 < \beta_3^2(\omega) \rho_1^2
\]

(10a)

and

\[
(\delta c/\omega)^2 < \beta_4^2(\omega) \rho_1^2
\]

(10b)

then no instability will occur for those values of \( \omega \).

Finally, one can replace \( \beta_3(\omega) \) and \( \beta_4(\omega) \) with variables \( \gamma_1 \) and \( \gamma_4 \), as before, provided \( \gamma_1^2 + \gamma_4^2 \leq 1 \). This results in the following:

**Assertion 2**: If, for all \( \omega \geq 1 \),

\[
[\delta m - (\delta k/\omega^2)]^2 < \gamma_2^2 \rho_2^2
\]

(11a)

and

\[
(\delta c/\omega)^2 < \gamma_4^2 \rho_2^2
\]

(11b)

where

\[
\gamma_1^2 + \gamma_4^2 \leq 1
\]

(11c)

then no instability will occur for those values of \( \omega \).

From Assertions 1 and 2 the following condition can be obtained:

**Assertion 3**: Given any \( \gamma_1, \gamma_2, \gamma_3, \) and \( \gamma_4 \) satisfying \( \gamma_1^2 + \gamma_2^2 \leq 1 \) and \( \gamma_3^2 + \gamma_4^2 \leq 1 \), and with \( \rho_1, \rho_2, \) and \( \rho_m \) as previously defined, the following condition implies system stability:

\[
| \delta k | < \min \{ \rho_1, \gamma_1 \rho_3 - | \delta m |, \gamma_4 \rho_4 - | \delta m | \}
\]

\[
| \delta c | < \min \{ \rho_1, \gamma_3 \rho_4, \gamma_4 \rho_3 \}
\]

By using Assertion 3, one can obtain guarantees of real variations in umbilical stiffness, umbilical damping, and payload mass that can occur simultaneously without violating stability specifications. Upon obtaining \( \rho_1, \rho_2, \) and \( \rho_m \) from the appropriate structuring-singular-value plots, the analyst chooses \( \gamma_2 \) and \( \gamma_4 \) based on the percentage error expected in \( c \). Next, he chooses an expected percentage error in \( m \) (such that \( | \delta m | < \rho_m \)). Then \( \gamma_1 \) and \( \gamma_4 \) are found from the equations \( \gamma_1^2 + \gamma_2^2 = 1 \) and \( \gamma_3^2 + \gamma_4^2 = 1 \), and finally, guarantees are determined on the allowable \( \delta k \). This provides the analyst with a guarantee on the allowable maximum magnitude of stiffness variation \( \delta k \), as a function of the assumed maximum magnitudes of simultaneous variations in the umbilical damping \( \delta c \) and payload mass \( \delta m \). For any combination of real parametric variations remaining within this real parameter space, the analyst can guarantee that the system will be stable.

This analysis procedure depends on the ability to arrange the microgravity isolation system model and complex-feedback-uncertainty block(s) in the appropriate form. Application to the one-dimensional case is found in Ref. 2. The method was found to give useful results for stability-robustness analysis, but the performance-robustness results were excessively conservative. For more complicated geometries the application will be more difficult and may not always be possible. Nonetheless, the use of feedback uncertainty \( \Delta \) block(s) can provide the engineer with at least a relative measure of closed-loop-system robustness to umbilical and payload parametric variations.

**Concluding Remarks**

Analysis is a vital part of controller design for a microgravity vibration-isolation problem. Because of the many competing requirements for a microgravity isolation system, the design of an effective controller involves a studied, iterative process of synthesis and analysis. Each synthesized controller must be subjected to a series of analytical checks, to verify that it will perform satisfactorily under even the most pessimistic combination of possible model inaccuracies.

Most of the types of uncertainty that are of concern with a microgravity isolation system can be modeled by complex uncertainty blocks, appropriately placed in the nominal system's transfer-function block diagram. This includes uncertainties in amplifier, actuator, and sensor models, as well as unmodeled higher modes of the system. The powerful methods of complex-\( \mu \) analysis permit the analyst to obtain guarantees on the amount of uncertainty of each type that can be tolerated without compromising stability or violating performance constraints.

Guarantees on allowable variations in real system parameters (such as umbilical stiffness and damping, and payload mass and moments of inertia) may be found by similar, if more sophisticated, techniques. These techniques generally require mutual isolation of selected parameter uncertainties in the uncertainty-block structure. The analyst then obtains the desired guarantees by using either real-, mixed-, or complex-\( \mu \) methods, in increasing order of conservatism. Alternatively, the form of the microgravity isolation problem permits an approach involving the use of complex \( \mu \) with a feedback uncertainty block having a much simplified structure. The particular relationships between the parametric uncertainties, which are represented intermingled in the feedback uncertainty block, have been exploited to determine allowable combinations of real-parameter variations.

**Appendix: Generalization to MIMO Systems**

The familiar PM and GM can be generalized to apply to multiplicative input (or output) uncertainties for MIMO systems. Dailey's
discussion is adapted below. Let the $i$th control-input channel have a multiplicative phase rotation $\theta_i$, i.e., $1 + \Delta_i = e^{j\theta_i}$, and let the various phase rotations be independent of each other. The input MIMO PM is defined as the largest real (unique) interval $\text{Int} = [-\theta, \theta]$ such that for all simultaneous independent phase rotations $\theta_i \in \text{Int} (i = 1, \ldots, n)$ the system remains stable. The output MIMO PM is defined similarly. An input MIMO GM is defined as a real (nonunique) interval $\text{Int} = [G_L, G_U]$ such that for all simultaneous independent GV $G_i$, satisfying $1 + G_i \in \text{Int} (i = 1, \ldots, n)$ the system remains stable. The output MIMO GM is similarly defined.

Let $S_i$ and $T_i$ be the sensitivity and complementary sensitivity transfer matrices, respectively, at the plant input. Then, in terms of the complementary sensitivity function $T_i$, a guaranteed lower bound for the MIMO PM is given by

$$\text{MIMO PM} \supseteq [-\theta, +\theta] \quad (A1a)$$

where

$$\theta = 2 \sin^{-1}(r_{\min}/2) \quad (A1b)$$

for

$$r_{\min} = \inf_{\omega \in \mathbb{R}} \|T_i(j\omega)\| \quad (A1c)$$

A valid MIMO GM is given by

$$\text{MIMO GM} = [1 - r_{\min}, 1 + r_{\min}] \quad (A2)$$

In terms of the sensitivity function $S_i$,

$$\text{MIMO PM} \supseteq [-\theta, +\theta] \quad (A3a)$$

where

$$\theta = 2 \sin^{-1}(r_{\min}/2) \quad (A3b)$$

for

$$r_{\min} = \inf_{\omega \in \mathbb{R}} \|S_i(j\omega)\| \quad (A3c)$$

and

$$\text{MIMO GM} = \left[ \frac{1}{1 + r_{\min}}, 1 \right] \quad (A4)$$

If instead of $\delta$ the structured singular value $\mu$ is used in Eqs. (A1c) and (A3c) above, the lower bounds on the stability margins will be improved. For the analogous output stability-margin guarantees, one merely substitutes the output sensitivity and complementary sensitivity transfer matrices $S_i$ and $T_i$, respectively, into the appropriate equations given above.

Application to cases where performance specifications are included as $\Delta$ blocks can be made readily using structured singular values. In these cases, however, the terms MIMO PV and MIMO GV are more appropriate, for obvious reasons.

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