Webgames and Strategy: Recipes for Interactive Learning

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Webgames and Strategic Behavior: 
Recipes for Active Learning

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Preface

This book is a collection of chapters, each containing a single game and the associated analysis of the results. The games set up simple economic situations, e.g. a market or auction, which highlight several related economic ideas. Each chapter provides the reading for a particular class, in a one-a-day approach. The reading can serve either as a supplement to other material, or (preferably) it can be assigned in conjunction with an in-class “experiment” in which students play the game with each other. Doing the experiments before the assigned reading enhances the teaching value of these experiments. Many of the games can be run in class “by hand,” with dice or playing cards. The appendix contains instructions for hand-run games in many cases.

For those with student computer access, all games are available online at the Vecon site:

http://vecon.econ.virginia.edu/admin.htm (for the instructor),
http://vecon.econ.virginia.edu/login.htm (for participants to log on and begin).

These web-based games can be set up and run from any standard browser (Internet Explorer or Netscape) that is connected to the internet, without loading any software. For instructions, see the link on the “admin” page above. The web-based programs have fully integrated instructions that automatically conform to the features selected by the instructor in the setup. Web-based games are quicker to administer, and the instructor data displays can provide records of decisions, earnings, round-by-round data averages, and in some cases, theoretical calculations. These displays can be printed or projected for post-experiment discussions. There is an extensive menu of setup options for each game that lets the instructor select parameters, e.g. the numbers of buyers, sellers, decision rounds, fixed payments, payoffs, etc. For example, the private-value auction setup menu allows one to choose the range of randomly determined private values, the number of bidders, the number of rounds, the pricing rule (first-price or second-price, and “winner-pays” or “all-pay”). I have even taught classes where students design their own experiments and run them on the others in the class, with a formal (Power-Point) presentation of results in the following class.

The first several chapters contain an example of an individual decision, a simple two-person game, and a market with buyers and sellers. These initial chapters raise a few methodological issues, such as whether and when financial incentives are needed for research and/or teaching experiments. In addition, some basic notions of decision making and equilibrium are introduced. The focus in these chapters is on the basics; discussion of anomalies and alternative theories is
deferred until later. After these chapters have been covered, there is a lot of flexibility in terms of the order of coverage of the remaining chapters, which are divided into groups: individual decisions, games, auctions, markets, bargaining, public choice, and asymmetric information. It is possible to pick and choose, based on the level and subject matter of the course.

This book provides an organizing device for courses in game theory, topics in microeconomics, and introductory experimental economics. For an upper-level or graduate course in experimental economics, Davis and Holt’s (1993) *Experimental Economics* has the advantage of being organized around the main classes of experiments, with presentations of the associated theory and methodological concepts. This book, in contrast, is organized around particular games, one per chapter, with a limited amount of theory and methodology that is mainly presented in the context of specific examples. These chapters are relatively self-contained, which makes it possible to choose a selection tailored for a particular course, e.g. public economics. The book could also be integrated into courses in microeconomics, managerial economics, or strategy at the M.B.A. level. Many of the experimental designs may be of interest to non-economists, e.g. students of political science, anthropology, and psychology, as well as anyone interested in behavioral finance or behavioral law and economics.

Mathematical arguments are simple, since experiments are typically based on parametric cases that distinguish alternative theories. The mathematical calculations are sometimes illustrated with spreadsheet programs that constructed in a step-by-step process. Then a process of copying blocks of cells results in iterative calculations that converge to equilibrium solutions. Calculus is avoided, except in a couple of places in the chapters on auctions. These optional sections are preceded by discrete examples that provide the intuition behind the more general results.

I have tried to keep the text simple. There are no footnotes. References to other papers are very limited, and most are confined to an “extensions and further reading” section at the end of each chapter. The book contains no extensive surveys of related literature on research experiments. Such surveys can be found in Davis and Holt (1993), Kagel and Roth’s (1995) *Handbook of Experimental Economics*, and Hey’s (1994) *Experimental Economics*, which are pitched at a level appropriate for advanced undergraduates, graduate students, and researchers in the field. For more discussion of methodology, see Friedman and Sunder’s (1994) *Experimental Methods*. The references in all of these books are somewhat out of date, since the number of published economics papers using laboratory methods has increased by about a third since 1995, when the last of these books was published. This is an increase of about 700 publications, and there are many more unpublished working papers (see Figure 1.1 in Chapter 1). These new publications are listed and categorized by keyword on the bibliography of
I would especially like to thank my coauthor Lisa Anderson and her William and Mary students for many helpful suggestions on this book and on the software that it utilizes. Much of what I know about these topics is due to joint research projects with Jacob Goeree (UVA - University van Amsterdam), and with others: Lisa Anderson (William and Mary), Simon Anderson (Virginia), Sheryl Ball (Virginia Tech), Jordi Brandts (Barcelona), Monica Capra (Washington and Lee), Ron Cummings (Georgia State), Catherine Eckel (Virginia Tech), Jean Ensminger (Caltech), Roland Fryer (Chicago/Harvard), Rosario Gomez (Malaga), Susan Laury (Georgia State), David Lucking-Reiley (Arizona), Tom Palfrey (Cal Tech), Al Roth (Harvard), Roger Sherman (Houston), and Rick Wilson (Rice). In addition, I was fortunate to have an unusually talented and enthusiastic group of Virginia students who read parts of the manuscript: Jeanna Composti, Kari Elasson, Katie Johnson, and Loren Pitt. Another student, Joe Monaco, set up the Lenux server and procedures for running the programs on a network of hand-held, wireless “pocket” PCs with color, touch-sensitive screens. Imagine a game theory class, outside on the University of Virginia “lawn,” with students competing in an auction via wireless PDA’s!
Part I. Basic Concepts: Decisions, Game Theory, and Market Equilibrium

The next several chapters provide an introduction to the three main types of experiments: individual decisions, games, and markets. Each chapter introduces some key concepts, such as expected value, risk aversion, Nash equilibrium, and market efficiency. Knowledge of these concepts makes it possible to pick and choose among the remaining chapters based on the goals and level of the course. All chapters in this section should be completed before proceeding to the sections that follow.

The first part of Chapter 1 provides an introduction to the use of experimental games for teaching and research, which is followed by an overview of the topics covered in the book. The final two sections of this chapter contain a very brief discussion of methodology and a history of the development of experimental economics. These final two sections are optional for students in most courses (other than experimental economics).

Chapter 2 introduces students to a “pit market” that corresponds to the trading of futures contracts in a trading pit, with a free intermingling of prospective buyers and sellers. The chapter describes a design that illustrates the role of prices in achieving an efficient set of trades. The best procedure is to run a class pit market experiment before discussing the chapter material. Instructions for this are provided in the appendix, and the instructor is free to use playing cards to design a setup that will complement or supplement the lessons that can be derived from the market design presented in the chapter. The hand-run experiment is a recommended way to stimulate a discussion of how markets work. However, some with computer access may prefer to set up the experiment using the Double Auction program, which is coded as DA on the Vconlab instructor “admin” and participant “login” menus.

Chapter 3 presents a simple game that can also be run by letting students select which of several playing cards to play in each round. Instructions are provided, or a web-based version of the game can be used. The games (prisoner’s dilemma and coordination) are used to introduce the notion of a Nash equilibrium.

The fourth chapter shifts attention to individual decisions in situations with random elements that affect money payoffs. This permits a discussion of expected money value for someone who is “neutral” towards risk. Non-neutral attitudes (risk aversion or risk seeking) may also arise in some situations.

The notions of decision making in the presence of random elements are used in Chapter 5 to analyze simple matrix games in which the equilibrium may involve randomization. We consider games of chicken, battle-of-sexes, and
matching pennies. The principles used to find equilibria in these simple games
will be applied in more complex situations encountered later, such as the choice
of price when there is danger of being undercut slightly by competitors.
Chapter 1. Introduction

I. Origins

Like other scientists, economists observe naturally occurring data patterns and then try to construct explanations. Then the resulting theories are evaluated in terms of factors like plausibility, generality, and predictive success. As with other sciences, it is often difficult to sort out cause and effect when many factors are changing at the same time. Thus, there may be several reasonable theories that are roughly consistent with the same observations. As Keynes noted, without a laboratory to control for extraneous factors, economists often “test” their theories by gauging reactions of colleagues. In such an environment, theories may gain support on the basis of mathematical elegance, persuasion, and focal events in economic history like the Great Depression. Theories may fall from fashion, but the absence of sharp empirical tests leaves an unsettling clutter of plausible alternatives. For example, economists are fond of using the word equilibrium preceded by a juicy adjective (e.g. proper, perfect, divine, or universally divine). This clutter is often not apparent in refined textbook presentations.

The development of sophisticated econometric methods has added an important discipline to the process of devising and evaluating theoretical models. Nevertheless, any statistical analysis of naturally occurring economic data is typically based on a host of auxiliary assumptions. Economics has only recently moved in the direction of becoming a laboratory science in the sense that key theories and policy recommendations are suspect if they cannot provide intended results in controlled experiments. This book provides an introduction to the study of economic behavior, organized around games that can be played in class.

The first classroom market games were conducted in a Harvard class by Edward Chamberlin (1948). He had proposed a new theory of “monopolistic” competition, and he used experiments to highlight failures of the standard model of perfect competition. Students were given buyer and seller roles and instructions about how trades could be arranged. For example, a seller would be given a card with a “cost” in dollars. If the seller were to find a buyer who agreed to pay a price above this cost, the seller would earn the difference. Similarly, a buyer would be given a card with a “resale value,” and the buyer could earn the difference if a purchase could be arranged at a price below this resale value. Different sellers may be given cards with different cost numbers, and likewise, buyers may receive different values. These values and costs are the key elements that any theory of market price determination would use to derive predictions, as explained in Chapter 2. Without going into detail, it should be clear that it is possible to set up a laboratory market, and to provide strong financial incentives
by using “large” value and cost numbers and by paying subjects their earnings in cash.

Chamberlin’s classroom markets produced some inefficiencies, which he attributed to the tendency for buyers and sellers in a market to break off and negotiate in small groups. Vernon Smith, who attended Chamberlin’s class, later began running classroom markets with an enforced central clearinghouse for all offers to buy and sell. This trading institution is called a “double auction” since sellers’ ask prices tend to decline at the same time as buyers’ bid prices rise. A trade occurs when the bid-ask spread closes and someone accepts another’s offer to buy or sell. Smith observed efficient competitive outcomes, even with as few as 6-10 traders. This result was significant, since the classical “large numbers” assumptions were not realistic approximations for most market settings.

These classroom markets can be quite useful, because students learn what an economic situation is like “from the inside” before standard presentations of the relevant economic theory. A classroom game (followed by structured question-and-answer discussion) can let students discover the relevant economic principle for themselves, which enhances the credibility of seemingly abstract economic models. Each of the chapters that follow will be built around a single game that can be run in class, using a web-based software suite and/or simple props like dice and playing cards.

II. Overview

Individual Decisions

A market typically involves a relatively complex set of interactions between multiple buyers and sellers over time. Sometimes it is useful to isolate key aspects of the behavior of individuals in isolation. For example, stock markets involve major risks of gains and losses, but such risks depend on anticipated behavior of others in the market. It is straightforward to set up a simple decision experiment by giving a person a choice between gambles or “lotteries,” e.g. between a sure $10 and a coin flip that yields $25 in the event of heads. Many of the earlier chapters pertain to such individual decisions, where payoffs may depend on random events but do not depend on the decisions made by others. Some of the topics in these early chapters, like expected values and risk aversion, are useful in the analysis of interactive situations in subsequent chapters.

Game Theory

The simplest strategic interactions among economic agents have been modeled as games. In a “matching-pennies” game, for example, each player chooses heads or tails with the prior knowledge that one will win a sum of money when the coins match, and the other will win when the coins do not match. Each
person’s optimal decision in such a situation depends on what the other player is expected to do. The systematic study of such situations began with John Von Neumann and Oscar Morgensern’s (1944) *Theory of Games and Economic Behavior*. They asserted that standard economic theory of competitive markets did not apply to the bilateral and small-group interactions that make up a significant part of economic activity. Their “solution” was incomplete, except for the case of “zero-sum” games in which one person’s loss is another’s gain. While the zero-sum assumption may apply to some extremely competitive situations, like sports contests or matching pennies games, it does not apply to situations where players might all prefer some outcomes to others. In particular, economists and mathematicians at the RAND Corporation in Santa Monica, California were trying to apply game-theoretic reasoning to military tactics at the dawn of the Cold War. In many nuclear scenarios it is easy to imagine the “winner” may be much worse off than would be the case in the absence of war, which results in a non-zero-sum game. At about this time, a young graduate student at Princeton entered Von Neumann’s office with a notion of equilibrium that applies to a wide class of games, including the special case of those that satisfy the zero-sum property. John Nash’s notion of equilibrium and the half-page proof that is generally exists were recognized by the Nobel Prize committee about 50 years later. With the Nash’s equilibrium as its keystone, game theory has recently achieved the central role that Von Neumann and Morgenstern envisioned. Indeed, with the exception of supply and demand, the “Nash equilibrium” is probably used as often today as any other construct in economics.

Intuitively speaking, a Nash equilibrium is a set of strategies, one for each player, with the property that nobody would wish to deviate from their planned action given the strategies being used by the other players. Consider a “coordination” game in which two sellers must decide independently whether to sell in a black market, or in an alternative (“red”) market. There is not enough room for two entrants in either market; they both earn zero for any red/red or black/black outcome. With a red/black outcome, the entrant in the black market earns 10 while the entrant in the red market only earns 5. In this game, the congested outcomes (red/red or black/black) that yield zero payoffs are not Nash equilibria, since each person would wish to deviate from their decision given the other’s decision to enter the same market. By this time you should have realized that the Nash equilibrium is a red/black outcome, since each person earns a positive amount (5 or 10), so neither would want to switch to the other’s market and end up earning 0.

This analysis does not settle the issue of which person gets to enter the more profitable black market, and many developments in game theory are focused on the issue of how to predict which of several Nash equilibria will have more predictive power a particular game. Such games are easy to set up as experiments...
by letting each person play a red card (Hearts or Diamonds) or a black card (Clubs or Spades). When existing theory makes no prediction, observed behavioral tendencies in such experiments can provide important guideposts for the development of new theories. Indeed, the mathematicians and economists at the RAND Corporation began running game experiments at about the same time as Chamberlin’s classroom market experiments. The second set of chapters in this book pertains to simple games and the nature of equilibrium behavior.

**Bargaining**

Economic decisions in small-group settings often raise issues of fairness since earnings may be inequitable. In a simple “ultimatum bargaining game,” one person proposes a way to divide a fixed amount of money, say $100, and the other may either agree to the division, which is implemented, or may reject the division, which results in zero earnings for both. Attitudes about inequity are more difficult to model than the simpler selfish money-seeking motives that may dominate in impersonal market situations. In this case, research experiments can provide insights in areas where theory is silent or less developed. Many topics in Law and Economics involve bargaining (e.g. bankruptcy, pre-trial settlements), and experiments can provide useful insights. Experiments such as these also have been used by anthropologists (Ensminger, 2001) to study attitudes about fairness in primitive societies in Africa and South America. The third set of chapters in this book pertains to bargaining and other classes of games where fairness, equity, and other interpersonal factors seem to matter.

**Auctions**

The advent of web-based communications has greatly expanded the possibilities for setting up auctions that connect large numbers of geographically dispersed buyers and sellers. For example, several economists were recently involved in designing and running an auction whereby farmers made offers to suspend irrigation on designated tracts of land in southwest Georgia during the 2001 growing season. Bids were collected from 8 sites and were ranked and processed by a web-based program. The interim results were viewed online hundreds of miles away by Georgia Environmental Protection Division officials who had to decide when to terminate bidding (Cummings, Holt, and Laury, 2001). Web-based auctions have also been used with great success in the sale of communications bandwidth in the U.S. and in Europe.

There are numerous decisions that must be made in setting up an auction, for example, whether to allow bid revisions during the auction. Experiments can be used to “testbed” alternative sets of rules. In the Georgia Irrigation Reduction Auction, for example, economists at Georgia State began running experiments as soon as the law passed. State officials observed some of these experiments before
drafting the actual auction procedures. A large-scale dry run with over a hundred bidders at 5 southwest Georgia locations preceded the actual auction, which involved about 200 farmers and $5 million dollars in irrigation reduction payments. The auction had much of the look and feel of an experiment, with the reading of instructions, a round-by-round collection of bids, and web-based bid collection and processing. The fourth set of chapters that follow pertains to auctions, including a web-based irrigation reduction auction.

**Markets**

The next group of chapters covers several ways that economists have modeled market interactions, with firms choosing prices, production quantities, or entry decisions. Some markets have distinct groups of buyers and sellers, and others more closely resemble stock markets in which purchase and resale is common. Market experiments can be used to assess the antitrust implications of mergers, contracts, and other market conditions. One goal of such experiments is to assess factors that increase the extent to which markets achieve all possible gains from trade, i.e. the efficiency of the market. Markets with enough price flexibility and good information about going prices tend to be highly efficient.

**Public Choice**

Inefficiencies can occur when some costs and values are not reflected in prices. For example, it may be difficult to set up a market that allows public goods like national defense to be provided by decentralized contributions. Another source of potential inefficiency occurs when one person’s activity has a negative impact on others’ well being, as is the case with pollution or the over-use of a freely available, shared resource. Non-price allocation mechanisms often involve the dedication of real resources to lobbying. No individual contestant would want the value of their effort to exceed the value of the object being sought, but the aggregate lobbying costs for a number of individuals may be large relative to the value of the prize. The Public Choice chapters pertain to such situations; topics include voluntary contributions and the use of a “common-pool” resource.

**Information**

Markets may also fail to generate efficient results when prices do not convey private information. With limited information, individuals may rely on “signals” like educational credentials or ethnic background. Informational asymmetries can produce interesting patterns of conformity (“herds”) that may have large effects on stock prices, hiring patterns, etc. Experiments are particularly useful in these cases, since the effect of informational disparities is to
produce many Nash equilibria. The information chapters include experiments on signaling, discrimination, and information “cascades.”

III. A Brief Comment on Methodology

This section and the next are optional for students who are primarily interested in microeconomics and game theory, as opposed to research in experimental economics.

Treatments

In an experiment, a treatment is a completely specified set of procedures, which includes instructions, incentives, rules of play, etc. Just as scientific instruments need to be calibrated, it is useful to calibrate economics experiments, which typically involves establishing a “baseline treatment” for comparisons. For example, suppose that individuals are given sums of money that can either be invested in an “individual” or a “group” account, where investments in the group account have a lower return to the individual, but a higher return to all group members. If the typical pattern of behavior is to invest half in each account, then this might be attributed either to the particular investment return functions used in the experiment or to “going fifty-fifty” in an unfamiliar situation. In this case, a pair of treatments with differing returns to the individual account may be used. Suppose that the investment rate for the individual account is fifty percent in one treatment, which could be attributed to confusion or uncertainty, as indicated above. The importance of the relevant economic incentives could be established if this investment rate falls, say to ten percent, when the return for investing in the individual account is reduced.

Next, consider a market example. High prices may be attributed to small numbers of sellers or to the way in which buyers are constrained from requesting private discounts. These issues could be investigated by changing the number of sellers, holding buyers’ shopping rules constant, or by changing buyers’ shopping opportunities holding the number of sellers constant. Many economics experiments involve a “2x2” design with treatments in each of the four cells: low numbers with buyer shopping; low numbers with no shopping; high numbers with no shopping, high numbers with shopping.

Treatment Structure: Between Versus Within

Each of the chapters that follow is based on a single experiment. In order to preserve time for discussion, the experiments typically involve a pair of treatments. One issue is whether half of the people are assigned to each treatment, which is called a “between subjects” design. The alternative is to let each person make decisions in each treatment, which results in a larger sample size but a
shorter duration per treatment. This is called a “within subjects” design, since each person’s behavior in one treatment is compared with the same person’s behavior in the other treatment. Each method has its advantages. If behavior is slow to converge or if many observations are required to measure what is being investigated, then the between-subjects design may be preferred since it will generate the most observations in the limited time available. Moreover, subjects are only exposed to a single treatment, which avoids sequence effects. For example, a market experiment that lets sellers discuss prices may result in some successful collusive arrangements, with high prices that may carry over even if communication is not allowed in a second treatment. These sequence effects may be avoided with a between subjects design. The advantage of a within-subjects design is that individual differences are controlled for by letting each person serve as their own control. For example, suppose that you have a group of eight adults, and you want to determine whether they run as fast wearing blue jeans as with running shorts. In any group of eight adults, running speeds may vary by factors of 2 or 3, depending on weight, age, health, etc. In this case it would be desirable to time running speeds for each person under both conditions unless the distance were so great that fatigue would cause major sequence effects. In general, a within subjects design is more appealing if there is a high behavioral variability across individuals.

Incentives

Economics experiments typically involve monetary decisions like prices, costly efforts, etc. Most economists are suspicious of results of experiments done with hypothetical incentives, and therefore real cash payments are generally used in research experiments. As we shall see, sometimes such incentives matter a lot and sometimes not much at all. For example, people have been shown to be more generous in offers to others when such offers are hypothetical than when generosity has a real cost. For purposes of teaching, it is typically not possible or even desirable to use monetary payments. Nevertheless, the results of class experiments can provide a useful learning experience as long as the effects of payments in research experiments are provided when important incentive effects have been documented. Therefore, the presentations in the chapters that follow will be based on a mixture of class and research experiments. When the term “experiment” or “research experiment” is used, this will mean that all earnings were at reasonable levels and were paid in cash. The term “classroom experiment” indicates that payoffs were basically hypothetical. However, for non-market classroom experiments discussed in this book, I would pick one person at random ex post and pay a small fraction of earnings, usually several dollars. This procedure is not generally necessary, but it was followed here to avoid unexpected differences between classroom and research data.
Replication

One of the main advantages of an experimental analysis is the ability to repeat the same setup numerous times in order to determine average tendencies that are relatively insensitive to individual or group effects. Replication requires that instructions and procedures be carefully documented. For example, it is essential that instructions to subjects be written as a script that is followed in exactly the same manner with each cohort that is brought to the laboratory. Having a set of written instructions helps ensure that “biased” terminology is avoided, and it permits other researchers to replicate the reported results. The general rule is that enough detail should be reported so that someone else could redo the experiment in a manner that the original author(s) would accept as being valid, even if the results turned out to be different. For example, if the experimenters provide a number of examples of how prices determine payoffs in a market experiment, and if these examples are not contained in the written instructions, the different results in a replication may be due to differences in the way the problem is presented to the subjects.

Control

A second main advantage of experimentation is the ability to control the factors that may be affecting observed behavior, so that extraneous factors are held constant (controlled) as the treatment variable changes. The most common cause of loss of control is to change more than one factor at the same time, so that it is difficult to determine the cause of any observed change. For example, a well-known study of lottery choice compared the tendency of subjects to choose a sure amount of money with a lottery that may yield higher or lower payoffs. In one treatment, the payoffs were in the zero to $10 range, and in another treatment the payoffs were in the thousands of dollars. Then the author concluded that there were no incentive effects, since there was no observed difference in the tendency to choose the safe payoff. The trouble with this conclusion is that the high-payoff treatment was conducted under hypothetical conditions, so two factors were being changed: the scale of the payoffs, and whether or not they were real or hypothetical. This conclusion turns out to be questionable, given results of experiments that change one factor at a time. For example, Holt and Laury (2001) report that scaling up hypothetical choices has little effect on the tendency to select the safer option in their experiments. Thus, choice patterns with low real payoffs do look like choice patterns with both low and high hypothetical payoffs. However, scaling up the payoffs in the real (non-hypothetical) treatments to hundreds of dollars causes a sharp increase in the tendency to choose the safer option.
Control is also lost when procedures make it difficult to determine the incentives that participants actually faced in an experiment. If people are trading physical objects like university sweatshirts, differences in individual valuations make it hard to reconstruct the nature of demand in a market experiment. There are, of course, situations where non-monetary rewards are desirable, such as experiments designed to test whether ownership of a physical object makes it more desired (“the endowment effect”). Thus control should always be judged in the context of the purpose of the experiment.

**Fatal Errors**

Professional economists often look to experimental papers for data patterns that support existing theories or that suggest desirable properties of new theories. Therefore, the researcher needs to be able to distinguish between results that will replicate from those that are artifacts of improper procedures. Even students in experimental sciences should be sensitive to procedural matters so that they can evaluate others’ results critically. Experiments also provide a rich source of topics for term papers, senior dissertations, etc. Students and others who are new to experimental methods in economics should be warned that there are some fatal errors that can render useless the results of economics experiments. As the above discussion indicates, these include: inappropriate incentives, non-standardized instructions and procedures, loss of control, and the failure to provide a calibrated baseline treatment.

A more detailed treatment of methodology can be found in Chapters 1 and 9 of Davis and Holt (1993) *Experimental Economics* and Friedman and Sunder (1994) *Experimental Methods*.

**IV. A Brief History of Experimental Economics**

Figure 1.1 below shows the trends in published papers in experimental economics. The first papers by Chamberlin and some of the game theorists at RAND were written in the late 1940’s and early 1950’s. In addition, some of the early interest in experimental methods was generated by the work of Fouraker and Siegel (1962, 1963). Siegel was a psychologist with high methodological standards; some of his work on “probability matching” will be discussed in a later chapter. In the late 1950’s, some business school faculty at places like Carnegie-Mellon became interested in business games, both for teaching and research. And Vernon Smith’s early market experiments were published in 1962. Even so, there were less than 10 publications per year before 1965, and less than 30 per year before 1975. Some of the interesting work during this period was being done by Reinhard Selten and other Germans, and there was an international conference on experimental economics held in Germany in 1973. During the late 1970’s, Vernon Smith was a visitor at Caltech, where he began working with Charles
Plott, who had studied at Virginia under James Buchanan and was interested in public choice issues. Plott’s (1979) first voting experiments stimulated work on voting and agendas by political scientists in the early 1980’s. Other interesting work included the Battalio, Green, and Kagel (1981) experiments with rats and pigeons, and Al Roth’s early bargaining experiments, e.g. Roth and Malouf (1979). There were still fewer than 50 publications in the area before 1985. At this time, my former thesis advisor and colleague, Ed Prescott, told me that “Experimental economics was dead end in the 1960’s and it will be dead end in the 1980’s.”

In the 1980’s, Vernon Smith and his colleagues and students at Arizona established the first large laboratory and began the process of developing computerized interfaces for experiments. In particular, Arlington Williams wrote the first posted-offer program. After a series of conferences in Tucson, the Economic Science Association was founded in 1986, and the subsequent presidents constitute a partial list of key contributors (Smith, Plott, Battalio, Hoffman, Holt, Forsythe, Palfrey, Cox, Schotter, Camerer, and Fehr).

![Figure 1.1. Numbers of Published Papers in Experimental Economics](image)

Vernon Smith and Mark Isaac begin editing *Research In Experimental Economics*, a series of collected papers appearing about every other year since 1979. The first comprehensive books in this area were published in the 1990’s, e.g. Davis and Holt’s (1993) and Hey (1994). The 1995 *Handbook of Experimental Economics*, edited by Kagel and Roth, contains survey papers on key topics like auctions, bargaining, public goods, etc, and these are still good sources for reference. The first specialty journal, *Experimental Economics*, was
started in 1998. The strong interest among Europeans is indicated by the fact that one of the founding coeditors is Arthur Schram from Amsterdam. The Economic Science Association now has an annual international meeting and additional regional meetings in the U.S. and Europe. These developments have resulted in over a hundred publications in this area every year since 1990, with a high of 233 in 1999.

These numbers indicate the growing acceptance of experimental methods in economics. Since the total number of publications in all areas of economics has increased, it is natural to consider papers published in the top journals, where the total article count is roughly constant. In the *American Economic Review*, for example, there was one experimental economics article in the 1950’s, 3 articles in the 1960’s, 11 in the 1970’s, 40 in the 1980’s, and 86 articles in the 1990’s. Similarly, the number of publications in *Econometrica* went from 2 in the 1970’s to 12 in the 1980’s to 27 in the 1990’s. A searchable bibliography of over 2,500 papers in experimental economics and related social sciences can be found in the author’s *Y2K Bibliography of Experimental Economics*: [http://www.people.virginia.edu/~cah2k](http://www.people.virginia.edu/~cah2k), which also contains HTML reading lists for specific topics.

There are lots of exciting developments in this field in recent years. Economics experiments are being integrated into introductory courses and the workbooks of some major texts. Theorists are looking at laboratory results for applications and tests of their ideas, and policy makers are increasingly willing to look at how proposed mechanisms perform in controlled tests. Experimental methods have been used to design large auctions (e.g. the FCC auction for bandwidth and the Georgia Irrigation Auction) and systems for matching people with jobs (e.g. medical residents and hospitals). Two of the recipients of the 1994 Nobel Prize in Economics (Nash and Selten) are game theorists who conducted some of the first economics experiments, and recent conferences in Sweden indicate that more recognition may be on the horizon. Economics is well on its way to becoming an experimental science.
Chapter 2. A Pit Market

When buyers and sellers can communicate freely and openly as in a trading “pit,” the price and quantity outcomes can be predictable and efficient. Deviations tend to be relatively small and may be due to informational imperfections. The discussion should be preceded by a class experiment, either using playing cards and the instructions provided in the Appendix, or using the Vecon web-based double auction program (“DA”).

I. A Simple Example

Chamberlin (1948) set up the first market experiment by letting students with buyer or seller roles negotiate trading prices. The purpose was to illustrate systematic deviations from the standard theory of perfect competition. Ironically, this experiment is most useful today in terms of what factors it suggests are needed to promote efficient, competitive market outcomes.

Each seller was given a card with a dollar amount or “cost” written on it. For example, one seller may have a cost of $2, and another may have a cost of $8. The seller earns the difference between the sale price and the cost, so the low-cost seller would be searching for a price above $2 and the high-cost seller would be searching for a price above $8. The cost is not incurred unless a sale is made, i.e. the production is “made to order.” The seller would not want to sell below cost, since the resulting loss is worse than the zero earnings from no sale.

Similarly, each buyer was given a card with a dollar amount or “value” written on it. A buyer with a value of $10, for example, could earn the difference if a price below this amount could be negotiated. A buyer with a lower value, say $4, would refuse prices above that level, since a purchase above value would result in negative earnings.

The market is composed of groups of buyers and sellers who can negotiate trades with each other, either bilaterally or in larger groups. For example, suppose that the market structure is: Buyers 1 and 2 have values of $10 and buyers 3 and 4 have values of $4. Sellers 5 and 6 have costs of $2, and sellers 7 and 8 have costs of $8:

<table>
<thead>
<tr>
<th>Values</th>
<th>Costs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buyer 1</td>
<td>$10</td>
</tr>
<tr>
<td>Seller 5</td>
<td>$2</td>
</tr>
<tr>
<td>Buyer 2</td>
<td>$10</td>
</tr>
<tr>
<td>Seller 6</td>
<td>$2</td>
</tr>
<tr>
<td>Buyer 3</td>
<td>$4</td>
</tr>
<tr>
<td>Seller 7</td>
<td>$8</td>
</tr>
<tr>
<td>Buyer 4</td>
<td>$4</td>
</tr>
<tr>
<td>Seller 8</td>
<td>$8</td>
</tr>
</tbody>
</table>
In addition to the “structural” elements of the market (numbers of buyers and sellers, and their values and costs), we must consider the nature of the market price negotiations. A “market institution” is a full specification of the rules of trade. For example, one might let sellers “post” catalogue prices and then let buyers contact sellers if they wish to purchase at a posted price, with discounts not permitted. This institution, known as a “posted offer auction,” is sometimes used in laboratory studies of retail markets. The asymmetry, with one side posting and the other responding, is common when there are many people on the responding side and few on the posting side. Posting on the thin side may conserve on information costs, and agents on the thin side may have the “power” to impose prices on a “take-it-or-leave-it” basis. In contrast, Chamberlin used an institution that was symmetric and less structured; he let buyers and sellers mix together and negotiate bilaterally or in small groups, much as traders of futures contracts interact in a trading “pit.” Sometimes he announced transactions prices as they occurred, much as the market officials watching from a “pulpit” over the trading pit would key in contract prices that are posted electronically and flashed to other markets around the world. At other times, Chamberlin did not announce prices as they occurred, which may have resulted in more decentralized trading negotiations.

II. A Classroom Experiment

Figure 2.1 shows the partial results of a classroom pit market experiment with 4 buyers and 4 sellers, using the values and costs from the example given above (two values at $10, two buyers with values at $4, two sellers with costs of $2, and two sellers with costs of $8). When a buyer and a seller agreed on a price, they came together to the recording desk, where the price was checked to ensure that it was no lower than the seller’s cost and no higher than the buyer’s value, as required by the instructions given in the appendix of this chapter. (The prohibition of trading at a loss was used since the earnings in this classroom experiment were only hypothetical.) Notice that the quantity of trades tends to be two and the prices are in the $5-$7 range after the first couple of periods.

Consider the question of why the prices stayed in the $5-$7 range, with a transactions quantity of two units per period. Notice that the quantity could have been as high as four. For example, suppose that the two high-value ($10) buyers negotiated with high-cost ($8) sellers and agreed on prices of $9. Similarly, suppose that the negotiated prices were $3 for sales from the low-cost ($2) sellers to the low-value ($4) buyers. In this scenario, all of the sellers’ units sell, the quantity is four, and each person earns $1 for the period. Prices, however, are quite variable. These patterns are not observed in the price sequence of trades shown in Figure 2.1. In particular, prices are in the $5-$7 range that prevents high-cost sellers from selling at a profit, and that prevents high-value buyers from
buying at a profit. The result is that only the two low-cost units are sold to the two high-value buyers.

Figure 2.1. Contract Price Sequence

The intuitive reason for the low price dispersion is fairly obvious. At a price of $9, all four sellers would be willing (and perhaps eager) to sell, but only the two high-value buyers would be willing to buy, and perhaps not so eager given the low buyer earnings at that high price. This creates a competitive situation in which sellers may try to lower prices to get a sale, causing a price decline. Conversely, suppose that prices began in the $3 range. At these low prices, all four buyers would be willing (and perhaps eager) to buy, but only the two low-cost sellers would be willing to sell. This gives sellers the power to raise price without losing sales.

Finally, consider what happens with prices in the intermediate range, from $4 to $8. At a price of $6, for example, each low cost seller earns $4 (= $6 - $2), and each high-value buyer earns $4 (= $10 - $6). Together, the four people who make trades earn a total of $16. These earnings are much higher than $1 per person that was earned with price dispersion, for a total of $1 times 8 people = $8. In this example, the effect of reduced price dispersion is to reduce quantity by half and to double total earnings. The reduced price dispersion benefits buyers and sellers as a group, since total earnings go up, but the excluded high-cost sellers and low-value buyers are worse off. Nevertheless, economic efficiency has increased by these exclusions. The sellers have high costs because the opportunity costs of the resources they use are high, i.e. the value of the resources
they would employ (perhaps inefficiently) is higher in alternative uses. Moreover, the buyers with low values are not willing to pay an amount that covers the opportunity cost of the extra production needed to serve them.

One way to measure the efficiency of a market is to compare the actual earnings of all participants with the maximum possible earnings. It is straightforward to verify that $16 is the highest total earnings level that can be achieved by any combination of trades in this four-buyer and four-seller market. The efficiency was 100% for both periods shown in Figure 1.1, since the two units that traded involved low costs and high values. If the third or fourth unit had traded, the aggregate earnings must go down since the high costs ($8) for the third and fourth units exceeds the low values ($4) for these units. Thus the earnings total goes down by the difference $4 for each of these two units, reducing earnings from the maximum of $16 to $8. In this case, the outcome with price dispersion and four units trades only has an efficiency level of 50 percent. Of course, it is possible that some dispersion results in a third unit being traded, but not a fourth, in which case earnings are only reduced by $4 from the maximum, for an efficiency of 75 percent.

The operation of this market can be illustrated with the standard supply and demand graph. First consider a seller with a cost of $2. This seller would be unwilling to supply any units at prices below this cost, and would offer the entire capacity (1 unit) at higher prices. Thus the seller’s individual supply function has a “step” at $2. The total quantity supplied by all sellers in the market is zero for prices below $2, but market supply jumps to two units at slightly higher prices as the two low-cost sellers offer their units. The supply function has another step at $8 when the two high-cost sellers offer their units at prices slightly above this high-cost step. This resulting market supply function, with steps at each of the two cost levels, is shown by the solid line in Figure 2.2. The market demand function is constructed analogously. At prices above $10, no buyer is willing to purchase, but the quantity demanded jumps to two units at prices slightly below this value. The demand function has another step at $4, as shown by the dashed line in Figure 2.2. This curve is vertical at prices below $4, since all four buyers will wish to purchase at any lower price. The supply and demand functions overlap at a quantity of 2 in the range of prices from $4 to $8. At any price in this region, the quantity supplied equals the quantity demanded. At lower prices, there is excess demand, which would tend to drive prices up. At prices above the region of overlap, there is excess supply that would tend to drive prices back down.

The fact that the maximum aggregate earnings are $16 for this design can be seen directly from Figure 2.2. Suppose that the price is $6 for all trades. The value of the first unit on the left is $10, and since the buyer pays $6, the surplus value is the difference, or $4. The surplus on the second unit is also $4, so the
“consumers’ surplus” is the sum of the surpluses on individual units, or $8. Notice that consumer’s surplus is the area under the demand curve and above the price paid. Since sellers earn the difference between price and cost, the “producers’ surplus” is the area (also $8) above the supply curve and below the price. Thus the “total surplus” is the sum of these areas, which equals the area under demand and above supply (to the left of the intersection). This total surplus area is $10 - $2 + $10 - $2 = $16. The total surplus is actually independent of the particular price at which the units trade. For example, a higher price would reduce consumers’ surplus and increase producers’ surplus, but the total area would remain fixed at $16. Adding a third unit would reduce surplus since the cost ($8) is greater than the value ($4). (In this section, the discussion has been simplified by considering only 4 buyers and 4 sellers. In fact, the actual classroom experiment alluded to involved 9 buyers with values: $10, $10, $10, $10, $10, $4, $4, $4, $4, and 9 sellers with costs: $2, $2, $2, $2, $2, $8, $8, $8, $8. These parameters yield a supply and demand configuration that has the “box” shape in Figure 2.2, with a vertical overlap from $4 to $8 at a quantity of 5 units. These 5 units sold in both periods, at prices in the competitive range: $5, $4, $5, $4, $5 in period 1 and $6, $5, $5, $6, $5 in period 2. In both periods, the 4 high cost units did not sell, and the 4 low-value units were not purchased, so the market was 100% efficient.)

These types of surplus calculations do not depend on the particular form of the demand and supply functions in Figure 2.2. Individual surplus amounts on
each unit are the difference between the value of the unit (the height of demand) and the cost of the unit (the height of supply). Thus the area between demand and supply to the left of the intersection represents the maximum possible total surplus, even if demand and supply have more steps than the example in Figure 2.2.

Figure 2.3 shows the results of a classroom experiment with 9 buyers and 9 sellers, using a design with more steps and an asymmetric structure. Notice that demand is relatively “flat” on the left side, and therefore that the competitive price range (from $7 to $8) is relatively high. For prices in the competitive range, the consumers’ surplus will be much smaller than the producer’s surplus. The right side shows the results of two periods of pit market trading, with the prices plotted in the order of trade. Prices start at about $5, in the middle of the range between the lowest cost and the highest value. The prices seem to be converging to the competitive range from below, which could be due to buyer resistance to increasingly unequal earnings. In both periods, all seven of the higher value ($10 and $9) units were purchased, and all seven of the lower cost ($2 and $7) units were sold. As before, prices stayed in a range needed to exclude the high-cost and low-value units, and efficiency was 100% in both periods.

Figure 2.3. Demand, Supply, and Transactions Prices for a Pit Market

The asymmetric structure in Figure 2.3 was used to ensure that prices did not start in the equilibrium range, in order to illustrate some typical features of price adjustments. The first units that traded in each period where the ones with $10 values and $2 costs on the left side of the supply and demand figure. After
these initial transactions in the $5-$7 range, the remaining traders were closer to
the competitive “margin.” These marginal buyers were those with values of $9
and $4. The marginal sellers were those with costs of $7 and $8. Clearly these
units will have to sell for prices above $7, which forces the prices closer to the
competitive prediction at the end of the period. When sellers who sold early in
the period see these higher prices, they may hold out for higher prices in the next
period. Similarly, buyers will come to expect prices to rise later in the period, so
they will scramble to buy early, which will tend to drive prices up earlier in each
subsequent period. To summarize, the convergence process is influenced by the
tendency for the highly profitable units (on the left side of demand and supply) to
trade early, leaving traders “at the margin” where price negotiations tend to be
near competitive levels. Price dispersion is narrowed in subsequent periods as
traders come to expect the higher prices at the end of the period.

III. Chamberlin’s Results and Vernon Smith’s Reaction

The negotiations in the class experiment mostly took place in a central
area that served as the trading pit, although some people tended to break off in
pairs to finalize deals. Even so, the participants could hear the public offers being
made by others, a process which should reduce price dispersion. A high-value
buyer, who is better off paying up to $10 instead of not trading, may not be
willing to pay such high prices when some sellers are making lower offers.
Similarly, a low-cost seller will be less willing to accept a low price when other
buyers are observed to pay more. In this manner, good market information about
the going prices will tend to reduce price dispersion. A high dispersion is needed
for high-cost sellers and low-value buyers to be able to find trading partners, so
less dispersion will tend to exclude these “extra-marginal” traders.

Chamberlin did report some tendency for the markets to yield “too many”
trades relative to competitive predictions, which he attributed to the dispersion
that can result from small group negotiations. In order to evaluate this conjecture,
he took the value and cost cards from his experiment and used them to simulate a
decentralized trading process. These simulations were not laboratory experiments
with student traders; they were mechanical, the way one would do computer
simulations today.

For groups of size two, Chamberlin would shuffle the cost cards and the
value cards, and then he would match one cost with one value and “make” a trade
at an intermediate price if the value exceeded the cost. Cards for trades that were
not made were returned to the deck to be reshuffled and rematched. In our first
example from Table 2.1, this process would result in two trades when the low cost
units were matched with the high value units. It would result in four trades when
both low cost units were matched with the low value units. The intermediate cases
would result in three trades, for example when the value/cost combinations are:
$10/$2, $10/$8, $4/$2, and $4/$8. The three value/cost pairs that result in a trade are shown in bold. To summarize, random matches in this example produce a quantity of trades of either two, three, or four, and some simulations should convince you that on average there will be three units traded.

For groups larger than two traders, Chamberlin would shuffle and deal the cards into groups and would calculate the competitive equilibrium quantity for each group. In the four-buyer/four-seller example from the previous section, there is only one group of size 4, i.e. all four traders, and the equilibrium for all four is the competitive quantity of 2 units. This illustrates Chamberlin’s general finding that quantity tended to decrease with larger group size in his simulations.

Notice the relationship between the use of simulations and laboratory experiments with human participants. The experiment provided the empirical regularity (excess quantity) that motivated a theoretical model (competitive equilibrium for subgroups), and the simulation confirmed that the same regularity would be produced by this model. In general, computer simulations can be used to derive properties of models that are too complex to solve analytically, which is often the case for models of out-of-equilibrium behavior and dynamic adjustment. The methodological order can, of course, be reversed, with computer simulations being used to derive predictions that can be tested with laboratory experiments using human participants.

The simulation analysis given above suggests that market efficiency will be higher when price information is centralized, so that all traders know the “going” levels of bid, ask, and transactions prices. Vernon Smith, who attended some of Chamberlin’s experiments when he was a student at Harvard, used this intuition to design a trading institution that promoted efficiency. After some classroom experiments of his own, he began using a “double auction” in which buyers made bids, sellers made offers (or “asks”), and all could see the highest outstanding bid and the lowest outstanding ask. Buyers could raise the current best bid at any time, and sellers could undercut the current best ask at any time. In this manner, the bid/ask spread would typically diminish until someone closed a contract by accepting the terms from the other side of the market, i.e. until a seller accepted a buyer’s bid or a buyer accepted a seller’s ask. This is called a double auction, since it involves both buyers (bidding up) and sellers (bidding down).

The table below shows a typical sequence of bids and asks in a double auction. Buyer 2 bids $3, and seller 5 offers to sell for $8. Seller 6 comes in with an ask of $7.50, and buyer 1 bids $4 and then $4.50. At this point seller 7 offers at $6, which buyer 2 accepts. Notice that this auction is “double” in the sense that buyers are bidding up, as in a normal auction for something like an antique. At the same time, sellers are bidding down by undercutting each other’s ask prices.
A trade occurs when these processes meet, i.e. when a buyer accepts a seller’s ask or when a seller accepts a buyer’s bid.

Table 2.2. A Price Negotiation Sequence

<table>
<thead>
<tr>
<th>Bid</th>
<th>Ask</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buyer 2</td>
<td>$3.00</td>
</tr>
<tr>
<td></td>
<td>$8.00</td>
</tr>
<tr>
<td></td>
<td>$7.50</td>
</tr>
<tr>
<td>Seller 5</td>
<td></td>
</tr>
<tr>
<td>Buyer 1</td>
<td>$4.00</td>
</tr>
<tr>
<td>Buyer 1</td>
<td>$4.50</td>
</tr>
<tr>
<td></td>
<td>$6.00</td>
</tr>
<tr>
<td>Seller 7</td>
<td></td>
</tr>
<tr>
<td>Buyer 2</td>
<td>Accepts $6.00</td>
</tr>
</tbody>
</table>

In a double auction, there is always good public information about the bid/ask spread and about past contract prices. This information creates a large-group setting that tends to diminish price variability and increase efficiency. Smith (1962, 1964) reported that this double auction resulted in efficiency measures of over ninety percent, even with relatively small numbers of traders (4-6) and with no information about others’ values and costs. The double auction tends to be somewhat more centralized than a pit market, which does not close off the possibility of bilateral negotiations that are not observed by others. Double auction trading more closely resembles trading for securities on the New York Stock Exchange, where the specialist collects bids and asks, and all trades come across the “ticker tape.”

Figure 2.4 shows the results of a double auction market conducted under research conditions, with earnings determined by value-price differences for buyers and price-value differences for sellers. In addition, each person received a “commission” of 5 cents for each unit that they bought or sold. The four buyers begin with four units each, for a total of 16 units, all with values of $6.80. The four sellers have a total of 11 units, all with costs of $5.70. The left side of the figure shows the resulting supply function, which is vertical at a quantity of 11 where it crosses demand. The intersection is at a price of $6.80, and at lower prices there is excess demand. The period-by-period price sequences are shown on the right side of the figure. Notice that first-period prices start in the middle of the range between values and costs, and then rise later in the period. Prices in period 2 start higher and rise again. This upward trend continues until prices reach the competitive level of about $6.80 in period 4. At this point, buyers are only earning pennies (e.g. the commission) on each transaction. A second treatment, not shown, began in period 6 with the total number of buyer units reduced to 11 and the number of seller units increased to 16. Buyers, who were
already earning very little on each unit, were disappointed at having fewer units. But prices started to fall immediately; the very first trading price in period 6 was only $6.65. Prices declined steadily, reaching the new competitive price of $5.70 by period 10. This session illustrates the strong convergence properties of the double auction, where excess demand or supply pressures can push prices to create severe earnings inequities that are unlikely to arise in bilateral bargaining situations.

Figure 2.4. Demand, Supply, and Transactions Prices for a Double Auction

IV. Extensions

The simple experiment presented in this chapter can be varied in numerous useful directions. For example, an upward shift in demand, accomplished by increasing buyers’ values, should raise prices. An increase in the number of sellers would tend to shift supply outward, which has the effect of lowering prices. The imposition of a $1 per-unit tax on buyers would (in theory) shift demand down by $1. To see this, note that if one’s value is $10 but a tax of $1 must be paid upon purchase, then the net value is only $9. All demand steps would shift down by $1 in this manner. Alternatively, a $1 tax per unit imposed on sellers would shift supply up by $1, since the tax is analogous to an $1 increase in cost. Some extensions will be considered in the chapter on double auctions that is in the market experiments section of this book.

We have discussed several alternative market institutions here. Chamberlin’s *pit market* corresponds most closely to trading of futures contacts in
a trading pit, whereas Smith’s *double auction* is more like the trading of securities on the New York Stock Exchange. A *posted-offer auction* (where sellers set prices on a take-it-or-leave-it basis) is more like a retail market with many buyers and few sellers. These different trading institutions have different properties and applications, and outcomes need not match competitive predictions. Nevertheless, it is useful to consider outcomes in terms of market efficiency, measured as the percentage of maximum earnings achieved by the trades that are made. In considering alternative designs for ways to auction off some licenses, for example, one might want to consider the efficiency of the allocations along with the amounts of revenue generated for the seller(s).

The theoretical predictions discussed in this chapter are derived from an analysis of supply and demand. In the competitive model, all buyers and sellers take price as given. This model may not provide good predictions when some traders perceive themselves as being price makers, with “power” to push prices in their favor. A single-seller monopoly may be able to raise price above competitive levels, and a small group of sellers with enough of a concentration of market capacity may be able to raise prices as well. The exercise of this type of market power is discussed in the chapter on posted offer auctions, in which sellers post prices on a take-it-or-leave-it basis. The exercise of market power by price-setting sellers is a strategic decision, and the relevant theoretical models are those of *game theory*, which is the analysis of interrelated strategic decisions. The word “interrelated” is critical here, since the amount that one firm may wish to raise price depends on how much others are expected to raise prices. We turn to a simple introduction of game theory in the next chapter. The discussion will be in terms of simple games with two players and two decisions, but the same principles will be applied later in the book to the analysis of more complex games with many sellers and many possible price levels.

**Questions:**

1. Suppose that all of the numbered Hearts and Spades from a deck of cards (excluding Ace, King, Queen, and Jack) are used to set up a market. The Hearts determine demand, e.g. a 10 represents a buyer with a redemption value of $10. Similarly, the Spades determine supply. There are 9 buyers and 9 sellers, each with a single card.

   a) Graph supply and demand and derive the competitive price and quantity predictions.

   b) What is the predicted effect of removing the 3, 4, and 5 of Spades?

2. When people are asked to come up with alternative theories about why prices in a pit market stabilize at a particular level observed in class, they sometimes suggest taking the average of all buyers’ values and sellers’ costs.
a) Devise and graph two experimental designs (list all buyers’ values and all sellers’ costs) that have the same competitive price prediction but different averages of values and costs.

b) Devise and graph two experimental designs that have the same average of all values and costs, but which have different competitive price predictions.
Chapter 3. Matrix Games: Competition and Coordination

A game with two players and two decisions can be represented by a 2x2 payoff table or “matrix.” Such games often highlight the conflict between incentives to compete or cooperate. This chapter introduces two such games, the prisoner’s dilemma and the coordination game. Discussion can be preceded with an experiment, either using playing cards (with instructions provided in the Appendix) or using the Vconlab web-based version (game MG).

I. Game Theory and the Prisoner’s Dilemma

The Great Depression, which was the defining economic event of the 20th century, caused a major rethinking of existing economic theories that represented the economy as a system of self-correcting markets needing little in the way of active economic policy interventions. On the macroeconomic side, John Maynard Keynes’ *The General Theory of Employment, Interest, and Money* focused on psychological elements (“animal spirits”) that could cause a whole economy to become mired in a low-employment equilibrium, with no tendency for self-correction. An equilibrium is a state where there are no net forces causing further change, and Keynes’ message implied that such a state may not necessarily be good. On the microeconomics side, Edward Chamberlin argued that markets may not yield efficient, competitive outcomes, as noted in the previous chapter. At about the same time, von Neumann and Morgenstern (1944) published *Theory of Games and Economic Behavior*, which was motivated by the observation that a major part of economic activity involves bilateral and small-group interactions, where the classical assumption of non-strategic, price-taking behavior (used in the previous chapter) is not realistic. In light of the protracted Depression, these new theories met with a quick acceptance. Chamberlin’s models of “monopolistic competition” were quickly incorporated into textbooks as alternative models, and von Neumann and Morgenstern’s book on game theory received front-page coverage in the *New York Times*.

A game is a mathematical model of a strategic situation in which players’ payoffs depend on their own and others’ decisions. A *game* is characterized by the players, their sets of feasible decisions, the information available at each decision point, and the payoffs (as functions of all decisions and random events). A key notion is that of a *strategy*, which is essentially a complete plan of action that covers all contingencies. For example, a strategy in an auction could be an amount to bid for each possible estimate of the value of the prize. Since a
strategy covers all contingencies, even those that are unlikely to be faced, it could be given to a hired employee to be played out on behalf of the player in the game. An equilibrium is a set of strategies that is stable in some sense, i.e. with no inherent tendency for change. Economists are interested in notions of equilibrium that will provide good predictions after behavior has had a chance to “settle down.”

As noted in Chapter 1, a Princeton graduate student, John Nash, corrected a major shortcoming in the Neumann and Morgenstern analysis by developing a notion of equilibrium for non-zero-sum games. Nash showed that this equilibrium existed under general conditions, and this proof caught the attention of researchers at the RAND Corporation headquarters in Santa Monica. In fact, two RAND mathematicians immediately conducted a laboratory experiment designed to test Nash’s new theory. Nash’s thesis advisor was in the same building when he noticed the payoffs for the experiment written on a blackboard. He found the game interesting and made up a story of two prisoners facing a dilemma of whether or not to make a confession. This story was used in a presentation to the psychology department at Stanford, and the “prisoner’s dilemma” became the most commonly discussed paradigm in the new field of game theory.

In a prisoner’s dilemma, the two suspects are separated and offered a set of threats and rewards that make it best for each to confess and essentially “rat” on the other person, whether or not the other person confesses. For example, the prosecutor may say: “if you do not confess and the other person rats on you, then I can get a conviction and I will throw the book at you, so you are better off ratting on them.” When the prisoner asks what happens if the other does not confess, the prosecutor replies: “Even without a confession, I can frame you both on a lesser charge, which I will do if nobody confesses. If you do confess and the other person does not, then I will reward you with immunity and book the holdout on the lesser charge.” Thus each prisoner is better off implicating the other person even if the other remains silent. Both prisoners, aware of these incentives, decide to “rat” on the other, even though they would both be better off if they could somehow form a binding code of silence. This analysis suggests that two prisoners might be bullied into confessing to a crime that they did not commit, which is a scenario from *Murder at the Margin*, written by two mysterious economists under the pseudonym Marshall Jevons (1977).

A game with a prisoner’s dilemma structure is shown in the table below. The row player’s Bottom decision corresponds to confession, as does the column player’s Right decision. The Bottom/Right outcome, which yields payoffs of 3 for each, is worse than the Top/Left outcome, where each receive payoffs of 8. (A high payoff here corresponds to a light penalty.) The dilemma is that the “bad” confession outcome is an equilibrium in the following sense: if either
person expects the other to talk, then their own best response to this belief is to confess as well. For example, consider the Row player, whose payoffs are listed on the left side of each outcome cell in the table. If Column is going to choose Right, then Row either gets 0 for playing Top or 3 for playing Bottom, so Bottom is the best response to Right. Similarly, column’s Right decision is the best response to a belief that Row will play Bottom. There is no other cell in the table with this stability property. For example, if Row thinks column will play Left, then Row would want to play Bottom, so Top/Left is not stable. It is straightforward to show that the diagonal elements, Bottom/Left and Top/Right are also unstable.

Table 3.1  A Prisoner’s Dilemma (Row’s payoff, Column’s payoff)

<table>
<thead>
<tr>
<th>Row Player:</th>
<th>Column Player:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top</td>
<td>Left</td>
</tr>
<tr>
<td>Top</td>
<td>8, 8</td>
</tr>
<tr>
<td>Bottom</td>
<td>10, 0</td>
</tr>
</tbody>
</table>

II. A Prisoner’s Dilemma Experiment

The payoff numbers in the previous prisoner’s dilemma experiment can be derived in the context of a simple example in which both players must choose between a low effort (0) and a higher effort (1). The cost of exerting the higher effort is $10, and the benefits to each person depend on the total effort for the two individuals combined. Since each person can choose an effort of 0 or 1, the total effort must be 0, 1, or 2, and the benefit per person is given:

<table>
<thead>
<tr>
<th>Total effort</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benefit per person</td>
<td>$3</td>
<td>$10</td>
<td>$18</td>
</tr>
</tbody>
</table>

To see the connection between this “production” function and the prisoner’s dilemma payoff matrix, let Bottom and Right correspond to zero effort. Thus the Bottom/Right outcome results in 0 total effort and payoffs of 3 for each person, as shown in the payoff matrix in Table 3.1. The Top/Left cell in the matrix is relevant when both choose efforts of 1 (at a cost of 8 each). The total effort is 2, so each earns 18 − 10 = 8, as shown in the Top/Left cell of the matrix. The Top/Right and Bottom/Left parts of the payoff table pertain to the case where one person receives the benefit of 10 at no cost, the other receives the benefit of 10 at a cost of 10, for a payoff of 0. Notice that each person has an incentive to “free ride” on the other’s effort, since one’s own effort is more costly ($10) than the
marginal benefit of effort, which is 7 (= 10 – 3) for the first unit of effort and 8 (= 18 – 10) for the second unit.

Cooper, De Jong, Forsythe, and Ross (1996) conducted an experiment using the prisoner’s dilemma payoffs in the above matrix, with the only change being that the payoffs were (3.5, 3.5) at the Nash equilibrium. Their matching protocol prevented individuals from being matched with the same person twice, or from being matched with anyone who had been matched with them or one of their prior partners. This “no-contageon” protocol is analogous to going down a receiving line at a wedding and telling everyone the same bit of gossip about the bride and groom. Even if this story is so curious that everyone you meet in line repeats it to everyone they meet, you will never encounter anyone who has heard the story, since all of the people who have heard the story will be behind you in the line. In an experiment, the elimination of any type of repeated matching, either direct or indirect, means that nobody is able to send a “message” or to punish or reward others for cooperating. Even so, cooperative decisions (Top or Left) were fairly common in early rounds (43%), and the incidence of cooperation declined to about 20% in rounds 15-20. A recent classroom experiment using the Veconlab program with the Table 3.1 payoffs and random matching produced cooperation rates of about 33 percent, with no downward trend. A second experiment with a different class produced cooperation rates starting at about 33% and declining to zero by the fourth period. This latter group behaved quite differently when they were matched with the same partner for 5 periods; cooperation rates stayed level in the 33-50% range until the final period. The “end-game effect” is not surprising since cooperation in earlier periods may consist of an effort to signal good intentions and stimulate reciprocity. These forward-looking strategies are not available in the final round.

The results of these prisoner’s dilemma experiments are representative. There is typically a mixture of cooperation and defection, with the mix being somewhat sensitive to payoffs and procedural factors, with some variation across groups. In general, cooperation rates are higher when individuals are matched with the same person in a series of repeated rounds. In fact, the first prisoner’s dilemma experiment run at the RAND Corporation over fifty years ago lasted for 100 periods, and cooperative phases were interpreted as evidence against the Nash equilibrium.

A strict game-theoretic analysis would have people realizing that there is no reason to cooperate in the final round, and knowing this, nobody would try to cooperate in the next-to-last round with the hope of stimulating final-round cooperation. Thus, there should be no cooperation in the next-to-last round, and hence there is no reason to try to stimulate such cooperation. Reasoning “backwards” from the end in this manner, one might expect no cooperation even in the very first round, at least when the total number of rounds is finite and
known. Nash responded to the RAND mathematicians with a letter maintaining that it is unreasonable to expect people to engage in this many levels of iterative reasoning in an experiment with many rounds. There is an extensive literature on related topics, e.g., the effects of punishments, rewards, adaptive behavior, and various “tit-for-tat” strategies in repeated prisoner’s dilemma games.

Finally, it should be noted that there are many ways to present the payoffs for an experiment, even one as simple as the prisoner’s dilemma. Both the Cooper et al. and the Veconlab experiment used a payoff matrix presentation. Alternatively, the instructions could be presented in terms of the cost of effort and the table showing the benefit per person for each possible level of total effort. This presentation is perhaps less neutral, but the economic context makes it less abstract and artificial than a matrix presentation. The Appendix for this chapter takes a different approach by setting up the prisoner’s dilemma game as one where each person chooses which of two cards to play (Capra and Holt, 2000). For example, suppose that each person has an 8 of Spades and a 6 of Diamonds. Playing the Diamond “pulls” $6 to one’s own earnings, and playing the Spade “pushes” $8 to the other person’s earnings. If they both pull the $6, earnings are $6 each. Both would be better off if they played the Spade, yielding $8 for each. Pulling $6, however, is better from a selfish perspective, regardless of what the other person does. This is clearly a prisoner’s dilemma. The card presentation is not neutral enough for a research experiment, but it is quick and easy to implement in class, where students hold the cards played against their chests, and the instructor picks people in pairs to reveal their cards. A Nash equilibrium survives an “announcement test” in that neither would wish they had played a different card given the card played by the other. In this case, the Nash equilibrium is for each to play the Spade.

III. A Coordination Game

Many production processes have the property that one person’s effort increases the productivity of another’s effort. For example, a mail order company must take orders by phone, produce the goods, and ship them. Each sale requires all three services, so if one process is slow, the effort of those in other activities is to some extent wasted. In terms of our two-person production function, recall that 1 unit of effort produced a benefit of $10 per person and 2 units produced a benefit of $18. Suppose that the second unit of effort makes the first one more productive in the sense that the benefit is more than doubled when a second unit of effort is added. In the table that follows, per-person benefit is $30 for 2 units of effort:

<table>
<thead>
<tr>
<th>Total effort</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benefit per person</td>
<td>$3</td>
<td>$10</td>
<td>$30</td>
</tr>
</tbody>
</table>

37
If the cost of effort remains at $10 per unit, then each person earns $30 - $10 = $20 in the case where both supply a unit of effort, as shown in the revised payoff matrix below:

Table 3.2  A Coordination Game
(Row’s payoff, Column’s payoff)

<table>
<thead>
<tr>
<th>Column Player:</th>
<th>Left</th>
<th>Right</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top</td>
<td>↑ 20, 20 ↔ 0, 10</td>
<td></td>
</tr>
<tr>
<td>Bottom</td>
<td>10, 0 ↓ 3, ⇒ 3</td>
<td></td>
</tr>
</tbody>
</table>

As before, the zero effort outcome (Bottom/Right) is a Nash equilibrium, since neither person would want to change from their decision unilaterally. For example, if Row knows that Column will choose Right, then Row can get $0 from Top and $3 from Bottom, so Row would want to choose Bottom, as indicated by the downward pointing arrow in the lower-right box. Similarly, Right is a best response to Row’s decision to use Bottom, as indicated by the right arrow in the lower-right box.

There is a second Nash equilibrium in the Top/Left box of this modified game. To verify this, think of a situation in which we “start” in that box, and consider whether either person would want to deviate, so there are two things to check. Step 1: if Row is thought to choose Top, then Column would want to choose Left, as indicated by the left arrow in the Top/Left box. Step 2: if Row expects Column to choose Left, then Row would want to choose Top, as indicated by the up arrow. Thus Top/Left survives an “announcement test” and would be stable. This second equilibrium yields payoffs of 20 for each, far better than the payoffs of 3 each in the Bottom/Right equilibrium outcome. This is called a “coordination game,” since the presence of multiple equilibria raises the issue of how players will guess which one will be relevant. (In fact, there is a third Nash equilibrium in which each player chooses randomly, but we will not discuss such strategies until Chapter 5.)

It used to be common for economists to assume that rational players would somehow coordinate on the best equilibrium, if all could agree which is the best equilibrium. While it is apparent that the Top/Left outcome is likely to be the most frequent outcome, one example does not justify a general assumption that players will always coordinate on an equilibrium that is better for all. This assumption can be tested with laboratory experiments, and it has been shown to be false. In fact, players sometimes get driven to an equilibrium that is worst for all concerned (Van Huyck, Battalio, and Beil; 1988). Examples of data from such
coordination games will be provided in a subsequent chapter on coordination problems. For now, consider the intuitive effect of increasing the cost of effort from $10 to $19. With this increase in the cost of effort, the payoffs in the high-effort Top/Left outcome are reduced to $30 - $19 = $11. Moreover, the person who exerts effort alone only receives a $10 benefit and incurs a $19 cost, so the payoffs for this person are -$9, as shown:

<table>
<thead>
<tr>
<th>Column Player:</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Top</td>
<td>↑↑ 11, 11 ⇐</td>
<td>-9, 10</td>
</tr>
<tr>
<td>Bottom</td>
<td>10, -9</td>
<td>⇓ 3, ⇐ 3</td>
</tr>
</tbody>
</table>

Notice that Top/Left is still a Nash equilibrium: if Column is expected to play Left, then Row’s best response is Top and vice versa, as indicated by the directional arrows in the upper-left corner. But Top and Left are very risky decisions, with possible payoffs of $11 and -$9, as compared with the $10 and $3 payoffs associated with the Bottom and Right strategies that yield the other Nash equilibrium. While the absolute payoff is higher for each person in the Top/Left outcome, each person has to be very sure that the other one will not deviate. In fact, this is a game where behavior is likely to converge to the Nash equilibrium that is worst for all. These payoffs were used in a Vconlab experiment that lasted 5 periods with random matching of 12 participants. Participants were individuals or pairs of students at the same computer. By the fifth period, only a quarter of the decisions were Top or Left. The same people then played the less risky coordination game shown in Table 3.2, again with random matching. The results were quite different; all but one of the pairs ended up in the good (Top, Left) equilibrium outcome in all periods.

To summarize, it is not appropriate to assume that behavior will somehow converge to the best outcome for all, even when this is a Nash equilibrium as in the coordination games considered here. With multiple equilibria, the one with the most “attraction” may depend on payoff features (e.g., the effort cost) that determine which equilibrium is riskier. These issues will be revisited in the chapter on coordination.

**IV. Extensions**

The prisoner’s dilemma game discussed in this chapter can be given an alternative interpretation, i.e. that of a “public good.” This is because the each
person’s benefit depends on the total effort, including that of the other person. In effect, neither person can appropriate more than half of the benefit of their own effort, and in this sense the benefit is publicly available to both, just as national defense or police protection are freely available to all. We will consider the public goods provision problem in more detail in a later chapter in the Public Choice section of this book. The setup is one where the number of possible effort levels is greater than two, and the effects of changes in costs and other incentives will be considered.

The prisoner’s dilemma has a Nash equilibrium that is worse than the outcome that results when individuals cooperate and ignore their private incentives to defect. The dilemma is that the equilibrium is the bad outcome, and the good outcome is not an equilibrium. In contrast, the coordination game considered above has a good outcome that is also an equilibrium. Even so, there is no guarantee that the better equilibrium will be realized. A more formal analysis of the “risk” associated with alternative equilibria is presented in a subsequent chapter on coordination. The main point is that there may be multiple Nash equilibria, and which equilibrium is observed may depend on intuitive factors like the cost of effort. In particular, a high effort cost may cause players to “get stuck” in a Nash equilibrium that is worse for all concerned, as compared with an alternative Nash equilibrium. The coordination game provides a paradigm in which an equilibrium may not be desirable. Such a possibility has concerned macroeconomists like Keynes, who worried that a market economy may become mired in a low-employment, low-effort state.

In all of the equilibria considered thusfar, each person chooses a decision without any randomness. It is easy to think of games where it is not good to be perfectly predictable. For example, a soccer player in a penalty kick situation should sometimes kick to the left and sometimes to the right, and the goalie should also avoid a statistical preference for diving to one side or the other. Such behavior in a game is called a “randomized strategy.” Before considering randomized strategies (in Chapter 5), it is useful to introduce the notions of probability, expected value, and other aspects of decision making in uncertain situations. This is the topic of the next chapter, where we consider the choice between simple lotteries over cash prizes.
Questions:

1. Suppose that the effort cost is raised from $10 to $25 for the example based on the effort-value table shown below. Is the resulting game a coordination game or a prisoner’s dilemma? Explain, and find the Nash equilibrium (or equilibria) for the new game.

<table>
<thead>
<tr>
<th>Total effort</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benefit per person</td>
<td>$3</td>
<td>$10</td>
<td>$30</td>
</tr>
</tbody>
</table>

2. What is the payoff table for the prisoner’s dilemma game described in the text based on “pulling” $6 or “pushing” $8?

3. Each player is given a 6 of Hearts and an 8 of Spades. The two players select one of their cards to play. If the suit matches, then they each are paid a number of dollars that equals the number of the card ($6 for two Hearts and $8 for two Spades). Earnings are zero in the event of a mismatch. Is this a coordination game or a prisoner’s dilemma? Show the payoff table and find all Nash equilibria (that do not involve random play).

4. Suppose that the payoffs for the original prisoner’s dilemma are altered as follows. The cost of effort remains at $10, but the per-person benefits are given in the table below. Recalculate the payoff matrix, find all Nash equilibria (that do not involve random play), and explain whether the game is a prisoner’s dilemma or a coordination game.

<table>
<thead>
<tr>
<th>Total effort</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benefit per person</td>
<td>3</td>
<td>5</td>
<td>18</td>
</tr>
</tbody>
</table>
Chapter 4. Risk and Decision Making

In this chapter, we consider decisions in risky situations. Each decision has a set of money consequences or “prizes” and the associated probabilities. In such cases, it is straightforward to calculate the expected money value of each decision. A person who is neutral to risk will select the decision with the highest expected payoff. A risk-averse person is willing to accept a lower expected payoff in order to reduce risk. A simple lottery choice experiment is used to illustrate the concepts of expected value maximization and risk aversion. Variations on the experiment can be conducted prior to class discussions, either with the instructions included in the appendix or with the Veconlab software (select experiment LC).

I. Introduction

Many decision situations involve consequences that cannot be predicted perfectly in advance. For example, suppose that you are a contestant on the television game show “Who Wants to Be a Millionaire?” You are at the $500,000 point and the question is on a topic that you know nothing about. Fortunately, you have saved the “fifty-fifty” option that rules out two answers, leaving two that turn out to be unfamiliar. At this point, you figure you only have a one-half chance of guessing correctly, which would make you into a millionaire. If you guess incorrectly, you receive the safety level of $32,000. Or you can fold and take the sure $500,000. In thinking about whether to take the $500,000 and fold, you decide to calculate the expected money value of the guess option. With probability 0.5 you earn $32,000, and with probability 0.5 you earn $1,000,000, so the expected value of a guess is:

$$0.5($32,000) + 0.5($1,000,000) = $16,000 + $500,000 = $516,000.$$  

This expected payoff is greater than the sure $500,000 one gets from stopping, but the trouble with guessing is that you either get 32K or one million dollars, not the average. So the issue is whether the $16,000 increase in the average payoff is worth the risk, which is considerable if you guess. If you love risk, then there is no problem; take the guess. If you are neutral to risk, the extra 16K in the average payoff should cause you to take the risk. Alternatively, you might reason that the 32K would be gone in 6 months, and only a prize of 500K or greater would be large enough to bring about a change in your lifestyle (new SUV, tropical vacation, etc.), which may lead you to fold. To an economist, the person who folds would be classified as being risk averse in this case, because the risk for this
person is sufficiently bad that it is not worth the extra 16K in average payoff associated with the guess.

Now consider an even more extreme case. You have the chance to secure a sure 1 million dollars. The alternative is to take a coin flip, which provides a prize of 3 million dollars for Heads, and nothing for Tails. If you believe that the coin is fair, then you are choosing between two lotteries:

- **Safe Lottery:** 1 million for sure;
- **Risky Lottery:** 3 million with probability 0.5, 0 with probability 0.5.

As before, the expected money value for the risky lottery can be calculated by multiplying the probabilities with the associated payoffs, yielding an expected payoff of 1.5 million dollars. When asked about this (hypothetical) choice, most students will select the safe million. Notice that for these people, the extra $500,000 in expected payoff is not worth the risk of ending up with nothing. An economist would call this risk aversion, but to a layman the reason is intuitive: the first million provides a major change in lifestyle. A second million is an equally large money amount, but it is hard for most of us to imagine how much additional benefit this would provide. Roughly speaking, the additional (marginal) utility of the second million dollars is much lower than the utility of the first million, and the marginal utility of the third million is likely to be even lower. A utility function with a diminishing marginal utility is one with a curved “hill” shape, as shown in Figure 4.1.

This utility function is the familiar “square root” function, where utility (in millions) is the square root of the payoff (in millions). Thus the utility of 0 is 0, the utility of 1 million dollars is 1, the utility of 4 million dollars is 2, the utility of 16 million is 4, etc. In fact, each time you multiply the payoff by 4, the utility only increases by a factor of 2, which is indicative of the diminishing marginal utility. This feature is also apparent from the fact that the slope of the utility function near the origin is high, and it diminishes as we move to the right. The more curved the utility function, the faster the utility of an additional million diminishes.
The diminishing-marginal-utility hypothesis was first suggested by Daniel Bernoulli (1738). There are many functions with this property. One is the class of power functions: \( U(x) = x^{1-r} \), where \( x \) is money income and \( r \) is a measure of risk aversion that is less than 1. Notice that when \( r = 0 \), we have the linear function: \( U(x) = x \), which has no curvature, and hence no diminishing marginal utility for income. For a person whose choices are represented by this function, the second million is just as good as the first million, etc., so the person is neutral to risk. Alternatively, if the risk aversion measure \( r \) is increased from 0 to 0.5, we have \( U(x) = x^{0.5} \), which is the square root function in the figure. Further increases in \( r \) result in more curvature, and in this sense \( r \) is a measure of the extent to which marginal utility of additional money income decreases. This measure is often called the coefficient of relative risk aversion. Next consider the results of a lottery choice experiment designed to evaluate risk attitudes.

II. A Simple Lottery-Choice Experiment

In all of the cases discussed above, the safe lottery is a sure amount of money. In this section we consider a case where both lotteries have random outcomes, but one is riskier than the other. In particular, suppose that Option A pays either $40 or $32, each with probability one half, and that Option B pays either $77 or $2, each with probability one half. First, we calculate the expected values:
Option A: \(0.5(40) + 0.5(32) = 20 + 16 = 36.00\)

Option B: \(0.5(77) + 0.5(2) = 38.50 + 1 = 39.50\).

In this case, option B has a higher expected value, by $3.50, but a lot more risk since the payoff spread from $77 to $2 is almost ten times as large as the spread from $32 to $40.

This choice was on a menu of choices used in an experiment (Holt and Laury, 2001). There were about 200 participants from several universities, including undergraduates, about 80 MBA students and about 20 business school faculty. Even though Option B had a higher expected payoff, eighty-four percent of the participants selected the safe option, which indicates some risk aversion.

The payoff probabilities in this experiment were determined by the throw of a ten-sided die. This allowed the researchers to alter the probability of the high payoff in one-tenth increments. A part of the menu of choices is shown in Table 4.1, where the probability of the high payoff ($40 or $77) is one tenth in Decision 1, four tenths in Decision 4, etc. Notice that Decision 10 is a kind of rationality check, where the probability of the high payoff is 1, so it is a choice between $40 for sure and $77 for sure. The subjects indicated a preference for all ten decisions, and then one decision was selected at random, *ex post*, to determine which decision would be relevant. After all decisions were made, we threw a 10-
sided die to determine which decision is relevant for that subject, and then we threw the ten-sided die again to determine the subject’s earnings for the selected decision. This procedure has the advantage of providing data on all ten decisions without any “wealth effects.” Such wealth effects could come into play, for example, if a person wins $77 on one decision and this makes them more willing to take a risk on the subsequent decision. The disadvantage is that incentives are diluted, which was compensated for by raising payoffs.

Table 4.2. Optimal Decisions for Risk Neutrality and Risk Aversion

<table>
<thead>
<tr>
<th>Probability of High Payoff</th>
<th>Risk Neutrality Expected Payoffs</th>
<th>Risk Aversion ((r = 0.5)) Expected Utilities</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Safe ($40) or $32)</td>
<td>Safe ($40) or $32)</td>
</tr>
<tr>
<td>0.1</td>
<td>$32.80</td>
<td>5.72</td>
</tr>
<tr>
<td>0.2</td>
<td>$33.60</td>
<td>5.79</td>
</tr>
<tr>
<td>0.3</td>
<td>$34.40</td>
<td>5.86</td>
</tr>
<tr>
<td>0.4</td>
<td>$35.20</td>
<td>5.92</td>
</tr>
<tr>
<td>0.5</td>
<td>$36.00</td>
<td>5.99</td>
</tr>
<tr>
<td>0.6</td>
<td>$36.80</td>
<td>6.06</td>
</tr>
<tr>
<td>0.7</td>
<td>$37.60</td>
<td>6.12</td>
</tr>
<tr>
<td>0.8</td>
<td>$38.40</td>
<td>6.19</td>
</tr>
<tr>
<td>0.9</td>
<td>$39.20</td>
<td>6.26</td>
</tr>
<tr>
<td>1.0</td>
<td>$40.00</td>
<td>6.32</td>
</tr>
</tbody>
</table>

The expected payoffs associated with each possible choice are shown under “Risk Neutrality” in the second and third columns of the Table 4.2. First, look in the fifth row where the probabilities are 0.5. This is the choice previously discussed in this chapter, with expected values of \$36 and \$39.50 as calculated above. The other expected values are calculated in the same manner, by multiplying probabilities by the associated payoffs, and adding up these products. The first choice is obvious, since the safe decision also has a higher expected value, or \$32.80 (versus \$9.50 for the risky lottery). In fact, 98 percent of the subjects selected the safe lottery (Option A) in this choice. Similarly, the last choice is between sure amounts of money, and all subjects chose Option B in this case. The expected payoffs are higher for the top four choices, as shown by the bolded numbers. Thus a risk neutral person, who by definition only cares about expected values regardless of risk, would make 4 safe choices in this menu. In fact, the average number of safe choices was 6, not 4, which indicates some risk aversion. As can be seen from the 0.6 row of the table, the typical safe choice for this option involves giving up about \$10 in expected value in order to reduce the risk. About two-thirds of the people made the safe choice for this decision, and 40 percent chose safe in Decision 7.
The percentages of safe choices are shown by the thick solid line in Figure 4.1, where the decision number is listed on the horizontal axis. The behavior predicted for a risk-neutral person is represented by the dashed line, which stays at the top (100 percent safe choices) for the first four decisions, and then shifts to the bottom (no safe choices) for the last six decisions. The thick line representing actual behavior is generally above this dashed line, which indicated the tendency to make more safe choices. There is some randomness in actual choices, however, so that choice percentages do not quite get to 100 percent on the left side.

![Graph of safe choices](image)

**Figure 4.2. Percentages of Safe Choices with Real Incentives and Hypothetical Incentives**

This real-choice experiment was preceded by a hypothetical choice task, in which subjects made the same ten choices with the understanding that they would not be paid their earnings for that part. The data averages for the hypothetical choices are plotted as points on the thin line in the figure. Several differences between the real and hypothetical choice data are clear. The thin line lies below the thick line, indicating less risk aversion when choices have no real impact. The average number of safe choices was about 6 with real payments, about 5 with hypothetical payments. Even without payments, subjects were a little risk averse as compared with the risk-neutral prediction of 4 safe choices, but they could not imagine how they really would behave when they had to face real consequences. Second, with hypothetical incentives, there may be a tendency for people to think less carefully, which may produce “noise” in the data. In
particular, for decision 10, the thin line is a little higher, corresponding to the fact that 2 percent of the people chose the sure $40 over the sure (but hypothetical) $77.

Any increased noise with hypothetical incentives is not surprising, but the shift in decisions is more interesting. This hypothetical bias was also observed to an even greater extent in two other treatments with much higher payoffs (the high prize was $192.50 in one treatment and $346.50 in the other). The only treatment where there was no hypothetical bias was when payoffs were 1/20 as high, with a high payoff of $3.85 (as in the instructions contained in the Appendix to this chapter). With low hypothetical payoffs of several dollars, people were better able to anticipate how they would behave with real payoffs, which is intuitive since low payoffs are “closer” to being hypothetical.

The issue of whether or not to pay subjects is one of the key issues that divides research in experimental economics from some work on similar issues by psychologists (see Hertwig and Ortmann, 2001, for a provocative survey on practices in psychology, with about 30 comments and an authors’ reply). One justification for using high hypothetical payoffs is realism. Two prominent psychologists, Kahneman and Tversky, justify the use of hypothetical incentives:

Experimental studies typically involve contrived gambles for small stakes, and a large number of repetitions of very similar problems. These features of laboratory gambling complicate the interpretation of the results and restrict their generality. By default, the method of hypothetical choices emerges as the simplest procedure by which a large number of theoretical questions can be investigated. The use of the method relies on the assumption that people often know how they would behave in actual situations of choice, and on the further assumption that the subjects have no special reason to disguise their true preferences. (Kahneman and Tversky, 1979, p. 265)

Of course, there are many documented cases where hypothetical and real-incentive choices coincide quite closely. But in the absence of a widely accepted theory about when they do and do not coincide, it is dangerous to assume that real incentives are not needed. In particular, the lottery choice data summarized in this chapter indicate the fallacy of observing that low real payoffs and hypothetical payoffs yield similar decisions, and then concluding that these patterns will be unchanged with high real payoffs.

III. Risk Aversion and Incentive Effects

The intuitive effect of risk aversion is to diminish the utility associated with higher earnings levels, as can be seen from the curvature for the “square
root” utility function in Figure 4.1. With nonlinear utility, the calculation of a person’s expected utility is analogous to the calculation of expected money value. For example, the safe option A for Decision 5 is a 1/2 of $40 and a 1/2 chance of $32. The expected payoff is found by adding up the products of money prize amounts and the associated probabilities:

$$
\text{Expected payoff} = 0.5(40) + 0.5(32) = 20 + 16 = 36.
$$

The expected utility of this option is obtained by replacing the money amounts, $40 and $32, with the utilities of these amounts, which we will denote by $U(40)$ and $U(32)$. If the utility function is the square-root function, then $U(40) = (40)^{1/2} = 6.32$ and $U(32) = (32)^{1/2} = 5.66$. Since each prize is equally likely, we take the average of the two utilities:

$$
\text{Expected utility} = 0.5\ U(40) + 0.5\ U(32) = 0.5(6.32) + 0.5(5.66) = 5.99.
$$

This is the expected utility for the safe option when the probabilities are 0.5, as shown in the 0.5 row of Table 4.2 in the column under “Risk Aversion” for the Safe Option. The other expected utilities for all ten decisions are shown in the two right-hand columns of the table. The Safe Option has the higher expected utility for the top six decisions, so the theoretical prediction for someone with this utility function is that they choose six safe options before crossing over to the risky option. Recall that the analogous prediction for risk neutrality (see the left side of the table) is 4 safe choices, and that the data exhibit 6 safe choices on average. In this sense, the square-root utility function provides a better fit to the data than the linear utility function that corresponds to risk neutrality.

Since this is a setting where the payoff scale matters, the results of classroom experiments can be a little misleading. Nevertheless, there is typically some risk aversion if even a small amount is being paid. Table 4.3 shows the results of two classroom Veconlab experiments, an experimental economics class of 20 and a game theory class of 29, which can be compared with the Holt and Laury research experiments. The columns correspond to payoff scale levels, where the payoffs for the treatment in the “20x” column match those in Table 4.1, with a high payoff of $77. In the “1x” column, the high payoff was $3.85, and in the “90x” column the high payoff was $346.50! The top row shows the average number of safe choices when money payments were actually made. Notice that there are more safe choices as the payoffs are scaled up, so risk aversion is higher at these high payoff levels. While we say that an $r$ value of 0.5 works well for the 20x treatment, we would only need about a 0.3 level of $r$ to explain average behavior in the low payoff (1x) treatment, but an $r$ value of about 0.9 would be needed to predict well for the 90x treatment.
Table 4.3. Average Numbers of Safe Choices: Differing Payment Conditions

<table>
<thead>
<tr>
<th>Subjects</th>
<th>Payoffs</th>
<th>1x</th>
<th>20x</th>
<th>50x</th>
<th>90x</th>
</tr>
</thead>
<tbody>
<tr>
<td>Holt and Laury (2001)</td>
<td>Full cash for 1 of 10 decisions</td>
<td>5.2</td>
<td>6.0</td>
<td>6.8</td>
<td>7.2</td>
</tr>
<tr>
<td>Holt and Laury (2001)</td>
<td>Hypothetical</td>
<td>4.9</td>
<td>5.1</td>
<td>5.3</td>
<td></td>
</tr>
<tr>
<td>W&amp;M Game Theory (Fall 2001)</td>
<td>5% for randomly selected person</td>
<td>5.5</td>
<td></td>
<td>6.0</td>
<td></td>
</tr>
<tr>
<td>UVA Experimental Economics</td>
<td>5% for randomly selected person</td>
<td>4.7</td>
<td></td>
<td>6.1</td>
<td></td>
</tr>
</tbody>
</table>

The second row of the table shows that hypothetical choices did not respond to payoff scale effects, and hence are subject to “hypothetical bias.” The payment method in the two classroom experiments was to select one person randomly ex post and pay a small percentage of earnings. Even with these weak incentives, there are some payoff scale effects, although the right-hand column indicates that these effects are not as strong as those observed in the Holt and Laury research experiments with full cash payments.

One interesting question is whether there are systematic demographic effects. The subjects in the Holt and Laury experiment included about 60 MBA students, about 30 business school faculty (and a Dean), and over a hundred undergraduates from three universities (University of Miami, Central Florida University, and Georgia State University). There is a slight tendency for people with higher incomes to be less risk averse. The women were more risk averse than men in the low-payoff (1x) condition, as observed in some previous studies, but this effect disappeared for higher payoff scales. All of that bravado went away with high stakes. There was no white/non-white difference, but Hispanics in the sample were a little less risk averse. It is not appropriate to make broad inferences about demographic effects from a single study. For example, the Hispanic effect could be an artifact of the fact that most Hispanic subjects were M.B.A. students in Miami, many of whom are from families that immigrated from Cuba.

**IV. Extensions**

There are, of course, other possible values for the risk aversion coefficient of the power function, which correspond to more or less curvature. And there are functions with the curvature of Figure 4.1 that are not power functions, e.g. the natural logarithm. Finding the function that provides the best fit to the data involves a statistical analysis based on specifying an “error” term that incorporates random variations due to emotions, perceptions, interpersonal
differences, etc. Various functional forms for the utility function will be discussed in later chapters.

Another issue that is being deferred is whether utility should be a function of final wealth or of gains from the current wealth position. Here we treat utility as a function of gains only. There is some experimental and theoretical evidence for this perspective that will also be discussed in a subsequent chapter.

Questions

1. For the square root utility function, find the expected utility of the risky lottery for Decision 5, and check your answer with the appropriate entry in Table 4.2.

2. Consider the quadratic utility function $U(x) = x^2$. Sketch the shape of this function in a figure analogous to Figure 4.1. Does this function exhibit increasing or decreasing marginal utility? Is this shape indicative of risk aversion or risk seeking behavior? Calculate the expected utilities for the safe and risky options in decision 4 for Table 4.1, i.e. when the probabilities are equal to 0.4 and 0.6.

3. Suppose that a deck with all face cards (Ace, King, Queen, and Jack) removed is used to determine a money payoff, e.g. a draw of a 2 of Clubs would pay $2, a draw of a 10 of Diamonds would pay $10, etc. Write down the nine possible money payoffs and the probability associated with each. What is the expected value of a single draw from the deck, assuming that it has been well shuffled? How much money would you pay to play this game?
Chapter 5. Randomized Strategies

In many situations there is a strategic advantage associated with being unpredictable, much as a tennis player does not always lob in response to an opponent’s charge to the net. This chapter discusses randomized strategies in the context of simple matrix games (Matching Pennies and Battle of Sexes). The associated class experiments can be run using the instructions in the Appendix or with the Veconlab software (select the MG experiment from the Game Theory menu).

I. Symmetric Matching Pennies Games

In a matching pennies game, each person uncovers a penny showing either Heads or Tails. By prior agreement, one person can take both coins if the pennies match (two Heads or two Tails), and the other can take the coins if the pennies do not match. In this case, a person cannot afford to have a reputation of always choosing Heads, or of always choosing Tails, because being predictable will result in a loss every time. Intuition suggests that each person play Heads half of the time.

A similar situation may arise in a soccer penalty kick, where the goalie has to dive to one side or the other, and the kicker has to kick to one side or the other. The goalie wants a “match” and the kicker wants a “mismatch.” Again, any tendency to go in one direction more often than another can be exploited by the other player. If there are no asymmetries in kicking and diving ability for one side versus the other, then each direction should be selected about half of the time.

It is easy to imagine economic situations where people would not like to be predictable. For example, a lazy manager only wants to prepare for an audit if such an audit is likely. On the other side, the auditor is rewarded for discovering problems, so the auditor would only want to audit if it is likely that the manager is unprepared. The qualitative structure of the payoffs for this situation may be represented by Table 5.1.

<table>
<thead>
<tr>
<th>Column Player (auditor):</th>
<th>Row Player (manager):</th>
</tr>
</thead>
<tbody>
<tr>
<td>Left (audit)</td>
<td>Right (not audit)</td>
</tr>
<tr>
<td>Top (prepare)</td>
<td>1, –1 ⇒</td>
</tr>
<tr>
<td>Bottom (not prepare)</td>
<td>–1, 1 ↑</td>
</tr>
</tbody>
</table>

Table 5.1. A Matching Pennies Game  
(Row’s payoff, Column’s payoff)
First suppose that the manager expects an audit, so the payoffs on the left side of the table are more likely. Then the manager prefers to prepare for the anticipated audit to obtain the good outcome with a payoff of 1, which is greater than the payoff of −1 for getting caught unprepared. This preferred deviation is represented by the “up” arrow in the lower-left box. Alternatively, if the manager expects no audit (the right side of the table), then the manager prefers not to prepare, which again yields a payoff of 1 (notice the down arrow in the top-right box). Conversely, the auditor prefers to audit when the manager is not prepared, and to skip the audit otherwise.

Given the intuition about being unpredictable, it is not surprising that we typically observe a near-equal split on decisions for the symmetric matching pennies game in Table 5.1 (In laboratory experiments, the payoffs are scaled up, and a fixed payment is added to eliminate the possibility of losses, but the essential structure of the game is unchanged, as will be seen in Chapter 11.) Despite the intuitive nature of “fifty-fifty” splits for the game in Table 5.1, it is useful to see what behavior is stable in the sense of being a Nash equilibrium. Up to now, we have only considered strategies without random elements, but such strategies will not constitute an equilibrium in this game. First consider the Top/Left box in the table, which corresponds to audit/prepare. This would be a Nash equilibrium if neither player has an incentive to change unilaterally, which is not the case since the auditor would not want to audit if the manager is going to prepare. In all cells, the player with the lower payoff would prefer to switch unilaterally.

As mentioned in Chapter 1, Nash (1948) proved that an equilibrium always exists (for games in which each person has a finite number of strategies). Since there is no equilibrium in non-random strategies in the matching pennies game, there must be an equilibrium that involves random play. The earlier discussion of matching pennies already indicated that this equilibrium involves using each strategy half of the time. Think about the “announcement test.” If one person is playing Heads half of the time, then playing Tails will win half of the time; playing Heads will win half of the time, and playing a “fifty-fifty” mix of Heads and Tails will win half of the time. In other words, when one player is playing randomly with probabilities of one half, the other person cannot do any better than using the same probabilities. If each person were to announce that they would use a coin flip to decide which side to play, the other could not do any better than using the coin flip. This is the Nash equilibrium for this game. It is called a “mixed equilibrium” since players use a probabilistic mix of each of their decisions. In contrast, an equilibrium in which no strategies are random is called a “pure strategy” equilibrium, since none of the strategies are probability mixes.

Another perspective on the mixed equilibrium is based on the observation that a person is only willing to choose randomly if no decision is any better than
another. So the row player’s choice probabilities must keep the column player indifferent, and vice versa. The only way one person will be indifferent is if the other is using equal probabilities of Heads and Tails, which is the equilibrium outcome.

Even though the answer is obvious, it is useful to introduce a graphical device that will be useful in more complicated situations. This graph will show each person’s best response to any given beliefs about the other’s decisions. In Figure 5.1, the thick solid line shows the best response for the row player (manager). The horizontal axis represents what Row expects Column to do. These beliefs can be thought of as a probability of Right, going from 0 on the left to 1 on the right. If Column is expected to choose Left, then Row’s best response is to choose Top, so the best response line starts at the top-left part of the figure, as shown by the thick line. If Column is expected to choose Right, then Row should play bottom, so the best response line ends up on the bottom-right side of the graph. The crossover point is where the Column’s probability is exactly 0.5, since Row does better by playing Top whenever Column is more likely to choose Left.

![Figure 5.1. Row’s Best Response to Beliefs about Column’s Decision](image)

A mathematical derivation of the crossover point (where Row is indifferent and willing to cross over) requires that we find the probability of Right for which Row’s expected payoff is exactly equal for each decision. Let $p$ denote
Row’s beliefs about the probability of Right, so \( 1 - p \) is the probability of Left. Recall from the previous chapter that expected payoffs are found by adding up the products of payoffs and associated probabilities. From the top row of Table 5.1, we see that if Row chooses Top, then Row earns 1 with probability \( 1 - p \) and Row earns \(-1\) with probability \( p \), so the expected payoff is:

\[
\text{Row’s Expected Payoff for Top} = 1(1 - p) - 1(p) = 1 - 2p.
\]

Similarly, by playing Bottom, Row earns \(-1\) with probability \( 1 - p \) and 1 with probability \( p \), so the expected payoff is:

\[
\text{Row’s Expected Payoff for Bottom} = -(1 - p) + p = -1 + 2p.
\]

These expected payoffs are equal when:

\[
1 - 2p = -1 + 2p.
\]

Solving, we see that \( p = 2/4 = 0.5 \), which confirms the earlier conclusion that Row is indifferent when Column is choosing each decision with equal probability.

A similar analysis shows that Column is indifferent when Row is using probabilities of one half. Therefore the best response line will “cross over” when Row’s probability of Top is 0.5. To see this graphically, let’s change the interpretation of the axes in Figure 5.1 and let the vertical axis represent Column’s beliefs about what Row will do. And instead of interpreting the horizontal axis as a probability representing Row’s beliefs about Column’s action, let’s now interpret it in terms of Column’s actual best response. With this change, the dashed line that crosses at a height of one half is Column’s best response to beliefs on the vertical axis. If Column thinks Row will play Bottom, then Column wants to play Left, so this line starts in the bottom/left part of the figure. This is because the high payoff of 1 for Column is in the bottom/left part of the payoff table in Table 5.1. There is another 1 for Column in the top/right part of the table, i.e. when Column thinks Row will play Top, then Right is the best response. For this reason, the dashed best response line in Figure 5.1 ends up in the top/right corner.

In a Nash equilibrium, neither player can do better by deviating, so a Nash equilibrium must be on the best response lines for both players. The only intersection of the solid and dashed lines in Figure 5.1 is at probabilities of 0.5 for each player. If both players think the other’s move is like a coin flip, they would be indifferent themselves, and hence willing to decide by a flip of a coin. This equilibrium prediction works well in the symmetric matching pennies game, but we will consider qualifications in a later chapter. The focus here is on calculating
and interpreting the notion of an equilibrium in randomized strategies, not on summarizing all behavioral tendencies in these games.

II. Battle of the Sexes

Thus far, we have considered two types of games with unique Nash equilibria, the prisoner’s dilemma (Chapter 3) and the matching pennies game. The coordination game discussed in Chapter 3 had two equilibria in non-random strategies, one of which was preferred by both players. Next we consider another game with two equilibria in non-random strategies, the payoffs for which are shown in Table 5.2. Think of this as a game where two friends wish to meet in a park, with an East side and a West side. It is obvious from the payoffs that Column wishes to meet on the West side, and Row prefers the East side. But notice the zero payoffs in the miss-matched (West and East) outcomes, i.e. each person would rather be with the other than to be on the preferred side of the park alone. Games with the structure shown in Table 5.2 are generally known as “battle-of-the-sexes” games.

Table 5.2. A Battle-of-Sexes Game  
(Row’s payoff, Column’s payoff)

<table>
<thead>
<tr>
<th></th>
<th>East</th>
<th>West</th>
</tr>
</thead>
<tbody>
<tr>
<td>East</td>
<td>2, 1</td>
<td>0, 0</td>
</tr>
<tr>
<td>West</td>
<td>0, 0</td>
<td>1, 2</td>
</tr>
</tbody>
</table>

Think of what you would do in a repeated situation. Clearly most people would take turns. This is exactly what tends to happen when two people play the game repeatedly in controlled experiments with the same partner. In fact, coordinated switching often arises even when explicit communication is not permitted. Table 5.3 shows a decision sequence for a pair who were matched with each other for 6 periods, using the Vecon software and the payoffs from Table 5.2 with the $2 payoff replaced with $4. There is a “match” on East in the first period, and the Column player then switches to West in period 2, which results in a mismatch and earnings of $0 for both. Row switches to West in period 3, and they alternate in a coordinated manner in each subsequent period, which maximizes their joint earnings. Four of the six pairs alternated in this manner, and the other two pairs settled into a pattern where one earned $4 in each period.
Table 5.3. Alternating Choices in a Battle-of-Sexes Game with Fixed Partners

<table>
<thead>
<tr>
<th>Round</th>
<th>Row</th>
<th>Column</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>East ($4)</td>
<td>East ($0)</td>
</tr>
<tr>
<td>2</td>
<td>West ($1)</td>
<td>East ($4)</td>
</tr>
<tr>
<td>3</td>
<td>East ($4)</td>
<td>West ($1)</td>
</tr>
<tr>
<td>4</td>
<td>East ($4)</td>
<td>East ($4)</td>
</tr>
<tr>
<td>5</td>
<td>East ($4)</td>
<td>East ($1)</td>
</tr>
<tr>
<td>6</td>
<td>East ($4)</td>
<td>East ($1)</td>
</tr>
</tbody>
</table>

The problem is harder if the battle-of-sexes game is played only once, without communication, or if there is repetition with random matchings. Table 5.4 shows the results of a recent Veconlab classroom experiment conducted at the College of William and Mary. There were 30 students located in several different computer labs, with random matchings between those designated as row players and those designated as column players. The table shows the percentage of times that players chose the preferred location (East for Row and West for Column). Intuitively, one would expect that the percentage of preferred-location choices to be above one half, and in fact this percentage converges to 67%. This mix of choices does not correspond to either equilibrium in pure strategies.

Table 5.4. Percentage of Preferred-Location Decisions for the Battle-of-Sexes Game

<table>
<thead>
<tr>
<th>Round</th>
<th>Row Players</th>
<th>Column Players</th>
<th>All Players</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>80</td>
<td>87</td>
<td>83.5</td>
</tr>
<tr>
<td>2</td>
<td>87</td>
<td>93</td>
<td>90</td>
</tr>
<tr>
<td>3</td>
<td>87</td>
<td>60</td>
<td>73.5</td>
</tr>
<tr>
<td>4</td>
<td>67</td>
<td>67</td>
<td>67</td>
</tr>
<tr>
<td>5</td>
<td>67</td>
<td>67</td>
<td>67</td>
</tr>
</tbody>
</table>

The remarkable feature of Table 5.4 is that both types seem to be choosing their preferred location about two-thirds of the time. It is not surprising that this fraction is above one-half, but why two thirds? If you look at the payoff table, you will notice that Row gets either 2 or 0 for playing East, and either 0 or 1 for playing West, so in a loose sense East is more attractive unless Column is expected to play West with high probability. Similarly, from Column’s point of view, West (with payoffs of 0 or 2) is more attractive than East (with payoffs of 1 and 0), unless Row is expected to play East with high probability. This intuition only provides a qualitative prediction, that each person will choose their preferred decision more often than not. The remarkable convergence of the frequency of preferred decisions to 2/3 cries out for some mathematical explanation, especially
considering that each person’s 2 and 1 payoffs add up to 3, and two thirds of sum can only be obtained with the preferred decision.

Instead of looking for mathematical coincidences, let us calculate some expected payoff expressions as was done in the previous section. First consider Row’s perspective when Column is expected to play West with probability \( p \). Consider the top row of the payoff matrix in Table 5.2, where Row thinks the right column is relevant with probability \( p \). If Row chooses East, Row gets 2 when Column plays East (expected with probability 1 – \( p \)) and Row gets 0 when Column plays West (expected with probability \( p \)). When Row chooses West, these payoffs are replaced by 0 and 1. Thus Row’s expected payoffs are:

\[
\text{Row's expected payoff for East} = 2(1- p) + 0(p) = 2 - 2p \\
\text{Row's expected payoff for West} = 0(1- p) + 1(p) = 0 + p.
\]

The expected payoff for East is higher when \( 2 - 2p > p \), or equivalently, when \( p < 2/3 \). Obviously, the expected payoffs are equal when \( p = 2/3 \), and East provides a lower expected payoff when \( p > 1/3 \).

Since Row’s expected payoffs are equal when column’s probability of choosing West is 2/3, Row would not have a preference between the two decisions and would be willing to choose randomly. At this time, you might just guess that since the game looks symmetric, the equilibrium involves each player choosing their preferred decision with probability 2/3. This guess would be correct, as we shall show with Figure 5.1. As before, the solid line represents Row’s best response that we analyzed above. If the horizontal axis represents Row’s beliefs about how likely it is that Column will play West, then Row would want to “go to the top of the graph” (play East) as long as Column’s probability \( p \) is less than 2/3. Row is indifferent when \( p = 2/3 \), and row would want to “go to the bottom” when \( p > 2/3 \).

So far, we have been looking at things from Row’s point of view, but an equilibrium involves both players, so let’s think about column’s decision where the vertical axis now represents Column’s belief about what Row will do. At the top, Row is expected to play East, and Column’s best response is to play East in order to be in the same location as Row. Thus Column’s dashed best response line starts in the upper-left part of the figure. This line also ends up in the lower-right part, since when Row is expected to play West (coming down the vertical axis), Column would want to switch to the West location that is preferred. It can be verified by simple algebra that the switchover point is at 2/3, as shown by the horizontal segment of the dashed line.
Since a Nash equilibrium is a pair of strategies such that each player cannot do better by deviating, each player has to be making a best response to the other’s strategy. In the figure, a Nash equilibrium will be on both Row’s (solid) best-response line and on Column’s (dashed) best-response line. Thus the final step is to look for equilibrium points at the intersections of the best response lines. There are three intersections. The one in the upper-left corner of the figure is where both go East (Row earns 2, Column earns 1). Similarly, the lower-right intersection is where both go West (Row earns 1, Column earns 2). We already found these equilibria by looking at the payoff matrix directly, but the third intersection point in the interior of the figure is new. At this point, players choose their preferred decisions (East for Row and West for Column) with probability 2/3, which is what we see in the data.

The graph shows the equilibria clearly, but it is useful to see how the random strategy equilibrium would be found with only simple algebra, since the graphical approach will not be possible with more players or more decisions.
Step 1. First we need to summarize notation:
\[ p = \text{probability that Column chooses West} \]
\[ q = \text{probability that Row chooses East} \]

Step 2. Calculate expected payoffs for each decision.
- Row’s Expected Payoff for East = \( 2(1-p) + 0(p) = 2 - 2p \)
- Row’s Expected Payoff for West = \( 0(1-p) + 1(p) = 0 + p \)
- Column’s Expected Payoff for West = \( 0(q) + 2(1-q) = 2 - 2q \)
- Column’s Expected Payoff for East = \( 1(q) + 0(1-q) = q + 0 \)

Step 3. Calculate the equilibrium probabilities.
Equate Row’s expected payoffs to determine \( p \).
Equate Column’s expected payoffs to determine \( q \).

We already showed that the first part yields \( p = 2/3 \), and it is straightforward to show that \( q = 2/3 \).

The idea behind these calculations is that, in order to randomize willingly, a person must be indifferent between the decisions, and indifference is found by equating expected payoffs. The tricky part is that equating Row’s expected payoffs pins down Column’s probability, and vice versa.

Extensions
The data in Table 5.4 are atypical in the sense that such sharp convergence to an equilibrium in randomized strategies is not always observed. Often there is a little more bouncing around the predictions, due to noise factors (see Question 7). Remember that each person is seeing a series of other people, so people have different experiences, and hence different beliefs. This raises the issue of how people learn after observing others’ decisions, which will be discussed in a later chapter on Bayesian learning. Second, the games discussed in this chapter are symmetric in some sense; payoff asymmetries may cause biases, as will be discussed in a later chapter. Finally, the battle-of-sexes game discussed here was conducted under very low payoff conditions, with only one person of 30 people being selected ex post to be paid their earnings. High payoffs might cause other factors like risk aversion to become important, especially when there is a lot more variability in the payoffs associated with one decision than with another. If risk aversion is a factor, then the expected payoffs would have to be replaced with expected utility calculations.
Questions

1. Use the expected payoff calculations in Step 2 above to solve for an equilibrium level of \( q \) for the battle-of-sexes game.

2. The appendix to this chapter contains instructions for a game with playing cards.  a) Write out the payoff matrix for round 1 of this game.  b) Find all equilibria in non-random strategies and say what kind of game this is.  c) Find expressions for Row’s expected payoffs, one for each of Row’s decisions.  d) Find expressions for Column’s expected payoffs.  e) Find the equilibrium with randomized strategies, using algebra.  f) Illustrate your answer with a graph.

3. Suppose that the payoffs in Table 5.2 are changed by raising the \( 2 \) payoff to a \( 3 \), for both players. Answer all parts c), d) and e) of question 2 for this game.

4. Find the Nash equilibrium in mixed strategies for the coordination game in Table 3.2, and illustrate your answer with a graph.

5. Graph the best-response lines for the prisoner’s dilemma game in Table 3.1, and indicate why there is a unique Nash equilibrium.

6. The data in the table below were for the battle-of-sexes game shown in Table 5.2, except that the payoffs for East/East were \((4, 1)\) and the payoffs for West/West were \((1, 4)\). The 12 players were randomly matched, and the final 5 periods out of a 10-period sequence are shown. The game was run with an experimental economics class at the University of Virginia. Each “player” consisted of one or two people at the same PC, and one player was selected at random \textit{ex post} to be paid a third of earnings. Calculate the percentage of times that Row players chose East in all 5 periods, the percentage of times that Column players chose West, and the average of these two numbers. Then calculate the mixed-strategy Nash equilibrium.

<table>
<thead>
<tr>
<th>Round</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Row</td>
<td>6 East</td>
<td>3 East, 3 West</td>
<td>5 East, 1 West</td>
<td>5 East, 1 West</td>
<td>6 East</td>
</tr>
<tr>
<td>Column</td>
<td>2 East, 4 West</td>
<td>2 East, 4 West</td>
<td>1 East, 5 West</td>
<td>3 East, 3 West</td>
<td>2 East, 4 West</td>
</tr>
</tbody>
</table>

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Part II. Individual Decision Experiments

The next part of the book contains a series of chapters on games involving decision making when the payoff outcomes cannot be known for sure in advance. Uncertainty about outcomes is represented by probabilities, which can be used to calculate the expected value of some payoff or utility function. As indicated in Chapter 4, a risk-neutral person will choose the decision with the highest expected money value. Non-neutral attitudes towards risk can be represented as the maximization of a nonlinear utility function.

The notion of expected utility became widely accepted in economics after Von Neumann and Morgenstern (1944) provided a set of plausible assumptions or “axioms” which ensure that decisions will correspond to the maximization of the expected value of some utility function. This is, of course, an “as if” claim; you may sometimes see people calculate expected values, but it is rare to see a person actually multiplying out utilities and the associated probabilities. Today, expected utility is widely used in economics (and related areas like finance), despite some well-known situations where behavior is sensitive to biases and “behavioral” factors.

Chapter 6 pertains to the simplest two-way prediction task, e.g. rain or shine, when the underlying probabilities are fixed but unknown in repeated rounds. Each new observation provides more information about the relative likelihood of the two events, and the issue of “probability matching” concerns the relationship between the predictions and the underlying frequency of each event. This topic is also used to introduce some simple learning rules, e.g. “reinforcement learning.” Chapter 7 provides a closer look at exactly how new information is used to revise probabilistic beliefs (Bayes’ rule). Anomalies in lottery choice situations are discussed in Chapter 8, e.g. the “Allais paradox.” All decision problems considered up this point will have been “static,” i.e. without any time-dependent elements. The ninth chapter is organized around a dynamic situation involving costly search (e.g., for a high wage or a low price). Chapter 10 also pertains to a sequence of decisions, but in this case, they are made by different people. Those later in the sequence are able to learn from others’ earlier decisions. This raises the possibility of a type of bandwagon effect, which is known as an “information cascade.”
Chapter 6. Probability Matching

Perhaps the simplest prediction problem involves guessing which of two random events will occur. The probabilities of the two events are fixed but not known, so a person can learn about these probabilities by observing the relative frequencies of the two events. One of the earliest biases recorded in the psychology literature was the tendency for individuals to predict each event with a frequency that approximately matches the fraction of times that the event is observed. This bias, known as “probability matching,” was widely accepted as being evidence of irrationality until Siegel’s experiments done in the 1960’s. These experiments provide an important methodological lesson for how experiments should be conducted. The results are also used to begin a discussion of learning models that may explain paths of adjustment to some “steady state” where systematic changes in behavior have ceased. Binary prediction tasks can be run by hand (with instructions in the Appendix) or using the Veconlab program (PM).

I. Being Treated Like a Rat

Before the days of computers, the procedures for binary prediction tasks in psychology experiments sometimes seemed like a setup for rats or pigeons that had been scaled up for human subjects. I will describe the setup used by Siegel and Goldstein (1959). The subject was seated at a desk with a screen that separated the working area from the experimenter on the other side. The screen contained two light bulbs, one on the left and one on the right, and a third, smaller light in the center that was used to signal that the next decision must be made. When the signal light went on, the subject recorded a prediction by pressing one of two levers (left and right). Then one of the lights was illuminated, and the subject would receive reinforcement (if any) based on whether or not the prediction was correct. When the next trial was ready, the signal light would come on, and the process would be repeated, perhaps hundreds of times. No information was provided about the relative likelihood of the two events (Left and Right), although sometimes people were told how the events were generated, e.g. by using a printed list. In fact, one of the events would be set to occur more often, e.g. 75 percent of the time. The process generating these events was not always random, e.g. sometimes the events were rigged so that in each block of 20 trials, exactly 15 would result in the more likely event.

By the time of the Siegel and Goldstein experiments in the late 1950’s, psychologists had already been running such experiments for over twenty years. The results seemed to indicate a curious pattern: the proportion of times that subjects predicted each event roughly matched the frequency with which the events occurred. For example, if Left occurred three-fourths of the time, then
subjects would come to learn this by experience and then would tend to predict left three-fourths of the time.

II. What is Rational Behavior in This Task?

The astute reader may have already figured out what is the best thing to do in such a situation, but a formal analysis will help ensure that such a conclusion does not rely on hidden assumptions like risk neutrality. In order to evaluate the rationality of this matching behavior, let us assume that the events really were independent random realizations, and that the more likely event occurred with probability $p > 1/2$. Let $U_C$ denote the utility of the reward for a correct prediction, and let $U_I$ denote the utility of the reward for an incorrect prediction. The rewards could be “external” (money payments, food), “internal” (psychological self-reinforcement), or some combination. The only assumption is that there is some preference for making a correct prediction: $U_C > U_I$. These utilities may even be changing over time, depending on the rewards received thus far; the only assumption is that an additional correct prediction is preferred.

Once a number of trials have passed, the person will have figured out which event is more likely, so let $p$ denote the subjective probability that represents those beliefs, with $p > 1/2$. There are two decisions: predict the more likely event and predict the less likely event. Each decision yields a lottery:

\[
\begin{align*}
\text{Predict more likely event:} & & U_C \text{ with probability } p \\
& & U_I \text{ with probability } 1-p \\
\text{Predict less likely event:} & & U_I \text{ with probability } p \\
& & U_C \text{ with probability } 1-p
\end{align*}
\]

Thus the expected utility for predicting the more likely event is higher if

\[
pU_C + (1-p)U_I > pU_I + (1-p)U_C,
\]

or equivalently,

\[
(2p-1)U_C > (2p-1)U_I,
\]

which is always the case since $p > 1/2$ and $U_C > U_I$. Although animals may become satiated with food pellets and other physical rewards, economists have no trouble with a non-satiation assumption for money rewards. Note that this argument does not depend on any assumption about risk attitudes. The implication is that the more likely event should always be predicted, i.e. with
probability 1. In this sense, probability matching is irrational as long as there is no satiation, so \( U_C > U_1 \).

III. Siegel and Goldstein’s Experiments

Sidney Siegel is perhaps the psychologist who has had the largest impact on experiments in economics. His early work provides a high standard of careful reporting and procedures, appropriate statistical techniques, and the use of financial incentives where appropriate. His experiments on probability matching are a good example of this work. In one experiment, 36 male Penn State students were allowed to make predictions for 100 trials, and then 12 of these were brought back on a later day to make predictions in 200 more trials. The proportions of predictions for the more likely event are graphed in Figure 6.1, with each point being the average over 20 trials.

![Figure 6.1. Prediction Proportions for the Event with Frequency 0.75](source: Siegel, Siegel, and Andrews (1964) for Dark Lines and Holt (1992) for Light Lines)

The 12 subjects in the “no-pay” treatment were simply told to “do your best” to predict which light bulb would be illuminated. These averages are plotted as the heavy dashed line, which begins at about 0.5 as would be expected
in early trials with no information about which event is more likely. Notice that
the proportion of predictions for the more likely event converges to the level of
0.75 (shown by a horizontal line on the right) predicted by probability matching,
with a leveling off at about trial 100.

In the “pay-loss” treatment, 12 participants received 5 cents for each
correct prediction, and they lost 5 cents for each incorrect decision. The 20-trial
averages are plotted as the dark solid line in the figure. Notice that the line
converges to a level of about 0.9, as shown by the upper horizontal line on the
right. A third “pay” treatment offered a 5-cent reward but no loss for an incorrect
prediction, and the results (not shown) are in between the other two treatments,
and clearly above 0.75. Clearly, incentives matter, and probability matching is
not observed with incentives in this context.

It would be misleading to conclude that incentives always matter, or that
probability matching will be observed in no-pay treatments using different
procedures. The two thin lines in Figure 6.1 show 20-trial averages for an
experiment (Holt, 1992) in which 6 University of Virginia students in no-pay and
pay treatments made decisions using computers, where the events were
determined by a random number generator. The instructions matched those
reported in Siegel, Siegel, and Andrews (1964), except that colored boxes on the
screen were used instead of light bulbs. Notice that probability matching is not
observed in either the pay/loss treatment (with a reward of 10 cents and a penalty
of 10 cents, shown by the thin solid line) or the no-pay treatment (thin dashed
line). The results for a third treatment with a 20-cent reward and no penalty (not
shown) were similar. The reason that matching was not observed in the Holt no-
pay treatment is unclear. One conjecture is that the computer interface makes the
situation more anonymous and less like a matching pennies game. In the Siegel
setup with the experimenter on one side of the screen, the subject might
incorrectly perceive the situation as having some aspects of a game against the
experimenter. Recall that the equilibrium in a matching pennies game involves
equal probabilities for each decision. This might explain the lower choice
percentages for the more likely event reported in the non-computerized setup, but
this is only a guess.

IV. A Simple Model of Belief Learning

Although the probability matching bias is not taken seriously these days,
at least by economists, the experiments provide a useful data set for the study of
learning behavior. Given the symmetry of the problem, a person’s initial beliefs
ought to be that each event is equally likely, but the first observation should raise
the probability associated with the event that was just observed. One way to
model this learning process is to let initial beliefs for the probability of events L
and R be calculated as:
where $\alpha$ is a positive parameter to be explained below. Of course, $\alpha$ has no role yet, since both of the above probabilities are equal to 1/2.

If event $L$ is observed, then $Pr(L)$ should increase, so let us add 1 to the numerator for $Pr(L)$. To make the two probabilities sum to 1, we must add 1 to the denominators for each probability expression:

(6.2) \[ Pr(L) = \frac{\alpha + 1}{\alpha + 1 + \alpha} \quad \text{and} \quad Pr(R) = \frac{\alpha}{\alpha + 1 + \alpha} \quad \text{(after observing L).} \]

Note that $\alpha$ determines how quickly the probabilities respond to the new information; a large value of $\alpha$ will keep these probabilities close to 1/2. Continuing to add 1 to the numerator of the probability for the event just observed, and to add 1 to the denominators, we have a formula for the probabilities after $N_L$ observations of event $L$ and $N_R$ observations of event $R$. Let $N$ be the total number of observations to date. Then the resulting probabilities are:

(6.3) \[ Pr(L) = \frac{\alpha + N_L}{2\alpha + N} \quad \text{and} \quad Pr(R) = \frac{\alpha + N_R}{2\alpha + N} \quad \text{(after N observations).} \]

where $N = N_L + N_R$.

In the early periods, the totals, $N_L$ and $N_R$, might switch in terms of which one is higher, but the more likely event will soon dominate, and therefore $Pr(L)$ will be greater than 1/2. The prediction of this learning model (and the earlier analysis of perfect rationality) is that all people will eventually start to predict the more likely event every time. Any “unexpected” prediction switches would have to be explained by adding some randomness to the decision making, or by adding “recency effects” which make probability assessments more sensitive to the most recently observed outcomes. These kinds of modeling changes will be deferred until a later chapter. The point here is that a simple and intuitive learning model can be constructed, and that this model can explain a high proportion of predictions of the more likely event.

V. Reinforcement Learning

In a psychology experiment, the rewards and punishments are referred to as “reinforcements.” One prominent theory of learning associates changes in behavior to the reinforcements actually received. For example, suppose that the
person earns a reinforcement of \(x\) for each correct prediction, nothing otherwise. If one predicts event \(L\) and is correct, then the probability of choosing \(L\) should increase, and the extent of the behavioral change may depend on the size of the reinforcement. One way to model this is to let the choice probability be:

\[
Pr(\text{choose } L) = \frac{\alpha + x}{\alpha + x + \alpha} \quad \text{and} \quad Pr(\text{choose } R) = \frac{\alpha}{\alpha + x + \alpha}.
\]

Despite the similarity with equation (6.2), there are two important differences. The left side of (6.4) is a choice probability, not a probability that represents beliefs. With reinforcement learning, beliefs are not explicitly modeled, as is the case for the “belief learning” models of the type discussed in the previous section. The second difference is that the \(x\) in (6.4) represents a reinforcement, not an integer count as in (6.2).

Of course, reinforcement is a broad term, which can include both physical things like food pellets given to rats as well as psychological feelings associated with success or failure. One way that this model is implemented for experiments with money payments is to make the simplifying assumption that reinforcement is measured by earnings. Suppose that event \(L\) has been predicted \(N_L\) times and that the predictions have sometimes been correct and sometimes not. Then the total earnings for predicting \(L\), denoted \(e_L\), would be less than \(xN_L\). Similarly, let \(e_R\) be the total earnings from the correct \(R\) predictions. The choice probabilities would then be:

\[
(6.5) \quad Pr(\text{choose } L) = \frac{\alpha + e_L}{2\alpha + e_L + e_R} \quad \text{and} \quad Pr(\text{choose } R) = \frac{\alpha + e_R}{2\alpha + e_L + e_R}.
\]

Notice that the \(\alpha\) parameters again have the role of determining how quickly learning responds to the stimulus, which is the money reinforcement in this case.

This kind of model might also explain some aspects of behavior in probability matching experiments with financial incentives. The choice probabilities would be equal initially, but a prediction of the more likely event will be correct 75% of the time, and the resulting asymmetries in reinforcement would tend to raise prediction probabilities for that event, and the total earnings for this event would tend to be much larger than for the other event. If \(L\) is the more likely event, then \(e_L\) would be growing faster, so that \(e_R/\alpha e_L\) would tend to get smaller as \(e_L\) gets larger. Thus the probability of choosing \(L\) in (6.5) would tend to converge to 1.
VI. Extensions

Both of the learning models discussed here are somewhat simple, which is part of their appeal. The reinforcement model builds in some randomness in behavior and has the appealing feature that incentives matter. But it has less of a cognitive element; there is no reinforcement for decisions not made. For example, suppose that a person chooses L three times in a row (by chance) and is wrong each time. Since no reinforcement is received, the choice probabilities stay at 0.5, which seems like an unreasonable prediction. Obviously, people learn something in the absence of previously received reinforcement, since they realize that making a good decision may result in higher earnings in the next round. Camerer and Ho (1999) have developed a generalization of reinforcement learning that contains some elements of belief learning. Roughly speaking, outcomes that are observed received partial reinforcement even if nothing is earned.

The belief-learning model in section IV can be derived from statistical principles (Bayes’s rule to be discussed in the next chapter). Then beliefs determine the expected payoffs (or utilities) for each decision, which in turn determine the decisions made. In theory, the decision with the highest expected payoff is selected with certainty. In an experiment, however, some randomness in decision making might be expected if the expected payoffs for the two decisions are not too different. This randomness may be due to random emotions, calculation errors, selective forgetting of past experience, etc. Building some randomness into models that use belief learning (as in Capra, et al., 1999) is a useful way to go about explaining data from laboratory experiments, as we shall see in later chapters.

These learning models can be enriched in other ways to obtain better predictions of behavior. For example, the sums of event observations in the belief-learning model weigh each observation equally. It may be reasonable to allow for “forgetting” in some contexts, so that the observation of an event like L in the most recent trail may carry more weight than something observed a long time ago. This is done by replacing sums with weighted sums. For example, if event L were observed three times, $N_L$ in (6.2) would be 3, which can be thought of as $1+1+1$. If the most recent observation (listed on the right in this sum) is twice as prominent as the one before it, then the prior event would get a weight of one half, and the one before that would get a weight of one-fourth, etc. These and other enrichments will be discussed in later chapters.
Questions

1. The initial beliefs implied by (6.1) are that each event is equally likely. How might this equation be altered in a situation where a person has some reason to believe that one event is more likely than another, even before any draws are observed?

2. A recent class experiment used the probability matching (PM) Veconlab software with payoffs of $0.20 for each correct prediction, and in-class earnings averaged several dollars. There were 6 teams of 1-2 students, who made predictions for 20 trials only. (Participants were University of Virginia undergraduates in an experimental economics class who had not seen a draft of this chapter, but who had read drafts of the earlier chapters.) The more likely event was programmed to occur with probability 0.75. Calculate the expected earnings per trial for a team that follows perfect probability matching. How much more would a team earn per trial by being perfectly rational after learning which event is more likely?

3. Answer the two parts of question 2 for the case where a correct answer results in a gain of $0.10 and an incorrect answer results in a loss of $0.10.

4. For the gain treatment described in question 2, the more likely event actually occurred with probability 0.77 in the first 20 trials, averaged over all 6 teams. This event was predicted with a frequency of 0.88. Where would a data point representing this average be plotted in Figure 6.1?

5. For the gain/loss treatment described in question 3, the more likely event was observed with a frequency of about 0.75 in the first ten trials and 0.78 in the second 10 trials. Predictions were made by 6 teams of 1-2 students, and their earnings averaged about 25 cents per team. (To cover losses, each team began with a cash balance of $1, as did the 6 other teams in the parallel gains treatment.) In the gain/loss treatment, the more likely event was predicted with a frequency of 0.58 in the first 10 trials and 0.70 in trials 11-20. To what extent do these results provide evidence in support of probability matching?

6. (A 10-sided die is required.) The discussion of long-run tendencies for the learning models was a little loose, since there are random elements in these models. One way to proceed is to simulate the learning processes implied by these models. Consider the reinforcement learning model, with initial choice probabilities of one half each. You can simulate the initial choice by throwing the 10-sided die twice, where the first throw determines the “tens” digit and the second determines the “ones” digit. For example, throws of a 6 and a 2 would determine a 62. If the die is marked with numbers 0, 1, …9, then any integer from 0 to 99 is equally likely. The simulation could proceed by letting the person predict L if the
throw is less than 50, which would occur with probability 0.5. The
determination of the random event, L or R, could be done similarly, with
the outcome being L if the next two throws determine a number that is less
than 75. Given the prediction, the event, and the reward, say 10 cents, you
can use the formula in (6.4) to determine the choice probabilities for the
next round. This whole process can be repeated for a number of rounds,
and then one could start over with a new simulation, which can be thought
of as a simulation of the decision pattern of a second person. Simulations of
this type are easily done with computers, or even with the random number
features of a spreadsheet program. However you choose to generate the
random numbers, your task is to simulate the process for four rounds,
showing the choice probabilities, the actual choice, the event observed,
and the reinforcement for each round.
Chapter 7. Bayes’ Rule

Any careful study of human behavior has to deal with the issue of how people learn from new information. For example, an employer may have beliefs about a prospective employee’s value to the firm based on prior experience with those of similar backgrounds. Then new information is provided by a job interview or a test, and the issue is how to combine initial (“prior”) beliefs with new information to form final (“posterior”) beliefs. This chapter uses a simple ball counting rule-of-thumb or “heuristic” to explain Bayes’s rule, which is a mathematical formula for forming posterior beliefs. Actual behavior will never be perfectly described by a mathematical formula, and some typical patterns of deviation are discussed. The appendix contains instructions for a game in which the participant sees draws of colored balls from one of two cups and has to form an opinion about which of the two cups is being used.

I. Introduction

Suppose that you have just received a test result indicating that you have a rare disease. Unfortunately, the disease is life-threatening, but you have some hope because the test is capable of producing “false positives,” and the disease is rare. Your doctor tells you that the test is fairly accurate, with a false positive rate of only 1 percent. The rate of the disease for those in your socio-economic group is only one per 10,000. What are your chances of having the disease? (Please write down a guess, so that you do not forget.)

Confronted with this problem, most people conclude will that it is more likely than not that the person actually has the disease, but such a guess would be seriously incorrect. The 1 percent false positive rate means that testing 10,000 randomly selected people will generate about 100 positive results (1%), but on average only one person out of 10,000 actually has the disease. Thus the chances of having the disease are only one in a hundred, even after you have tested positive with a test that is correct 99 times out of 100. This example illustrates the dramatic effect of prior information about the “base rate” of some attribute in the population.

This chapter introduces Bayes’ rule, an optimal procedure for using prior information, like a population base rate, together with new information, like a test result. As the incorrect answers to the disease question may suggest, peoples’ decisions and inferences in such situations may be seriously biased. A related issue is the extent to which people are able to correct for potential biases in market situations where the incentives are high and one is able to learn from past
experience. And in some (but not all) situations, those who do not correct for biases lose business to those who do.

When acquiring new information, it is useful to think about the difference between the initial beliefs, the information obtained, and the new beliefs after seeing the information. If the initial (prior) beliefs are very strongly held, then the new (posterior) beliefs are not likely to change very much, unless the information is very good. For example, passing a lie detector test will not eliminate suspicion if the investigator is almost positive that the suspect is guilty. On the other hand, very good information may overwhelm prior beliefs, as would happen with the discovery of DNA evidence that clears a person who has already been convicted. Obviously, the final (posterior) beliefs will depend on the relative quality of the prior information and on the reliability of the test result. Bayes’ rule provides a mathematical way of handling diverse sources of information. The Bayesian perspective is useful because it dictates how to evaluate different sources of information, based on their reliability. This perspective may even be valuable when it is undesirable or illegal to use some prior information, e.g. in the use of demographic information about crime rates in a jury trial. In such cases, knowing how such prior information would be used (by a Bayesian) may make it easier to guard against a bias derived from such information.

II. A Simple Example and a Counting Heuristic (Anderson and Holt, 1997a)

The simplest informational problem is deciding which of two possible situations or “states of nature” is relevant, e.g. guilty or innocent, infected or not, defective or not, etc. To be specific, suppose that there are two cups or “urns” that contain equally sized Amber (a) and Blue (b) marbles, as shown in Table 7.1. Cup A has two Ambers and one Blue, and Cup B has one Amber and two Blues. We will use the flip of a fair coin to choose one of these cups. The cup selection is hidden, so your prior information is that each cup is equally likely. Then you are allowed to see several draws of marbles from the selected cup. Each time a draw is made and shown, it is then returned to the cup, so the draws are “with replacement” from a cup with contents that do not change.

Table 7.1. A Two Cup Example

<table>
<thead>
<tr>
<th>Cup A</th>
<th>Cup B</th>
</tr>
</thead>
<tbody>
<tr>
<td>a, a, b</td>
<td>a, b, b</td>
</tr>
</tbody>
</table>

Suppose the first draw is Amber and you are asked to report the probability that cup A is being used. An answer of 1/2 is disappointingly common, sometimes even among graduate students, since it can be justified by
the argument that each urn was equally likely to be selected. But if each cup was equally likely beforehand, what was learned from the draw? Another commonly reported probability for cup A following an Amber draw is 1/3, since this cup has a one half chance of being used, and if used, there is a 2/3 chance of drawing an Amber. Then we multiply 1/2 and 2/3 to get 1/3. This is clearly wrong, since the cups were equally likely \textit{ex ante}, and the Amber draw is more likely when cup A is used. Another problem with this answer is that analogous reasoning requires the probability of cup B to be 1/2 times 1/3, or 1/6. This yields an inconsistency, since if the probability of cup A is 1/3 and the probability of cup B is 1/6, where does the rest of the probability go? These probabilities (1/3 and 1/6) sum to 1/2, so we should double them (to 2/3 and 1/3), which are the correct probabilities for cups A and B after the draw of one Amber marble. This kind of scaling up of probabilities will be seen later as a part of the mathematical formula for Bayes’ rule.

A close look at where the Amber marbles are in Table 7.1 makes it clear why the probability of cup A being used is 2/3 after the draw of an Amber marble. There are two \textit{a} marbles on the left and one on the right. Why does this suggest that the right answer is 2/3? All six marbles are equally likely to be drawn before the die is thrown to select one of the cups. So, no Amber marble is more likely to be chosen than any other, and 2 of the 3 Amber marbles are in cup A. It follows that the posterior probability of cup A given an Amber draw is 2/3.

The calculations in the previous paragraphs are a special case of Bayes’ rule with equal prior probabilities for each cup. Suppose that the first marble drawn (Amber) is returned to the cup, and the decision maker is told that a second draw is to be made from the same cup originally selected by the throw of the die. Having already seen an Amber, the person’s beliefs before the second draw are that the probability of cup A is 2/3 and the probability of cup B is 1/3, so the person thinks that it is twice as likely that cup A is being used after observing one \textit{a} draw. The next step is to figure out how the previous paragraph’s method of counting balls (when the two cups were initially equally likely) can be modified for the new situation where one cup is twice as likely as the other one. In order to create a situation that corresponds to the new beliefs, we want to somehow get twice as many possible draws as coming from the cup that is twice as likely. Even though the physical number of marbles in each cup has not changed, we can represent these beliefs by thinking of cup A as having twice as many marbles as cup B, \textit{with each marble in either cup having the same chance of being drawn}. These posterior beliefs are represented in Table 7.2, where the proportions of Amber and Blue marbles are the same as they were in cups A and B respectively. Even though the physical number of marbles is unchanged at six, the prior corresponds to a case in which the imagined marbles in Table 7.2 are numbered from one to nine, with one of the nine marbles chosen randomly.
Table 7.2. A Mental Model after One Amber Draw  
(real marbles in bold, imagined marbles not bold)

<table>
<thead>
<tr>
<th>Cup A</th>
<th>Cup B</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a, a, b)</td>
<td>(a, b, b)</td>
</tr>
<tr>
<td>(a, a, b)</td>
<td></td>
</tr>
</tbody>
</table>

When the posterior beliefs after an Amber draw are represented in Table 7.2, it is clear that a Blue on the second draw is equally likely to have come from either urn, since each cup contains two \(b\) marbles. Thus the posterior probability for cup A after a Blue on the second draw is \(1/2\). This is consistent with intuition based on symmetry, since the prior probabilities for each urn were initially \(1/2\), and the draws of an Amber (first) and a Blue (second) are balanced. A mixed sample in the opposite order (Blue first, then Amber) would, of course, have the same effect.

Suppose instead that the two draws were Amber, with the two cups being equally likely to be used \textit{ex ante}. As before, the posterior belief after the first Amber draw can be represented by the cups in Table 7.2. Since four of the five (real or imagined) Amber marbles are in cup A, the posterior probability of cup A after seeing a second Amber draw is \(4/5\). After two Amber draws, cup A is, therefore, four times as likely as cup B, since \(4/5\) is four times as large as \(1/5\). To represent these posterior beliefs in terms of colored marbles that are equally likely to be drawn, we need to have four times as many rows on the Cup A side of Table 7.2 as there are on the Cup B side. Thus we would need to add two imagined more rows of three marbles under cup A in Table 7.2, holding the proportions of Amber and Blue marbles fixed.

Table 7.3. A Mental Model of the Situation  
After Two Amber Draws

<table>
<thead>
<tr>
<th>Cup A</th>
<th>Cup B</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a, a, b)</td>
<td>(a, b, b)</td>
</tr>
<tr>
<td>(a, a, b)</td>
<td>(a, a, b)</td>
</tr>
<tr>
<td>(a, a, b)</td>
<td>(a, a, b)</td>
</tr>
</tbody>
</table>

To test your understanding, you might consider the probability of cup A after seeing two Ambers and a Blue (exercise 1 at the end of the chapter). Up to this
point, the analysis has been intuitive, but it is now time to be a little more analytical.

III. Relating the Counting Heuristic to Bayes' Rule

To make the connection with Bayes’ rule, we will need a little notation. Suppose there are \( N \) marbles in each cup. The marbles will be Amber or Blue, and we will use the letter \( C \) to represent a specific color, so \( C \) can be either Amber or Blue. What we want to know is the probability of cup A given the draw of a marble of color \( C \). When \( C \) is Amber and the contents are shown in Table 7.1, we already know the answer (2/3), but our goal here is to find a general formula for the probability of cup A given a draw of a color \( C \) marble. This probability is denoted by \( \Pr(A|C) \), which reads “the probability of A given C.” This formula should be general enough to allow for different proportions of colored marbles, and for differences in the prior probability of each cup.

Consider \( \Pr(C|A) \), which reverses the order of the A and the C from the order used in the previous paragraph. Note that \( \Pr(C|A) \) is read as “the probability of color \( C \) given cup A.” Thus \( \Pr(C|A) \) denotes the fraction of marbles in cup A that are of color \( C \), where \( C \) is either Amber or Blue. Similarly, \( \Pr(C|B) \) is the fraction of marbles in cup B that are of color \( C \). For example, if there are 10 marbles in cup A and if \( \Pr(C|A) = 0.6 \), then there must be six marbles of color \( C \) in the cup. In general, there are a total of \( \Pr(C|A)N \) marbles of color \( C \) in cup A, and there are \( \Pr(C|B)N \) marbles of color \( C \) in cup B. If each cup is equally likely to be selected, then each of the \( 2N \) marbles in the two cups is equally likely to be drawn \textit{ex ante} (before the cup is selected). Suppose the marble drawn is of color \( C \). The posterior probability that a marble of color \( C \) was drawn from cup A, denoted \( \Pr(A|C) \), is just the ratio of the number of color \( C \) marbles in cup A to the total number of marbles of this color in both cups:

\[
\tag{7.1} \Pr(A|C) = \frac{\text{Number of Color } C \text{ Marbles in Cup } A}{\text{Number of Color } C \text{ Marbles in Both Cups}},
\]

which can be expressed:

\[
\tag{7.2} \Pr(A|C) = \frac{\Pr(C|A)N}{\Pr(C|A)N + \Pr(C|B)N}.
\]

It is worth emphasizing that this formula is only valid for the case of equal prior probabilities and equal numbers of marbles in each urn. Nothing is changed if we divide both numerator and denominator of the right side on (7.2) by \( 2N \), which is the total number of balls in both cups, which yields a formula for calculating the posterior when the priors are 1/2:
(7.3) \[ \Pr(A|C) = \frac{\Pr(C|A)(1/2)}{\Pr(C|A)(1/2) + \Pr(C|B)(1/2)} \] (for priors of 1/2).

A person who has seen one or more draws may not have prior probabilities of 1/2, so this formula must be generalized. This involves replacing the (1/2) terms on the right side of the with the new prior probabilities, denoted Pr(A) and Pr(B). This is Bayes' rule:

(7.4) \[ \Pr(A|C) = \frac{\Pr(C|A) \Pr(A)}{\Pr(C|A) \Pr(A) + \Pr(C|B) \Pr(B)} \] (Bayes' rule).

For the previous example with equal priors, Pr(A) = 1/2, Pr(a|A) = 2/3, and Pr(a|B) = 1/3, so equation (7.4) implies that the posterior following an Amber draw is: Pr(A|a) = (2/6)/(2/6 + 1/6) = 2/3. Similarly, the probability of cup B is calculated: Pr(B|a) = (1/6)/(2/6 + 1/6) = 1/3. Notice that the denominators in both of the previous calculations are 1/2, so dividing by 1/2 scales up the numerator by a factor of 2, which makes the probabilities add up to one.

Having arrived at the Bayes’ rule formula (for two events) as it appears in the textbooks, it is important to point out that the argument was intuitive but not rigorous. It helps, therefore, to relate the general formula for Bayes' rule back to the counting heuristic. Recall that the two cups were initially equally likely, and the first time we saw an a draw, the probability of cup A was 2/3, so the ratio Pr(A)/Pr(B) was 2. Then we took the \(N\) balls in cup A and imagined that there were twice as many, i.e. \(2N\) balls in cup A (\(N\) real balls and \(N\) imagined balls). After seeing two a draws, the probability of Cup A was 4/5, or four times as great as the 1/5 probability of Cup B, so we imagined that there were \(4N\) balls in Cup A instead of the original \(N\) balls. How can this approach be generalized for the case where Pr(A)/Pr(B) is something other than 2 or 4? Obviously, we calculate the ratio Pr(A)/Pr(B) and imagine that there are \(N\) times Pr(A)/Pr(B) balls in Cup A, holding the original proportions constant. To relate the Bayes’ rule formula in (7.4) to this counting heuristic, we need to figure out how to get terms involving \(N\) times Pr(A)/Pr(B) terms into (7.4). This can be done by dividing both numerator and denominator of (7.4) by Pr(B)/N, to obtain:

(7.5) \[ \Pr(A|C) = \frac{\Pr(C|A) \frac{\Pr(A)}{N}}{\Pr(C|A) \frac{\Pr(A)}{N} + \Pr(C|B)\frac{N}{N}}. \]
If there are \( N \) marbles in each cup, this equation shows that the basic Bayes' rule formula is equivalent to imagining that the \( N \) marbles in cup A are increased or decreased to a number \( M \), which is \( N \) times the prior “odds ratio,” \( \Pr(A)/\Pr(B) \). Take the representation in Table 7.2 for example, where the prior for cup A is 2/3 after seeing an \( a \) draw. Then \( \Pr(A) \) is now 2/3, \( \Pr(B) \) is 1/3, and the odds ratio is \((2/3)/(1/3) = 2\). With \( N = 3 \) marbles in each cup, the odds ratio times \( N \) is 2 times 3, or 6, which replaces the odds ratio times \( N \) in the numerator and the left side of the denominator of (7.5). This is as if we imagine that there are 6 marbles in cup A and only 3 in cup B, the 3 corresponding to the \( N \) term on the right side of the denominator in (7.5).

To summarize, if there is a prior probability of 1/2 that each cup is used and if the cups contain equal numbers of colored marbles, then the posterior probabilities can be calculated as ratios of numbers of marbles of the color drawn, as in equation (7.1). If the marble drawn is of color C, then the posterior that the draw was from cup A is the number of color C marbles in cup A divided by the total number of color C marbles in both cups. When the prior probabilities or numbers of marbles in the cups are unequal, then the 1/2 terms in (7.3) are replaced by the prior probabilities, as in Bayes' rule (7.4). Finally, the Bayes' rule formula is equivalent to imagining that the number \( N \) of marbles in cup A is changed by a factor \( \Pr(A)/\Pr(B) \), holding the proportion of color C marbles fixed at \( \Pr(C|A) \), with the counting heuristic that each of the actual and imagined marbles is equally likely to be chosen.

IV. Experimental Results

Nobody would expect that something so noisy as the formation of beliefs would adhere strictly to a mathematical formula, and experiments have been directed towards finding the nature of systematic biases. The disease example mentioned earlier suggests that, in some contexts, people may underweight prior information based on population base rates.

“Base rate bias” was the motivation behind some experiments reported by Kahneman and Tversky (1973), who gave subjects lists of brief descriptions of people who were either lawyers or engineers. The subjects were told that the descriptions had been selected at random from a sample that contained 70 percent lawyers and 30 percent engineers. Subjects were asked to report the chances out of 100 that the description pertained to a lawyer. A second group was given some of the same descriptions, but with the information that the descriptions had been selected from a sample that contained 30 percent lawyers and 70 percent engineers. Respondents had no trouble with descriptions that obviously described one occupation or another, but some were intentionally neutral with phrases like “he is highly motivated” or “will be successful in his career.” The modal response for such neutral descriptions involved probabilities of near a half,
regardless of the respondent’s treatment group. This behavior is insensitive to the prior information about the proportions of each occupation, a type of base rate bias.

Grether (1981) pointed out several potential procedural problems with the Kahneman and Tversky experiment. There was deception to the extent that the descriptions had been made up, and even if people “bought into” the context, they would have no incentive to think about the problem carefully. Moreover, the information conveyed in the descriptions is hard to evaluate in terms of factors that comprise Bayes’ rule formula. In other words, it hard to determine an appropriate guess about the probability of a particular description conditional on the occupation. Grether based his experiments on cups with two types of balls as described. One of the biases that he considered is known as “representativeness bias.” In the two cup example discussed earlier, a sample of three draws that yields two Ambers and one Blue has the same proportions as cup A, and in this sense the sample looks representative of cup A. We saw that the probability of cup A after such a sample would be 2/3, and a person who reports a higher probability, say 80 percent, may be doing so due to representativeness bias.

Notice that a sample of two Ambers and one Blue makes cup A more likely. If you ask someone which cup is more likely, an answer of A cannot distinguish between Bayesian behavior and a strong representativeness bias, which also favors cup A. Grether cleverly got around this problem by introducing some asymmetries which make it possible for representativeness to indicate a cup that is less likely given Bayes’ rule. For example, if you lower the prior probability of cup A to 1/8, then the prior odds ratio on the right side of (7.5) will be: \( \Pr(A)/\Pr(B) = 1/7 \), which will make \( \Pr(A|C) \) lower for each pattern of draws represented by C. In this manner, a sample of two Ambers and one Blue would look like the contents of cup A, but if the prior probability of A is small enough, the Bayesian probability of cup A would be less than one half. In this manner, representativeness and Bayes’ rule would have differing predictions when a person is asked which cup is more likely.

A binary choice question about which cup makes it easy to provide incentives: simply offer a cash prize if the cup actually used turns out to be the one the person said is more likely. This is the procedure that Grether used, with a $15 prize for a correct prediction and a $5 prize otherwise. When representativeness and Bayes’ rule gave different predictions, subjects tended to follow the Bayesian prediction more often than not, but a lot less often than when the two criteria matched, as with the symmetric example discussed in this chapter. When representativeness and Bayesian calculations indicated the same answer, subjects tended to give the correct answer about 80 percent of the time (with some variation depending on the specific sample). This percentage fell to about 60
percent when representativeness and Bayesian calculations suggested different answers.

V. Bayes’ Rule with Elicited Probabilities

Sometimes it is useful to ask subjects to report a probability instead of just saying which event is more likely. This can be phrased as a question about the “chances out of 100 that the cup used is A.” The issue here is how to provide incentives for people to think carefully. The instructions in the appendix provide one approach, which is complicated, but which is based on a simple idea. Suppose that you send your friends to a fruit stand, and they ask you whether you prefer red or yellow tomatoes in case both are available. You would have no incentive to lie about your preference, since telling the truth allows your friends to make the best decision on your behalf. This section describes a method of eliciting probabilities that is based on this intuition. Probability elicitation is useful when we need specific probability numbers to evaluate theoretical predictions.

In particular, suppose that you have seen some draws (say Amber and Blue) and have concluded that the probability of cup A is 0.5. If you are promised a $1,000 payment if cup A is really used, then you essentially have a “cup A lottery” that pays $1,000 with probability one half. The idea is to ask someone for the probability (in chances out of 100) that cup A is used, and to give them the incentive to tell the truth. This incentive will be provided by having the person running the experiment make a choice for the subject. In this context, the experimenter is like the friends going to the fruit stand, the subject needs to tell the truth about their preferences so that the experimenter will make the best choice on the subject’s behalf. In order to set up the right incentives to tell the truth, we will have the experimenter choose between that cup A lottery and another one that is constructed randomly, by using throws of 10-sided dice to get a number, $N$, between 0 and 100. This “dice lottery” pays $1,000 with chances $N$ out of 100. If the probability of Cup A is 1/2, this dice lottery is preferred if $N > 50$, and the cup A lottery is preferred if $N < 50$. The subject should answer that the chances of Cup A are 50 out of 100 so that the experimenter can make the right choice. This is really like the red and yellow tomato example discussed above, the experimenter is choosing between two lotteries on the subject’s behalf, and therefore needs to know the subject’s true value of the cup A lottery to make the right decision.

To convince yourself that the subject is motivated to tell the truth in this situation, consider what might happen otherwise. Suppose that the subject incorrectly reports that the chances of cup A are 75 out of 100, when the person really believes that each cup is equally likely. Thus, the cup A lottery provides a one-half chance of $1,000. If the experimenter then throws a 7 and a 0, then $N =
70, and the dice lottery would yield a 70% chance of $1,000, which is a much better prospect than the 50-50 chance based on the subject’s actual beliefs that each cup is equally likely. But since the subject incorrectly reported that the chances for cup A to be 75 out of 100, the experimenter would reject the dice lottery and base the subject’s earnings on the cup A lottery, which gives a 20 percent lower chance of winning. A symmetric argument can be made for why it is bad to report that the chances of cup A are less than would be indicated by the subjects’ true beliefs (question 3). For a mathematical derivation of the result that it is optimal to reveal one’s true probability, see question 5.

The advantage of direct probability elicitation is that you can often make stronger conclusions when specific numbers are available, as opposed to the qualitative data obtained by asking which event is more likely. The disadvantage is that the elicitation process itself is not perfect in the sense that the measurements may contain more “noise” or measurement error than we get with binary comparisons.

Table 7.4 shows some elicited probabilities for an experiment conducted by the author using University of Virginia subjects, and prize amounts of $1.00 instead of $1,000. The two cups, A and B, each contained three marbles with the contents as shown in Table 7.1. The experiment consisted of three parts. The first part was done largely to acquaint people with the procedures, which are admittedly complicated. Then there were 10 rounds with asymmetric probabilities (a two-thirds chance of using cup A) and 10 rounds with symmetric probabilities (a one-half chance of using cup A). The order of the symmetric and asymmetric treatments was reversed with different groups of people.

The information and decisions for subjects 1 and 2 are shown in the table, for rounds 21-27 where the prior probability was 1/2 for each cup. First consider subject 1 on the left. In round 21, there were no draws, and the None (0.50) in the Draw column indicates that the correct Bayesian probability of cup A is 0.50. The subject’s response in the Elicited Probability column was 0.49, as is also the case for this person in round 26 where the draws, AB, cancelled each other out. The other predictions can be derived using the ball counting heuristic or with Bayes’ rule, as described above. In round 22, for example, there was only one draw, a, and the Bayesian probability of cup A is 0.67, since two of the three a balls from both cups are located in cup A. Subject 1 reports a probability 0.65, which is quite accurate. This person was unusually accurate, with the largest deviation from the theoretical prediction being in round 24, where the reported probability is a little too low, 0.25 versus 0.33.
Subject 2 is considerably less predictable. The \(ab\) and \(ba\) samples, which leave each cup being equally likely, resulted in answers of 0.60 and 0.30. The average of these answers is not far off, but the dispersion is atypically large for such an easy inference task, as compared with others in the sample. This person was fairly accurate with single draws of \(b\) (round 22) and of \(a\) (round 25), but the three-draw samples show behavior that is consistent with “representativeness bias.” In round 27, for example, the sample of \(aab\) looks like cup A, and the elicited probability of 0.80 for cup A is much higher than the actual probability of 0.67. Subject a also seems to fall prey to this bias in round 24, where the sample, \(bab\), looks like cup B.

Figure 7.1 shows some aggregate results for the symmetric and asymmetric treatments, for all 22 subjects in the study. The horizontal axis plots the Bayesian probability of cup A, which varies from 0.05 for a sample of BBBB in the symmetric treatment (with equal priors) to 0.97 for a sample of aaaa in the asymmetric treatment (where the prior probability of cup A was 2/3). The elicited probabilities are shown in the vertical dimension. The 45-degree dotted line shows the Bayes’ prediction, the solid line connects the average of the elicited probabilities, and the dashed line connects the medians.
In the aggregate, Bayes’ rule does quite well. A slight upward bias on the left side of the figure and a slight downward bias on the right side might possibly be due to the fact that there is more “room” for random error in the upward direction on the left and in the downward direction on the right. This conjecture is motivated by the observation that the medians (solid dashed line) are generally closer to Bayesian predictions. For example, one person got confused and reported a probability of 0.01 for cup A after observing draws of aa in the symmetric treatment, which should lead to a posterior of 0.8. (The draws were of light and dark marbles, and were coded on the decision sheet as LL.) On the right side of the figure, there is more “room” for extreme errors in the downward direction. To see this, consider a vertical line above the point 0.8 on the horizontal axis of Figure 7.1. If you imagine some people with no clue who put marks more or less equally spaced on this vertical line, more of the marks will be below the 45-degree dashed line, since there is more room below. Random errors of this type will tend to pull average reported probabilities down below the 45-degree line on the right side of the figure, and the reverse effect (upward bias)
would occur on the left side. Averages are much more sensitive to extreme errors than are medians, which may explain why the averages show more of a deviation from the 45 degree dashed line.

The averages graphed in the figure mask some interesting patterns of deviation. In the symmetric treatment, for example, there are several posterior probability levels that can result from different numbers of draws. A posterior of 0.67 can be achieved by draws of either \( a \) or \( aab \). The average posterior for the \( a \) draw alone is 0.61, which a little too low relative to the Bayes’ prediction of 0.67, and the “representativeness” pattern of \( aab \) is actually closer to the mark at 0.66. Similarly, the posterior of 0.33 can be reached by draw patterns of \( b \) or \( bba \). The \( b \) pattern yielded an average of 0.42, whereas the \( bba \) pattern resulted in a lower average of 0.31. Representativeness would imply that the elicited probability should be lower than what is observed for the \( bba \) sample and higher than what is observed for the \( aab \) sample. It may be after several draws, some people are ignoring the prior information and reporting a probability that matches the sample average, which would be a natural type of heuristic.

VI. Extensions

Although there seem to be some systematic deviations from the predictions of Bayes’ rule, there is no widely accepted alternative model of how information is actually processed by people in these situations. Some biases seem to be due to the use of heuristics, and some of the bias may be due to asymmetries in the way the measurements are made. At present, economists tend to use Bayes’ rule to derive predictions, although there is some renewed interest in non-Bayesian models like that of reinforcement learning discussed in the previous chapter. Finally, it is useful to know that Bayes’ rule can be used to derive the previous chapter’s model of belief learning by making a specific assumption about the nature of the prior beliefs. This derivation is beyond the scope of this book (see DeGroot, 1970), but it is not very important since most people who use these models start making additional changes to the belief learning model that cannot be justified from Bayes’ rule.

Questions

1. What would the mental model with imagined balls in Table 7.3 look like after seeing two Amber draws and one Blue draw? What does this model imply that the posterior probability for cup A would be?

2. Answer question 1 for a sample of three Amber draws, and check your answer using Bayes’ rule. (Hint: the probability of getting three Amber draws from cup A is \((2/3)(2/3)(2/3) = 8/27\). You will also need to calculate the probability of getting three Amber draws from cup B.)
3. Suppose we are using that the elicitation scheme described in section V, and that the subject’s beliefs are that cups A and B are equally likely. Show why it would be bad for the person to report that the chances for cup A are 25 out of 100.

4. In the asymmetric treatment discussed in Section V, the prior probability of cup A is 2/3. Use the ball counting heuristic to calculate the probability of cup A after seeing a sample of: $b$. What is the probability of cup A after a sample of $aaa$?

5. Twenty-one University of Virginia students in an undergraduate Experimental Economics (Spring 2002) participated in the Bayes’ rule elicitation experiment, using the instructions in the Appendix to Chapter 7 (but with only 7 rounds). The experiment lasted for about 20 minutes, and cash payments amounted to about $3-$4 per person. Each cup was equally likely to be used. Participants claimed that they did have time to do mathematical calculations. The draw sequences and the average elicited probabilities are shown in the table below. A) Calculate the Bayes’ posterior for each case, and write it in the relevant box. B) How would you summarize the deviations from Bayesian predictions? Is there evidence for the representativeness bias in this context? Discuss briefly.

<table>
<thead>
<tr>
<th>Draw:</th>
<th>no draws</th>
<th>L</th>
<th>D</th>
<th>DD</th>
<th>LD</th>
<th>LDD</th>
<th>DDD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.5</td>
<td>0.68</td>
<td>0.33</td>
<td>0.24</td>
<td>0.47</td>
<td>0.28</td>
<td>0.13</td>
</tr>
<tr>
<td>Median</td>
<td>0.5</td>
<td>0.67</td>
<td>0.33</td>
<td>0.2</td>
<td>0.50</td>
<td>0.30</td>
<td>0.11</td>
</tr>
</tbody>
</table>

6. (Advanced and Tedious) The incentives for the method of eliciting probabilities discussed in Section V can be evaluated with calculus. Let $P$ denote the person’s true probability of Cup A, i.e. the probability that represents their beliefs after seeing the draws of colored marbles. Let $R$ denote the reported probability. (To simplify notation, both $R$ and $P$ are fractions between 0 and 1, not numbers between 0 and 100 as required for the “chances out of 100” discussion in the text.) a) Explain why the following statement is true for the elicitation mechanism discussed in Section V: “If the person reports $R$, then there is an $R$ probability of ending up with the Cup A lottery, which pays $1 with probability $P$.” b) Explain why the following statement is true: “Similarly, there is a 1-$R$ probability of ending up with the dice lottery, which pays $1 with a probability $N/100$, where $N$ is the outcome of the throw of the ten-sided
die twice.”  c) We know that \( N / 100 \) is greater than \( R \), since the dice lottery is only relevant if its probability of paying $1 is greater then the reported probability of Cup A. Use these observations to express the expected payoff as: \( 1/2 + PR - R^2/2 \).  d) Then show that this quadratic expression is maximized when \( R = P \), i.e. when the reported probability equals the probability that represents the person’s beliefs.  \textit{Hint}: Since the dice lottery is only relevant if \( N/100 > R \) and \( N/100 < 1 \), this dice lottery has an expected value that is halfway between \( R \) and 1, i.e. \( (1+R)/2 \).  \textit{Comment}: Probability elicitation methods of this type are sometimes called “scoring rules.” The particular method being discussed is a “quadratic scoring rule” since the function being maximized, \( 1/2 + PR - R^2/2 \), is quadratic.
Chapter 8. Lottery Choice Anomalies

Choices between lotteries with money payoffs may produce anxiety and other emotional reactions, especially if these choices may result in significant monetary gains and losses. Expected utility theory implies that such choices, even the difficult and stressful ones, can be modeled as the maximization of a mathematical function (a sum of products of probabilities and utilities). As would be expected, actual decisions sometimes deviate from these mathematical predictions, and this chapter begins with one of the more common anomalies: the Allais paradox. Other biases, such as the miss-perception of large and small probabilities are discussed. The web program LC can be used to evaluate the potentially different effects of gains and losses.

I. Introduction

The predominant approach to the study of individual decision making in risky situations is the expected utility model introduced in Chapter 4. Expected utility calculations are sums of probabilities times (possibly nonlinear) utility functions of the prize amounts. In contrast, the probabilities enter linearly, which precludes over-weighting of low probabilities, for example. The nonlinearities in utility permit an explanation of risk aversion. This model, which dates to Bernoulli (1738), received an formal foundation in Von Neumann and Morgensetern’s (1944) book on game theory. This book specified a set of assumptions (“axioms”) that imply behavior consistent with the maximization of the expected value of a utility function.

Almost from the very beginning, economists were concerned that behavior in some situations seemed to contradict the predictions of this model. The most famous contradiction, the Allais paradox, is presented in the next section. Such anomalies have stimulated a lot of work, theoretical and experimental, on developing alternative models of choice under risk. The most commonly mentioned alternative, “Prospect Theory,” is presented and discussed in the sections that follow.

A reader looking for a resolution of the key modeling issues will be disappointed; some progress has been made, but much of the evidence is mixed. The purpose of this chapter is to introduce the issues, and to help the reader interpret seemingly contradictory results obtained with different procedures. For example, it is not uncommon for the estimates of the environmental benefits of some policy to differ by a factor of 2, which may be attributed to the way the questions were asked and to a “willingness-to-pay/willingness-to-accept bias.” A familiarity with this and other biases is crucial for anyone interested in
interpreting the results of experimental and survey studies of situations where the outcomes are unknown in advance.

II. The Allais Paradox

Consider a choice between a sure 3,000 and a 0.8 chance of winning 4,000. This choice can be thought of as a choice between two “lotteries” that yield random earnings:

Table 8.1. A Lottery Choice Experiment (Kahneman and Tversky, 1979)

<table>
<thead>
<tr>
<th>Lottery S (selected by 80%)</th>
<th>Lottery R (selected by 20%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3,000 with probability 1.0</td>
<td>4,000 with probability 0.8</td>
</tr>
<tr>
<td>0 with probability 0.2</td>
<td>0 with probability 0.2</td>
</tr>
</tbody>
</table>

A lottery is an economic item that can be owned, given, bought, or sold. People may prefer some lotteries to others, and economists assume that these preferences can be represented by a utility function, i.e. that there is some mathematical function with an expected value that is higher for the lottery selected than for the lottery not selected. This is not an assumption that people actually think about utility or do such calculations, but rather, that choices can be represented by (and are consistent with) rankings provided by the utility function.

The simplest utility function is the expected money value, which would represent the preferences of someone who is risk neutral. An expected payoff comparison would favor Lottery R, since $0.8(4,000) = 3,200$, which is higher than the 3,000 for the safe option. In this situation, Kahneman and Tversky reported that 80% of the subjects chose the safe option, which indicates some risk aversion. (Payoffs, in Israeli pounds, were hypothetical.) A person who is not neutral to risk would have preferences represented by a utility function with some curvature. The decision of an expected utility maximizer who prefers the safe option could be represented:

\[(8.1)\quad U(3,000) > 0.8U(4,000) + 0.2U(0),\]

where the utility function represents preferences over money income.

Next consider some simple mathematical operations that can be used to obtain a prediction for how such a person would choose in a different situation. For example, suppose that there is a three-quarters chance that all gains from either lottery will be confiscated. To analyze this possibility, multiply both sides of (8.1) by 0.25, to obtain:

\[(8.2)\quad 0.25U(3,000) > 0.2U(4,000) + 0.05U(0).\]
In order to make the probabilities on each side sum to 1, we need to add the $0.75U(0)$ that corresponds to confiscation. This amount is added to both sides of (8.2) to obtain:

$$0.25U(3,000) + 0.75U(0) > 0.2U(4,000) + 0.8U(0).$$

The inequality implies that the same person (who initially preferred Lottery S to Lottery R in Table 8.1) would prefer a one-fourth chance of 3,000 to a one-fifth chance of 4,000. Any reversal of this preference pattern, e.g. preferring the sure 3,000 in the first choice and the 0.2 chance of 4,000 in the second choice would violate expected utility theory. A risk-neutral person, for example, would prefer the lottery with the possibility of winning 4,000 in both cases (we already checked the first case, and the second is assigned in question 1).

The intuition underlying these predictions can be seen by reexamining equation (8.3). The left side is the expected utility of a one-fourth chance of 3,000 and a three-fourths chance 0. Equivalently, we can think of the left side as a one-fourth chance of Lottery S (which gives 3,000) and a three-fourths chance of 0. Although it is not so transparent, the right side of (8.3) can be expressed analogously as a one-fourth chance of Lottery R and a three-fourths chance of 0. To see this, note that Lottery R provides 4,000 with probability 0.8, and one fourth of 0.8 is 0.2, as indicated on the right side of (8.3). Thus the implication of the inequality in (8.3) is that a one-fourth chance of Lottery S is preferred to a one-fourth chance of Lottery R. What the mathematics of expected utility implies is that if you prefer Lottery S to Lottery R as in (8.1), then you prefer a one-fourth chance of Lottery S to a one-fourth chance of Lottery R as in (8.3). The intuition for this prediction is that the “extra” three-fourths chance of 0 that was added to both sides of the equation in going from (8.2) to (8.3) is the same for both sides. This extra probability of 0 dilutes the chances of winning in both Lottery S and Lottery R, but this added probability of 0 is a common, and hence “irrelevant” addition. Indeed, one of the basic axioms used to motivate the use of expected utility is the assumption of “independence with respect to irrelevant alternatives.”

As intuitive as the argument in the previous paragraph may sound, a significant fraction of the Kahneman and Tversky subjects violated this prediction. Eighty percent chose Lottery S over Lottery R, but sixty-five percent chose the diluted version of Lottery R over the diluted version of Lottery S. This behavior is inconsistent with expected utility theory, and is known as an Allais Paradox, named after the French economist, Allais (1953), who first proposed these types of paired lottery choice situations. In particular, this is the “common-ratio” version of the Allais paradox, since probabilities of positive payoffs for both lotteries are diluted by a common ratio. Anomalous behavior in Allais
paradox situations has also been reported for experiments in which the money prizes were paid in cash (e.g., Starmer and Sugden, 1989, 1991). Battalio, Kagel, and Macdonald (1985) even observed similar choice patterns with rats that could choose between levers that provided food pellets on a random basis.

III. Prospect Theory: Probability Misperception

The Allais paradox results stimulated a large number of studies that were intended to develop alternatives to expected utility theory. The alternatives are typically more complicated to use, and none of them show a clear advantage over expected utility in terms of predictive ability. The possible exception to this conclusion is “prospect theory” proposed by Kahneman and Tversky (1979). Prospect theory is really a collection of elements that specify how a person evaluates a risky prospect in relation to some status quo position for the individual. Roughly speaking, there is a reference point, e.g. current wealth, from which gains and losses are evaluated, and gains are treated differently from losses. People are assumed to be averse to losses, but when making choices where payoffs are all losses, they are thought to be risk seeking. On the other hand, when payoffs are all gains, people are assumed to be risk averse in general. A final element is a notion that probabilities are not always correctly perceived, i.e. that low probabilities are over-weighted and high probabilities are under-weighted. These elements can be unbundled and evaluated one at a time, and modifications of expected utility theory that include one or more of these elements can be considered, which is the plan of the remainder of this chapter.

The possibility of probability misperception is suggested by Figure 7.1 in the previous chapter. Recall that this was a graph with a 45-degree line representing the Bayesian prediction from which deviations could be observed. There is a “reverse S-shaped” pattern to the deviations (over-weighting of low probabilities and under-weighting of high probabilities), although the biases there are somewhat small. Prospect theory predicts a probability weighting function which has the general shape observed in Figure 7.1, starting at the origin, rising above the 45-degree line for low probabilities, falling below for high probabilities, and ending up on the 45-degree line in the upper-right corner. The notion that the probability weighting function should cross the 45-degree line at the upper-right corner is based on the intuition that it is difficult to misperceive a probability of one.

To see how prospect theory may explain the Allais paradox, note that Lottery S on the left side of Table 8.1 is a sure thing, so no misperception of probability is possible. The 0.8 chance of a 4,000 payoff on the right, however, may be affected. The largest deviation between Bayesian predictions and average reported probabilities is at a posterior of 0.8 on the horizontal axis in Figure 7.1. Here, the average report was about seven-tenths. Now reconsider
Lottery R on the right side of Table 8.1, when the 0.8 probability of 4,000 is treated as if it were 0.7. This enhances the attractiveness of Lottery S, so that even a risk neutral person would prefer S (exercise 2). Next consider the choice that results when both lotteries are diluted by a three-fourths chance of a 0 payoff. It is apparent from (8.3) that the 0.25 probability of the 3,000 gain on the left is about the same as the 0.2 probability of the 4,000 on the right. In other words, 0.25 and 0.2 are located close to each other, so that a smooth probability weighting function will overweight them more or less by the same amount. Here a probability weighting function will have little effect, and a risk-neutral person will prefer the diluted version of Lottery R, even though the same person might prefer the undiluted version of Lottery S when the high probability of the 4,000 payoff for lottery R is under-weighted. Put differently, the certain payoff in the original Lottery S is not misperceived, but the 0.8 probability of 4,000 in Lottery R is under-weighted. So probability weighting tends to bias choices towards S. But the diluted S and R lotteries have probabilities in the same range (0.25 and 0.2) so no such bias would be introduced by a “smooth” probability weighting function.

This explanation of the Allais paradox is plausible, but it is not universally accepted. First, the evidence on probability weighting functions is mixed. Recall that the “reverse-S” pattern of deviations in Figure 7.1 could also be due to the tendency for random errors in the elicitation process to spread out on the side where there is “more room” for error. Support for this error-based explanation is provided in a recent paper by Eckel and Goeree (2002). They report data averages that suggest over-weighting low probabilities and under-weighting of high probabilities, but then estimate that there is no systematic misperception of probabilities when an explicit analysis of random error is introduced. A number of other recent studies have also failed to find the “reverse-S” pattern of probability misperceptions (Goeree, Holt, and Palfrey, 2000, 2001). Harbaugh, Krause, and Vesterlund (2002) found this “reverse S” pattern when probabilities were elicited by asking for subjects to assign prices to the lotteries, but the opposite pattern (S-shaped) was observed when the probabilities were elicited by giving people choices between lotteries. As intuitive as the probability weighting explanation for the Allais paradox seems, one would have to say that the evidence is inconclusive at this time.

IV. Prospect Theory: Gains, Losses, and “Reflection Effects”

A second part of prospect theory pertains to the notion of a reference point from which gains and losses are evaluated. In experiments with money payments, the most obvious candidate for the reference point is the current level of wealth, which includes earnings up to the present. The reference point is the basis for the notion of “loss aversion,” which implies that losses are given more weight in
choices with outcomes that involve both gains and losses. A second property of the reference point is known as the “reflection effect,” which pertains to cases where positive payoffs are multiplied by minus one in a manner that “reflects” them around 0. The reflection effect postulates that the risk aversion exhibited by choices when all outcomes are gains will be transformed into a preference for risk when all outcomes are losses.

A comparison of behavior in the gain and loss domains is difficult in a laboratory experiment for a number of reasons. First, the notion of a reference point is not precisely defined. For example, would “paper” earnings recorded up to the present point (but not actually paid) in cash be factored into the current wealth position? A second problem is that human subjects committees do not allow researchers to collect losses from subjects in an experiment, who cannot walk out with less money that they started with. One solution is to give people an initial stake of cash before they face losses, but it is not clear that this process will really change a person’s reference point. The notion that an initial stake is treated differently than hard-earned cash is called the “house-money effect.” There is some evidence that gifts (e.g. candy) tend to make people more willing to take risks in some contexts and less willing in others (Arkes et al. 1988, 1994). It is at least possible that the warm glow of a house-money effect may cause people to appear risk seeking for losses when this may not ordinarily be the case with earned cash. One solution is to give people identical stakes before both gain and loss treatments, which holds the house-money effect constant. And making people earn the initial stake through a series of experimental tasks is probably more likely to change the reference point.

Kahneman and Tversky (1979) presented strong experimental evidence for a reflection effect. The design involved taking all gains in a choice pair like those in Table 8.1 and reflecting them around zero to get losses, as in Table 8.2. Now the “safe” lottery, S*, involves a sure loss, whereas the risky lottery, R*, may yield a worse loss or no loss at all. The choice pattern in Table 8.1, with 80% safe choices, is reversed in Table 8.2, with only 8% safe choices.

| Table 8.2. A Reflection Effect Experiment (Kahneman and Tversky, 1979) |
|-----------------|-----------------|
| **Lottery S*** (selected by 8%) | **Lottery R*** (selected by 92%) |
| minus 3,000 with probability 1.0 | minus 4,000 with probability 0.8 |
| 0 with probability 0.2 |

The Kahneman and Tversky experiments used hypothetical payoffs, which raises the issue of whether this reflection effect will persist with economic incentives. (Recall from Chapter 4 that risk aversion was strongly affected by the use of high economic incentives, as compared with hypothetical payoffs).
Holt and Laury (2002) evaluate the extent of the hypothetical bias in a reflection experiment. They took a menu of paired lottery choices similar to that in Table 4.1 and reflected all payoffs around 0. Recall that this menu has safe lotteries on one side and risky lotteries on the other, and that the probability of the higher payoff number increases as one moves down the menu. Risk aversion is inferred by looking at the number of safe choices relative to the number of safe choices that would be made by a risk-neutral person (in this case 5). All participants made decisions in both the gains menu and in the losses menu, with the order of menu presentation alternated in half of the sessions. In their hypothetical payoff treatment, subjects were paid a fixed amount $45 in exchange for participating in a series of tasks (search, public goods) in a different experiment, and afterwards they were asked to indicate their decisions for the lottery choice menus with the understanding that all gains and losses would be hypothetical. When all payoffs were hypothetical gains, about half of the subjects were risk averse, and slightly more than 50% of those were risk seeking for losses. The modal pattern in this treatment was reflection, although other patterns (e.g. risk aversion for gains and losses) were also observed with some frequency. The choice frequencies for the hypothetical choices are shown in the left panel of Figure 8.1. The modal pattern of reflection is represented by the tall spike in the back-right corner of the right panel.

The real-incentive treatments for gains and losses were run in a parallel manner with the same choice menus. Participants were allowed to build up earnings of about $45 in a different experiment using the same tasks used under the hypothetical treatment. In contrast with earlier results, the modal pattern of behavior with real incentives did not involve reflection. The most common pattern was for people to exhibit risk aversion for both gains and losses. The real-payoff choice frequencies are shown in the right panel of Figure 8.1. The modal pattern of risk aversion in both cases is represented by the spike in the back-left side of this panel. There is a little more risk aversion with real payoffs than with hypothetical payoffs; sixty percent of the subjects exhibit risk aversion in the gain condition, and of these only about a fifth are risk seeking for losses. The rate of reflection with real payoffs is less than half of the reflection rate observed with hypothetical payoffs.

Despite the absence of a clear reflection effect, there is some evidence that gains and losses are treated differently. On average, people tended to be essentially risk neutral in the loss domain, but they were generally risk averse in the gain domain. This result provides some support for the notion of a reference point, around which gains and losses are evaluated, which suggests that laboratory data should be analyzed using utility as a function of earnings (gains and losses), not final wealth. In other words, if the net worth of a person’s assets is \( w \), and if a decision may produce earnings or losses of \( x \), then the analysis of expected utility
(with or without the probability weighting of prospect theory) should be expressed in terms of $U(x)$, not $U(w+x)$.

Figure 8.1. Inferred Risk Aversion for Hypothetical Payoffs (Left) and Real Payoffs (Right)

The absence of a clear reflection effect in the Holt and Laury data is a little surprising given the results of several other studies that found reflection with real money incentives (Camerer, 1989; Battalio et al. 1989). One difference is that instead of holding initial wealth constant in both treatments (at a level high enough to cover losses), these studies provided a high initial stake in the loss treatment, so the final wealth position is constant across treatments. For example, a lottery over gains of 4,000 and 0 could be replaced with an initial payoff of 4,000 and a choice involving losses of 4,000 and 0. Each presentation or “frame” provides the same possible final wealth positions (0 or 4,000), but the framing is in terms of gains in one treatment and in terms of losses in the other. A setup like this is exactly what is needed to document a “framing effect.” Such an effect is present since both studies report a tendency for subjects to be risk averse in the gain frame and risk seeking in the loss frame. Whether these results indicate a reflection effect is less clear, since the higher stake provided in the loss treatment may itself have induced more risk seeking behavior, just as gifts of candy and money tend to increase risk seeking in experiments reported by psychologists.

V. Extensions and Further Reading
The dominant method of modeling choice under risk in economics and finance involves expected utility, either applied to gains and losses or to final
wealth. The final wealth approach involves a stronger type of rationality in the sense that people can see past gains and losses and focus on the variable that determines consumption opportunities (final wealth). The Camerer (1989) and Battalio et al. (1989) experiments provide strong evidence that decisions are framed in terms of gains and losses, and that people do not “integrate” gains and losses into a final asset position. Indeed, there is little if any experimental evidence for such “asset integration.” Rabin (2000) and Thaler and Rabin (2001) also provide a theoretical argument against the use of expected utility as a function of final wealth; the argument being that the risk aversion needed to explain choices involving small amounts of money implies absurd levels of risk aversion for choices involving large amounts of money. Most analyses of risk aversion in laboratory experiments are, in fact, already done in terms of gains and losses (Binswanger, 1980; Kachelmeyer and Shehata, 1995; Goeree, Holt, and Palfrey, 2000, 2001).

Even if expected utility is modeled in terms of gains and losses, there is the issue of whether to incorporate other elements like nonlinear probability weighting that represents systematic misperceptions of probabilities. As noted above, the evidence on this method is mixed, as is the evidence for the reflection effect. Some, like Camerer (1995), have urged economists to give up on expected utility theory in favor of prospect theory and other alternatives. More recently, Rabin and Thaler (2001) have expressed the hope that they have written the final paper that discusses the expected utility hypothesis, referring to it as the “ex-hypothesis” with the same tone that is sometimes used in talking about an ex-spouse. Other economists like Hey (1995) maintain that the expected-utility model outperforms the alternatives, especially when decision errors are explicitly modeled in the process of estimation. In spite of this controversy, expected utility continues to be widely used, either implicitly by assuming risk neutrality or explicitly by modeling risk aversion in terms of either gains and losses or in terms of final wealth.

Some may find the mixed evidence on some of these issues to be worrisome, but to an experimentalist it provides an exciting area for further research. For example, there remains a lot of work to be done in terms of finding out how people behave in high-stakes situations. One way to run such experiments is to go to countries where using high incentives would not be so expensive. For example, Binswanger (1980) studied the choices of farmers in Bangladesh when the prize amounts sometimes involved more than a month’s salary. (He observed considerable risk aversion, which was more pronounced with very high stakes.) Similarly, Kachelmeier and Shehata (1992) performed lottery choice experiments in rural China. They found that the method of asking the question has a large impact on the way people value lotteries. For example, if you ask for a selling price (the least amount of money you would accept to sell the
lottery), people tend to give a high answer, which would seem to indicate a high value for the risky lottery, and hence a preference for risk. But if you ask them for the most they would be willing to pay for a risky lottery, they tend to give a much lower number, which would seem to indicate risk aversion. The incentive structure was such that the optimal decision was to provide a “true” money value in both treatments (much as the instructions for the previous chapter provided an incentive for people to tell the truth about their probabilities in a Bayes’ rule experiment). It seems that people go into a bargaining mode when presented with a pricing task, demanding high selling prices and offering low buying prices. The nature of this “willingness-to-pay/willingness-to-accept bias” is not well known, at least beyond the simple bargaining mode intuition provided here. Nevertheless, it is important for policy makers to be aware of the WTP/WTA bias, since studies of non-market goods (like air and water quality) may have estimates of environmental benefits that vary by 100% depending on how the question is asked. Given the strong nature of this WTP/WTA bias, it is usually advisable to avoid using pricing tasks to elicit valuations. For more discussion of this topic, see Shogren, et al. (1994).

A number of additional biases have been documented in the psychology literature on judgement and decision making. For example, there may be a tendency for people to be overconfident about their judgements in some contexts. Some of the systematic types of judgemental errors will be discussed at length in later chapters, such as the “winner’s curse” in auctions for prizes of unknown value. For further discussion of these and other anomalies, see Camerer (1995).

**Questions**

1. Show that a risk-neutral person would prefer a 0.8 chance of winning 4,000 to a sure payment of 3,000, and that the same person would prefer a 0.2 chance of winning 4,000 to a 0.25 chance of winning 3,000.
2. If the probability of the 4,000 payoff for lottery R in Table 8.1 is replaced by 0.7, show that a risk neutral person would prefer Lottery S.
Chapter 9. ISO (In Search of …)

When you go out to make a purchase, you probably do not check prices at all possible locations. In fact, you might stop searching as soon as you find an acceptable offer, even if you know that a better offer would probably turn up eventually. Such behavior may be optimal if the search process is costly, which makes it worthwhile to compare the costs and benefits of additional searching. The game discussed in this chapter is a search problem, where each additional offer costs a fixed amount of money. Observed behavior in such situations is often surprisingly rational, and the classroom game can be used to introduce a discussion of optimal search. As usual, the game can be done using ten-sided dice to generate the random offers, following the instructions in the Appendix, or it can be run on the Veconlab software (select SR).

I. Introduction

Many economists believed that the rise of e-commerce would lead to dramatically lower search costs and a consequent reduction in price levels and dispersion. Evidence to date, however, suggests that there is still a fair amount of price variability on the internet, and most of us can attest to the time costs of searching for low prices. Of course, a reduction in the cost of obtaining a price quote would cause one to get more quotes before deciding on a major purchase, but the total time spent searching may go either way, since this total is the product of the cost per search and the number of searches.

Issues of search and price dispersion are central to the study of how markets promote efficiency by connecting buyers and sellers. Search is also important on a macroeconomic level, since much of what is called “frictional” unemployment is due to workers searching for an acceptable wage offer. This chapter presents a particularly simple search problem, in which a person pays a constant cost for obtaining each new observation (e.g. a wage offer or price quote). Observations are independent draws from a probability distribution that is known. From a methodological point of view, the search problem is interesting because it is dynamic, i.e. decisions are made in sequence.

II. Search from a Uniform Distribution

The setup used here is based on a particular probability framework, the uniform distribution. Many situations of interest involve equal probabilities for the relevant events. For example, consider an airport with continuously circulating buses numbered from 1 to 20, which all pass by a certain central pickup point. If there is no particular pattern, the next bus to pass might be modeled as a uniform random variable. The uniform distribution is easy to
explain since all probabilities are equal, and the theoretical properties of models with uniform distributions are often quite simple. Therefore, the uniform distribution will be used repeatedly in other parts of this book, for example, in the chapters on auctions.

A uniform distribution is easily generated with a random device, like drawing ping pong balls with different numbers written on them, so that each number is equally likely to be drawn. Even if a computer random number generator is used, it is important to provide a physical example to explain what the distribution implies. One way to explain a uniform distribution on some interval, say from 0 to 99, is to imagine a roulette wheel with 100 stops, labeled 0, 1, 2, … 99, so that a hard spin is just as likely to stop at one point as at any other. A convenient and inexpensive way to generate the realization of a uniform distribution is with throws of a 10-sided die, which is generally labeled 0, 1, 2, …9. The first throw can determine the “tens” digit, and the second throw can determine the “ones” digit. In this manner, each of the 100 integers on the interval from 0 to 99 is equally likely.

Besides being easy to explain and implement, the uniform distribution has the useful property that the expected value is the midpoint of the interval. This midpoint property is due to the absence of any asymmetry around the midpoint that would pull expected value in one direction or the other. To see this, consider the simplest case, i.e. with two equally likely outcomes, 0 and 1. Since each occurs with probability 1/2, the expected value is \((1/2)(0) + (1/2)(1) = 1/2\), which is the midpoint of the interval. For a distribution from 0 to 99 determined by the die throws, each integer in the interval has the same probability, 0.01, and the expected value is \(0.01(0) + 0.01(1) + 0.01(2) + \ldots + 0.01(99)\), which is midpoint, 49.5.

The search instructions for the experiment discussed in this chapter present subjects with an opportunity to purchase draws from a distribution that is uniform on the interval from 0 to 90 (pennies). This distribution is generated by a computer random number generator for the web version (SR), and it can be determined by the throws of dice by ignoring all outcomes above 90. Each draw costs 5 cents, and there is no limit on the number of draws. The subject may decline to search, thereby earning zero. If the first draw is \(D_1\) pennies, then stopping at that point would result in earnings of \(D_1 - 5\) pennies. Suppose that the second draw is \(D_2\). Then the options are: 1) pay another nickel and search again, 2) stop and earn \(D_2 - 10\), or if going back is allowed, 3) accept the first draw and earn \(D_1 - 10\). We will say that there is “recall” if going back to take any previously rejected draw is permitted; otherwise we say that there is “no recall.” Recall may not be possible in some situations, e.g. when going through the classified (or personal ISO) ads.
Some typical search sequences for one person are shown in Table 9.1. The round number is in the left column, and the draws are listed in sequence in the middle column, with the accepted draw shown in boldface type. The search cost was 5 cents, and the resulting earnings are shown in the right column. This person, who had no prior practice, seems to stop as soon as an offer of about 45 cents is obtained.

<table>
<thead>
<tr>
<th>Round</th>
<th>Draw Sequence (cents)</th>
<th>Earnings</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>70</td>
<td>65</td>
</tr>
<tr>
<td>2</td>
<td>31, 43, <strong>63</strong></td>
<td>48</td>
</tr>
<tr>
<td>3</td>
<td>21, 8, 43, 43, <strong>51</strong></td>
<td>26</td>
</tr>
<tr>
<td>4</td>
<td><strong>53</strong></td>
<td>48</td>
</tr>
<tr>
<td>5</td>
<td>87</td>
<td>82</td>
</tr>
<tr>
<td>6</td>
<td><strong>69</strong></td>
<td>64</td>
</tr>
<tr>
<td>7</td>
<td>3, 8, 35, <strong>43</strong></td>
<td>23</td>
</tr>
<tr>
<td>8</td>
<td><strong>85</strong></td>
<td>80</td>
</tr>
<tr>
<td>9</td>
<td><strong>46</strong></td>
<td>41</td>
</tr>
<tr>
<td>10</td>
<td>22, 12, <strong>65</strong></td>
<td>50</td>
</tr>
</tbody>
</table>

The analysis of the optimal way to search is simplified if we assume that the person is risk neutral, which lets us determine a benchmark from which the effects of risk aversion can be evaluated. In addition, assume that the subject faces no cash constraint (wouldn’t it be nice) on the number of nickels (well that’s not so unreasonable) available to pay the search costs. In this case, the future always looks the same because the horizon is infinite and the payoff parameters (search cost, prize distribution) are fixed over time. Since the future opportunities are the same regardless of how many draws have been rejected thus far, any draw rejected at one point in time should never be taken at a later point. Thus each possible draw in the interval from 0 to 90 should either be always accepted or always rejected, and the boundary that separates the acceptable and unacceptable draws is a goal or “reservation prize.” The person who made the choices shown in Table 9.1 seems to have a reservation prize level of about 45 cents. It is useful to see a broader sample of how people behave in this situation before further analysis of the optimal manner of search.

**III. Experimental Data**

Figure 9.1 shows the results of a classroom experiment with two different search cost treatments. Each “search sequence” was a series of draws made until the subject stopped and accepted a draw, and there were ten such search sequences in each treatment. (The data are from the last 5 periods for the second
and third sequences, so some learning has already occurred. Using the final 5 periods takes out the inertia effects caused by slow convergence that may show up when the search cost is changed between treatments. As a result, the data in the figure are a little “cleaner” than is normally the case.)

Consider the 5-cent search cost treatment, shown on the left side of the figure, where the draw number is indicated on the horizontal axis. The marks
directly over the “1st” label on the axis represent the first-round draws, one per search sequence, which are fairly uniformly distributed from 0 to 90 as would be expected. Those accepted in the first round are labeled as “+” signs, and the rejected draws are labeled as dots. Each of the dots in the “1st” column becomes a mark in the “2nd” column, either a “+” if the second draw is accepted or a dot if not. The dots in the second column become marks (plus or dot) in the third column. The majority of search sequences ended by the fourth draw, so all data that ended in the fourth period and afterwards are shown in the “4-12th” column (nobody made more than 12 draws). As before, this last column contains plus marks for accepted draws and minus marks for rejected draws. Hence a draw that is rejected in period 4 and accepted in period 5 will show up as two marks in the same column. In addition, the 4-12th column contains several asterisk marks, which represent accepted draws that were initially rejected. Since the draw was both rejected and accepted, think of the asterisk as a combination of a plus sign and a minus sign. Such recalls are occasionally observed, even though they are irrational under the assumptions made above.

The divide between accepted and rejected offers for the 5-cent cost treatment is in the range from 55 to 60 cents for most people in this group. In contrast, the dramatic increase in search cost to 20 cents results a much larger acceptance region, as shown on the right side of the figure. With high search costs, there are fewer searches, and the divide is somewhere between 25 and 30 cents. The infrequency of recalls (asterisks) and the relatively clean divide between acceptance and rejection regions is roughly consistent with the notion of a reservation or goal level. The next step is to consider whether the observed divides can be explained with simple expected value calculations of benefits and costs.

IV. Optimal Search

First, consider the low-search-cost treatment. Obviously any amount above 85 cents should be accepted, since the potential gain from searching again is at most 5 cents (if a very lucky draw of 90 is obtained) and the cost of the new draw is 5 cents. Thus there is only a very low chance (1/91) that the new draw will cover the cost. At the opposite extreme, a low draw, say 0, would be rejected since the cost of another draw is only 5 cents and the expected value of the next draw is 45 cents (the midpoint of the distribution from 0 to 90). Notice the use of an expected value here, which is justified by the assumption of risk neutrality. To summarize, the benefits of further search exceed the cost when the best current draw is very low, and the cost exceeds the expected benefit when the best current draw is near the top of the range. The optimal reservation prize level is found by locating the point at which the expected benefits of another search are equal to the search cost.
Suppose that the best current draw is 60. There is about a 2/3 chance (61/91) that the next draw is at 60 or below, in which case the net gain is 0. This situation is represented in Figure 9.2, where the flat horizontal line represents the fact that each of the draws will be observed with the same probability. The area below this line represents probability. The line has a height of 1/91, so the area over each interval of unit width is 1/91, which is the probability that a single integer will be selected by a random throw of the dice. About one third of this probability is located to the right of 60. In other words, starting from 60, there is a 1/3 chance of obtaining better draw.

![Figure 9.2. A Uniform Distribution on the Interval [0, 90] (With One Third of the Probability to the Right of 60)](image)

Next consider the expected gains from search when the best current draw is 60 (with recall and a 5-cent search cost). A draw below 60 will not produce any gain, so the expected value of the gain will have a term that is the product: \((2/3)0 = 0\). The region of gain is to the right of the vertical line at 60 in Figure 9.2, and the area below the dashed line is about one third of the total area. Thus, there is essentially a 1/3 chance of an improvement, which on average will be half of the distance from 60 to 90, i.e. half of 30. The expected value of an improvement, therefore, is 15 cents. An improvement occurs with a probability about 1/3, so the expected improvement is approximately \((1/3)(15) = 5\) cents. Any lower current draw will produce an expected improvement from further search that is above 5 cents, and any higher current draw will produce a lower expected improvement. It follows that 60 is the reservation draw level for which the cost of another draw equals the expected improvement. (This argument ignored the fact that there are 91 possible draws instead of 90, and an exact calculation shows that all draws of 60 or below would be rejected and all draws above 60 would be accepted; see question 1.) The same reasoning applied to the 20-cent search cost treatment implies that the optimal reservation draw is 30 cents (see question 2).

The predicted reservation values of 60 and 30 are graphed as horizontal lines in Figure 9.1. The power of these predictions is quite clear. There is a break
between the plus signs indicating acceptance and the minus signs indicating rejection, with little overlap. This break is slightly below the 30 and 60 levels predicted for a risk-neutral person. Actually, adding risk aversion lowers the predicted cutoff. The intuition behind risk aversion effects is clear. While a risk-neutral person is approximately indifferent between searching again at a cost of 5 cents and stopping with a draw of 60, a risk-averse person would prefer to stop with 60, since it is a sure thing. In contrast, drawing again entails the significant risk (two thirds) of having to pay the cost without getting any improvement.

One implication of optimal behavior is that the reservation offer is essentially the value of having the opportunity to go through this search process. This is because one rejects all offers below the reservation offer, thereby indicating that the value of playing the search game is greater than those offers. Conversely, offers above the reservation offer level are accepted, indicating a preference for those sure amounts of money over a continuation of the search process. This implication was tested by Schotter and Braunstein (1981), who elicited a price at which individuals would be willing to sell the option to search. In one of their treatments, the draws were from a uniform distribution on the interval from 0 to 200 with a search cost of 5 cents. The optimal reservation offer for a risk neutral person is 155 (see question 3). This should be the value of being able to play the search game, assuming that there is no enjoyment derived from doing an additional search sequence after several have already been completed. The mean reported selling price was 157, and the average accepted offer was 170, which is about halfway between the theoretical reservation offer of 155 and the upper bound of 200. These and other results lead to the conclusion that the observed behavior is roughly consistent with the predictions of optimal search theory.

**Further Reading and Extensions**

The search game discussed here is simple, but it illustrates the main intuition behind the determination of the reservation prize level in a sequential search problem. There are many interesting and realistic variations of this problem. The planning horizon may not be infinite, e.g. when you have a deadline for finding a new apartment before you have to move out of the current one. In this case, the reservation prize level will tend to fall as the deadline nears and desperation takes over. In the Cox and Oaxaca (1989) experiments with a finite horizon, subjects stopped at the predicted point about three-fourths of the time, and the deviations were in the direction of stopping too soon, which would be consistent with risk aversion.

Another generalization is to allow people to learn about the distribution of offers as they search. Suppose that you think the offers will be in the range from 0 to 100, and you see an offer of 180. This would have been well above your
reservation level if the distribution from which draws were made had an upper limit of 100, but now you realize that you do not know what the upper limit really is. In this case, a high offer may be rejected in order to find out whether even higher offers are possible (Cox and Oaxaca, 2000).

Although the aggregate data discussed here are roughly consistent with theoretical predictions, this leaves open the issue of how people learn to behave in this manner. Certainly they do not generally do any mathematical analysis. Hey (1981) has analyzed individual behavior in search of heuristics and adaptive patterns that may explain behavior at the individual level. Also, see Hey (1987, when he is “still searching”). He specified a number of rules of thumb, like:

*One Bounce Rule:*
Buy at least two offers, and stop if an offer received is less than the previous offer.

*Modified One Bounce Rule:*
Buy at least two offers, and stop if an offer received is less than the previous offer plus the search cost.

The experiment involved treatments with and without recall, and with and without information about the distribution (a truncated normal distribution) from which offers were drawn. Behavior of some subjects in some rounds corresponded to one or more of these rules, but by far the most common pattern of behavior was to use a reservation offer level. In the fifth and final round, about three-fourths of the participants exhibited behavior that conformed to the optimal reservation offer rule alone.

**Questions**

1. If any integer draw between and including 0 and 90 is equally likely, then the chance of each draw is $1/91$. If the current draw is 60, then the chances that the next draw will be better are: a $1/91$ chance of a draw of 61 for a gain of 1 cent; a $1/91$ chance of a draw of 62 for a gain of 2; etc. Use a spreadsheet to find the expected value of the gain over 60 (not the expected value of the draw), and compare this with the search cost of 5 cents. Would a draw of 60 be rejected?

2. With a search cost of 20 cents per draw from a distribution that is uniform from 0 to 90, show that the person is approximately indifferent between searching again and stopping when the current best draw is 30.

3. With a search cost of 5 cents for draws from a uniform distribution on the interval from 0 to 200, show that the reservation offer is about 155.
4. Evaluate the data in Table 9.1 in terms of the “One Bounce Rule” and the “Modified One Bounce Rule.” Do these rules work well? Can you think of other rules of thumb that might be good when the distribution used is not known?
Chapter 10. Information Cascades

Suppose that individuals may receive different information about some unknown event, like whether or not a newly patented drug will be effective. This information is then used in making a decision, like whether or not to invest in the company that developed the new drug. If decisions are made in sequence, then the second and subsequent decision makers can observe and learn from earlier decisions. The dilemma occurs when one’s private information suggests a decision that is different from what others have done before. An “information cascade” forms when people follow the consensus decision regardless of their own private information. This game could be implemented with draws from cups and throws of dice, but the web version (CAS) is easier to administer in a manner that controls for unwanted informational signals.

I. “To do exactly as your neighbors do is the only sensible rule.”

Conformity is a common occurrence in social situations, as the section title implies (from Emily Post, 1927, Chapter 33). People may follow others’ decisions because they value conformity and fear social sanctions. For example, an economic forecaster may prefer the risk of being wrong along with others to the risk of having a deviant forecast that turns out to be inaccurate. Similarly, Keynes (1936) remarked: “Worldly wisdom teaches that it is better for reputation to fail conventionally than to succeed unconventionally.” Some experimenters have suggested that there is an irrational preference for the current state of affairs, i.e. a status quo bias. In some hypothetical choice experiments, Samuelson and Zeckhauser (1988) showed subjects two portfolios that could be used to invest money inherited from a rich uncle. There was a strong preference for the portfolio that was indicated to be the current investment profile, and a switch in the status quo designation caused a tendency to switch in preference (in a between-subjects design).

In some situations, people may be tempted to follow others’ decisions because of the belief that there is some wisdom or experience implicit in an established pattern. Maybe subjects in the status quo bias experiments concluded that the deceased uncle was wealthy because he had selected a good portfolio. The possible effects of collective wisdom may be amplified in a large group. For example, someone may prefer to buy a Honda or Toyota sedan thinking that the large market shares for those models signal a lot of customer satisfaction. This raises the possibility that a choice pattern started by a few individuals may set a precedent that results in a string of incorrect decisions. At any given moment, for example, there are certain classes of stocks that are to be considered good investments, and herd effects can be amplified by efforts of some to anticipate
where the next fads will lead. Anyone who purchased a wide range of “tech” stocks several years ago can attest to the dangers of following the bulls.

A particularly interesting type of bandwagon effect can develop when individuals make decisions in a sequence and can observe others’ prior decisions. For example, suppose that a person is applying for a job in an industry with a few employers who know each other well. If the applicant makes a bad impression in the first couple of interviews and is not hired, then the third employer who is approached may hesitate even if the applicant makes a good impression on the third try. This third employer may reason: “anyone may have an off day, but I wonder what the other two people saw that I missed.” If the joint information implied by the two previous decisions is deemed to be more informative than one’s own information, it may be rational to follow the pattern set by others’ decisions, in spite of contradictory evidence. A chain reaction started in this manner may take on a life of its own as subsequent employers hesitate even more. This is why first impressions can be important in the workplace. The effect of information inferred from a sequential pattern of conforming decisions is referred to as an “information cascade” (Bikhchandani, Hirschleifer, and Welch, 1992).

Since first impressions can be wrong, the interviews or tests that determine initial decisions may start an “incorrect” cascade that implies false information to those who follow. This possibility was raised in an article in the *Economist* about the popular drug Prozac: “Can 10 Million People be Wrong?” John Dryden is somewhat more poetic: “Nor is the people’s judgement always true, the most may err as grossly as the few.” This chapter considers how cascades, incorrect or otherwise, may form even if individuals are good Bayesians in the way that they process information.

II. A Model of Rational Learning from Others’ Decisions

Following Anderson and Holt (1997, 2002), the discussion of cascades will be based on a very stylized model in which the two events are referred to as Cup A with contents $a, a, b$, and Cup B with contents $a, b, b$. Think of these cups as containing amber or blue marbles, with the proportions being correlated with the cup label. Each cup is equally likely to be selected, with its contents then being emptied into an opaque container from which draws are made privately. Each person sees one randomly drawn marble from the selected cup, and then must guess which cup is being used. Decisions are made in a pre-specified sequence, so that the first person has nothing to go on but the color of the marble drawn. Draws are made with replacement, so the second person sees a draw from the unknown cup, and must make a decision based on two things: the first decision and the second (own) draw. Like the employers who cannot sit in on others’ interviews, each person can see prior decisions but not the private information that may have affected those decisions. There is no external
incentive to conform in the sense that one’s payoff depends only on guessing the cup being used; there is no benefit in conforming to others’ (prior or subsequent) decisions.

The first person in the sequence has only the observation, \(a\) or \(b\), and the prior information that each cup is \textit{ex ante} equally likely. Since two of the three \(a\) marbles are in cup A, the person should assess the probability of cup A to be \(2/3\) if an \(a\) is observed, and similarly for cup B (cf. Chapter 7). Thus, the first decision should reveal that person’s information. In the experiments discussed below, about 95% of the people made the decision that corresponded to their information when they were first in the sequence.

The second person then sees a private draw, \(a\) or \(b\), and faces an easy choice if the draw matches the previous choice. A conflict is more difficult to analyze. For example, if the first person predicts cup A, the second person who sees a \(b\) draw may reason: “the cups are now equally likely, since that was the initial situation and the \(a\) that I think the first person observed cancels out the \(b\) that I just saw.” In such cases, the second person should be indifferent, and might choose each cup with equal probability. On the other hand, the second person may be a little cautious, being more sure about what they just saw than about what they are inferring about what the other person saw. Even a slight chance that the other person made a mistake (deliberate or not) would cause the second person to “go with their own information” in the event of a conflict. In the experiments, about 95% of the second decision makers behaved in this manner in the experiment described below, and we will base subsequent discussion on the assumption that this is the case.

Table 10.1. Possible Inferences Made by the Third Decision Maker

<table>
<thead>
<tr>
<th>Prior Decisions</th>
<th>Own Draw</th>
<th>Inferred Pr(cup A)</th>
<th>Decision</th>
</tr>
</thead>
<tbody>
<tr>
<td>A, A</td>
<td>(a)</td>
<td>0.89</td>
<td>A (no dilemma)</td>
</tr>
<tr>
<td>A, A</td>
<td>(b)</td>
<td>0.67</td>
<td>A (start cascade)</td>
</tr>
<tr>
<td>A, B</td>
<td>(a)</td>
<td>0.67</td>
<td>A</td>
</tr>
<tr>
<td>A, B</td>
<td>(b)</td>
<td>0.33</td>
<td>B</td>
</tr>
</tbody>
</table>

Now the third person will have observed two decisions and one draw. There is no loss of generality in letting the first decision be labeled A, so the various possibilities are listed in Table 10.1. (An analogous table could be constructed when the first decision is B, and the same conclusions would apply.) In the top row of the table, all three pieces of information line up, and the standard Bayesian calculations would indicate that the probability of cup A is 0.89, making A the best choice (see question 1). The case in the second row is more interesting, since the person’s \textit{own} draw is at odds with the information inferred from other’s decisions. When there are two others, who each receive independent
draw “signals” that are just as informative as the person’s own draw, it is rational to go with the decision implied by the others’ decisions. (Recall from chapter 7 that an imbalance of one draw in favor of cup A will raise the probability of cup A to 0.67.) The decision in the final two rows is less difficult because the preponderance of information favors the decision that corresponds to one’s own information.

The pattern of following others’ behavior (seen in the top two rows of Table 10.1) may have a domino effect: the next person would see three A decisions and would be tempted to follow the crowd regardless of their own draw. This logic applies to all subsequent people, so an initial pair of matching decisions (AA or BB) can start an information cascade that will be followed by all others, regardless of their information. This logic also applies when the first two decisions cancel each other out (AB or BA) and the next two form an imbalance that causes the fifth person to decide to follow the majority. An example of such a situation would be: ABAA, in which case the fifth person should choose A even if the draw observed is b. Again, the intuition is that the first two decisions cancel each other and that the next two matching decisions are more informative than the person’s own draw.

Notice that there is nothing in this discussion that implies that the first two people will guess correctly. There is a 1/3 chance that the first person will see the odd marble drawn from the selected cup, and will guess incorrectly. There is a 1/3 chance that the same thing will happen to the second person, so in theory, there is a \((1/3)(1/3) = 1/9\) chance that both of the first two people will guess incorrectly and spark an incorrect cascade. Of course, cascades (incorrect or not) may initially fail to form and then may later form when the imbalance of draws is 2 or more in one direction. In this case, the aggregate information inferred from others’ decisions is greater than the information inherent in any single person’s draw.

### III. Experimental Evidence

Anderson and Holt (1997) used this setup in a laboratory experiment in which people earned $2 for a correct guess, nothing otherwise. Subjects were in isolated booths, so that they could not see others’ draws. Decisions (but not subject ID numbers) were announced by a third person to avoid having confidence or doubts communicated by the decision-maker’s tone of voice. The marbles were light or dark, with two lights and a dark in cup A, and with two darks and a light in cup B. The cup to be selected was determined by the throw of a six-sided die, with a 1, 2, or 3 determining cup A. The die was thrown by a “monitor” selected at random from among the participants at the start of the session. There were six other subjects, so each prediction sequence consisted of six private draws and six public predictions. For each group of participants, there
were 15 prediction sequences. The monitor used a random device to determine the order in which individuals saw their draws and made their predictions. The monitor announced the cup that had actually been used at the end of the sequence, and all participants who had guessed correctly added $2 to their cumulative earnings. The monitor received a fixed payment, and all others were paid their earnings in cash. This “ball and urn” setup was designed to reduce or eliminate preferences for conformity that were not based on informational considerations.

It is possible that an imbalance of signals does not develop, e.g. alternating \( a \) and \( b \) draws, making a cascade unlikely. An imbalance did occur in about half of the prediction sequences, and cascades formed about 70% of the time in such cases. A typical cascade sequence is:

Table 10.2. A Cascade

<table>
<thead>
<tr>
<th>Subject:</th>
<th>58</th>
<th>57</th>
<th>59</th>
<th>55</th>
<th>56</th>
<th>60</th>
</tr>
</thead>
<tbody>
<tr>
<td>Draw:</td>
<td>( b )</td>
<td>( b )</td>
<td>( a )</td>
<td>( b )</td>
<td>( a )</td>
<td>( a )</td>
</tr>
<tr>
<td>Prediction:</td>
<td>B</td>
<td>B</td>
<td>B</td>
<td>B</td>
<td>B</td>
<td>B</td>
</tr>
</tbody>
</table>

Sometimes people do deviate from the behavior that would be implied by a Bayesian analysis. For example, consider the sequence:

Table 10.3. A Typical “Error”

<table>
<thead>
<tr>
<th>Subject:</th>
<th>8</th>
<th>9</th>
<th>12</th>
<th>10</th>
<th>11</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Draw:</td>
<td>( a )</td>
<td>( a )</td>
<td>( b )</td>
<td>( a )</td>
<td>( b )</td>
<td>( a )</td>
</tr>
<tr>
<td>Prediction:</td>
<td>A</td>
<td>A</td>
<td>B</td>
<td>A</td>
<td>A</td>
<td>A</td>
</tr>
</tbody>
</table>

The decision of subject 12 in the third position is the most commonly observed type of error, i.e. basing a decision on one’s own information even when it conflicts with the information implied by others’ prior decisions. This type of error occurred in about a fourth of the cases in which it was possible. Notice that the cascade starts later with subject 11 in the fifth position.

Once a cascade begins, the subsequent decisions convey no information about their signals, since these people are just following the crowd. In this sense, patterns of conformity are based on a few draws, and consequently, cascades may be very fragile. In particular, one person who breaks the pattern by revealing their own information may alter the decisions of those who follow.

Finally, a number of incorrect cascades were observed. Cup B was actually being used for the sequence shown in Table 10.4, but the first two individuals were unfortunate and received misleading \( a \) signals. Their matching predictions caused the others to follow with a string of incorrect A predictions,
which would have been frustrating given that these individuals had seen private draws that indicated the correct cup.

Table 10.4. An Incorrect Cascade

<table>
<thead>
<tr>
<th>Subject:</th>
<th>11</th>
<th>12</th>
<th>8</th>
<th>9</th>
<th>7</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Draw:</td>
<td>a</td>
<td>a</td>
<td>B</td>
<td>b</td>
<td>b</td>
<td>b</td>
</tr>
<tr>
<td>Prediction:</td>
<td>A</td>
<td>A</td>
<td>A</td>
<td>A</td>
<td>A</td>
<td>A</td>
</tr>
</tbody>
</table>

To summarize, the general tendency was for individuals to let an emerging pattern of others’ decisions override their own private information. The resulting information cascades were common, but were sometimes broken by later deviant decisions.

V. Extensions and Further Reading

The model discussed in this chapter is based on an example in Bikhchandani, Hirschleifer, and Welch (1992), which contains a rich array of examples and applications. Hung and Plott (2001) discuss cascades in voting situations, e.g., when the payoff depends on whether the majority decision is correct. Some of the more interesting applications are in the area of finance. Keynes (1936) compared investment decisions with people in a guessing game who must predict which beauty contestant will receive the most votes. Each player in this game, therefore, must try to guess who is viewed as being attractive to the others, and on a deeper level, who the others will think that others will find more attractive. Similarly, investment decisions in the stock market may involve both an analysis of fundamentals and an attempt to guess what stocks will attract attention from other investors, and the result may be “herd effects” that may cause surges in prices and later corrections in the other direction. Some of these price movements may be due to psychological considerations, which Keynes compared with “animal spirits,” but herd effects may also result from attempts to infer information from others’ decisions. In such situations, it may not be irrational to follow others’ decisions during upswings and downswings in prices (Christie and Huang, 1995). Bannerjee’s (1992) model of herd behavior is motivated by an investment example. This model has been evaluated in the context of a laboratory experiment reported by Alsopp and Hey (2001). Other applications to finance are discussed in Devenow and Welch (1996) and Welch (1992).

It is very difficult to sort out all of the possible factors that affect prices in a stock market, so again laboratory experiments may be useful. Plott, Wit, and Yang (1997) ran some experiments with a “parimutuel” betting structure like that of a horse race where the purse is divided among those who bet on the winner, with payouts in proportion to the amounts of the winning bets. There were six
alternative assets, and only one of them would have a positive payout. The payout was divided equally among the investors in that asset. Each subject received a private signal about asset profitability, but the order of decision making was not exogenous as in the cascade experiments. Individuals could observe others’ decisions as they were made. In this manner, the information dispersed among the investors could become aggregated and incorporated into the prices of the six assets. In most cases, the asset prices ended up signaling which asset would actually produce a positive payout. In some cases, however, an initial surge of purchases for a particular asset would stimulate others to follow, and the result was that the market price rose for an asset that had no payout in the end, as would be the case with an incorrect cascade.

Questions

1. Use the ball counting heuristic from chapter 7 to verify the Bayesian probabilities in the right-hand column of Table 10.1 under the assumption that the first two decisions correctly reveal the draws seen.

2. (Advanced) The discussion in this chapter was based on the assumption that the second person in a sequence would make a decision that reveals their own information, even when this information differs from the draw inferred from the first decision. The result is that the third person will always follow the pattern set by two matching decisions, because the information implied by the first two decisions is greater than the informational content of the third person’s draw. In this question, consider what happens if we alter the assumption that the second person always makes a decision that reveals their draw. Suppose instead that the second person chooses randomly (with probabilities of 1/2) when the second draw does not match the draw inferred from the first decision. Use Bayes’ rule to show that two matching decisions (AA or BB) should start a cascade even if the third draw does not match. Then provide the intuition. (Hint: the information implied by the first decision is just as good as the information implied by the third draw, so the second draw can break a “tie” if it contains at least some information.)

3. In the Anderson and Holt experiments, the marbles were not labeled $a$ and $b$, but rather, were referred to as “light” and “dark,” since they were either translucent or a dark blue. What are the advantages of each labeling method? Which would you prefer to use in a research experiment, and why?
Part III. Game Theory

As one might expect, human behavior will not always conform tightly to simple mathematical models, especially in interactive situations. These models, however, can be extremely useful if deviations from the Nash equilibrium are systematic and predictable. This part presents several games in which behavior is influenced by intuitive economic forces in ways that are not captured by basic game theory. The applications are taken from Goeree and Holt (2001), “Ten Little Treasures of Game Theory and Ten Intuitive Contradictions.” The “treasures” are treatments where data conform to theory, and the contradictions are produced by payoff changes with strong behavioral effects, even though these changes do not affect the Nash predictions. Since game theory is so widely used in the study of strategic situations like auctions, mergers, and legal disputes, it is fortunate that there is progress in understanding these anomalies. We will consider a modification of the Nash equilibrium that injects some randomness or “noise” into players’ best response functions. You might think of the randomness as being due to un-modeled effects of emotions, attention lapses, partial calculations, and other factors that may vary from person to person and from time to time for the same person.

The simplest applications of game theory involve strong assumptions of perfect rationality and perfect predictions of others’ decisions. These assumptions imply that one always chooses the decision with the highest expected payoff, even if payoff differences are small. Such rationality leaves no room for computation errors, emotional or impulsive actions, and other random factors that are not embodied in the formal model of the game. In a simple decision, such noise elements tend to spread decisions around the optimal choice, with the most likely choice being near the optimal level, but with some probability of error in either direction. This would create a kind of “bell curve” pattern. With strategic interaction, however, even small amounts of noise in one person’s decisions may cause large biases in others’ decisions, especially if there is a lot of risk. Imagine a person trying to find the highest point on a hill, where the peak is on the edge of a steep cliff. The effect of this asymmetry in risk is that any slight chance of wind gusts may cause the person to move off of the peak. In a game, the payoff peaks depend on others’ decisions, i.e. the peaks move. In such cases, the effects of noise or small un-modeled factors may have a “snowball” effect, moving all decisions well away from the Nash predictions.

The generalized matching pennies game in Chapter 11 shows that behavioral deviations can be strongly influenced by payoff asymmetries. Existing data indicate that the Nash equilibrium only provides good, unbiased predictions by coincidence, i.e. when the payoffs for each decision exhibit a kind of balance
or symmetry. This effect is even more dramatic in the Traveler’s Dilemma game discussed in Chapter 12, where the data patterns and the Nash equilibrium may be on opposite ends of the range of possible decisions. Finally, the Coordination Game in chapter 13 has a whole series of Nash equilibria, and the issue is which one will have more drawing power. This game is important in developing an understanding of how people may become “stuck” in an equilibrium that is bad for all concerned.
Chapter 11. Generalized Matching Pennies

There are many situations in which a person does not want to be predictable. The equilibrium in such cases involves randomization, but to be willing to randomize, each player must be indifferent between the decisions over which they are randomizing. In particular, each player’s decision probabilities have to keep the other player indifferent. For this reason, changes in a player’s own payoffs are not predicted to affect the player’s own decisions. This counter-intuitive feature is contradicted by data from matching games with payoff imbalances, where “own-payoff effects” are systematic. These games can be run with minor modifications to the instructions used for Chapter 5, or with the program MG.

I. The Case of Balanced Payoffs

In a classic game of matching pennies, each person places a coin on a table, covering it so that the other person cannot see which side is up. By prior agreement, one person takes both pennies if the pennies match, and the other takes the pennies if they do not match. This is analogous to a soccer penalty kick, where the goalie must dive to one side or another before it is clear which way the kick will go, and the kicker cannot see which way the goalie will dive at the time of the kick. In this case, the goalie wants a match and the kicker wants a mismatch. Table 11.1 shows a game where the Row player prefers to match:

Table 11.1. A Modified Matching Pennies Game
(row’s payoff, column’s payoff)

<table>
<thead>
<tr>
<th>Row Player:</th>
<th>Column Player:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Left (heads)</td>
<td>72, 36 ⇒ 36, 72</td>
</tr>
<tr>
<td>Right (tails)</td>
<td>36, 72 ↑ ⇐ 72, 36</td>
</tr>
</tbody>
</table>

(It can be shown that this game is really equivalent to a matching pennies game in which there is always one player with a penny gain and another with a penny loss; see question 1). In each of the cells of the payoff table, there is one person who would gain by altering the placement of their penny unilaterally, as indicated by the arrows. For example, if they were both going to choose Heads, then the column player would prefer to switch to Tails, as indicated by the arrow pointing to the right in the upper/left box of the payoff table. The arrows in each box indicate the direction of a unilateral payoff-increasing move, so there is no equilibrium in non-random strategies. (Non-random strategies are commonly
called “pure strategies” because they are not probability-weighted mixtures of other strategies.)

The game in Table 11.1 is balanced in the sense that possible decision for each player has the same set of possible payoffs, i.e. 36 and 72. In games with this type of balance, the typical result is for subjects to play each decision with an approximately equal probability (Ochs, 1994; Goeree and Holt, 2001). This tendency to choose each decision with probability one half is not surprising given the simple intuition that one must not be predictable in this game. Nevertheless, it is useful to review the representation of the Nash equilibrium as an intersection of players’ best response functions, as explained in Chapter 5. First consider the Row player, who will choose Top if Column is expected to choose Left. Since Row’s payoffs are 72 and 36 in the top part of Table 11.1 and are 36 and 72 in the bottom part, it is apparent that Row would choose Top as long as Left is thought to be more likely than Right. This best-response behavior is represented by the solid line in Figure 11.1 that begins in the top/left corner, continuing along the top until there is an abrupt drop to the bottom when the probability of Right is 0.5. (Please ignore the curved dashed line for now.)

![Figure 11.1. Best Response Functions for a Symmetric Matching Pennies Game: The Effect of Noisy Behavior](image)

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The column player’s best response line is derived analogously and is shown by the thick dashed line that starts in the lower/left corner, rises to 0.5 and then crosses over horizontally to the right side, because Column would want to play Right whenever the probability of Top is greater than 0.5. The intersection of these two lines is in the center of the graph, at the point where each probability is 0.5, which is the Nash equilibrium in mixed strategies.

II. Noisy Best Responses

The previous analysis is not altered if we allow a little “noise” in the players’ responses to their beliefs. Noisy behavior of this type was noticed by psychologists who would show subjects two lights and ask which was brighter, or let them hear two sounds and ask which was louder. When the signals (lights or sounds) were not close in intensity, almost everybody would indicate the correct answer, with any errors being caused by mistakes in recording decisions. As the two signals became close in intensity, then some people would guess incorrectly, e.g. because of bad hearing, random variations in ambient noise, or distraction and boredom. As the intensity of the signals approached equality, the proportions of guesses for each signal would approach 1/2 in the absence of measurement bias. In other words, there was not a sharp break where the stronger signal was selected with certainty, but rather a “smooth” tendency to guess the stronger signal more as its intensity increased. The probabilistic nature of this behavior is captured in the phrase: “probabilistic choice” or “noisy best response.”

This probabilistic-choice perspective can be applied to the matching pennies game. The intuitive idea is that Row will choose Top with high probability when the expected payoff for Top is a lot larger than the expected payoff for Bottom, but that some randomness will start to become apparent when the expected payoffs for the two decisions are not that far apart. To proceed with this argument, we begin by calculating these expected payoffs. Let \( p \) denote Row’s beliefs about the probability of Right, so \( 1-p \) is the probability of Left. From the top row of Table 11.1, we see that if Row chooses Top, then Row earns 72 with probability \( 1-p \) and 36 with probability \( p \), so the expected payoff is:

\[
\text{Row’s Expected Payoff for Top} = 72(1-p) + 36(p) = 72 - 36p.
\]

Similarly, by playing Bottom, Row earns 36 with probability \( 1-p \) and 72 with probability \( p \), so the expected payoff is:

\[
\text{Row’s Expected Payoff for Bottom} = 36(1-p) + 72(p) = 36 + 36p.
\]

It follows that the difference in these expected payoffs is: \( (72 - 36p) - (36 + 36p) \), or equivalently:
Row’s Expected Payoff Difference (for Top Minus Bottom) = 36 – 72p.

When $p$ is near zero, this difference is 36, but as $p$ approaches 1/2 this difference goes to zero, in which case Row is indifferent and would be willing to choose either decision or to flip a coin.

If Row were perfectly rational (and responded to arbitrarily small expected payoff differences), then Top would be played whenever $p$ is even a little less than 1/2. The curved line in Figure 11.1 shows some departure from this rationality. Notice that the probability that Row plays Top is close to 1 but not quite there when $p$ is less than 1/2, and that the curved line deviates more from the best-response line as $p$ approaches 1/2.

The curved “noisy best-response line” for Row intersects Column’s dashed best-response line in the center of Figure 11.1, so a relaxation of the perfect rationality assumption for Row will not affect the equilibrium prediction. Similarly, suppose that we allow some noise in Column’s best response, which will “smooth off” the sharp corners for the thin dashed line that represents Column’s best responses, as shown by the upward sloping dashed line on the right side of Figure 11.2. This starts in the lower-left corner, rises with a smooth arc as it levels off at 0.5 in the center of the graph before curving upward along the upper-right boundary of the graph. Since this line will intersect Row’s downward sloping noisy response line in the center of the graph, we see that the “fifty-fifty” prediction is not affected if we let each player’s decision be somewhat noisy.

![Figure 11.2. Best Responses (Left Side) and Noisy Best Responses (Right Side)](image)
Finally, you should think about what would happen if the amount of randomness in behavior were somehow reduced. This would correspond to a situation where each person takes full advantage of even a slight tendency for the other to choose one decision even slightly more often than the other. These sharp responses to small probability differences would cause the curved lines on the right to become more like the straight-line best response functions, i.e. the “corners” on in the curved lines would become sharper. In any case, the intersection would remain in the center, so adding noise has no effect on predictions in the symmetric matching pennies game.

III. The Effects of Payoff Imbalances

As one would expect, the fact that the noisy best response lines always intersect in the center of Figure 11.2 is due to the balanced nature of the payoffs for this game (Table 11.1). An unbalanced payoff structure is shown in Table 11.2, where the Row player’s payoff of 72 in the Top/Left box has been increased to 360. Recall that the game was balanced before this change, and the choice proportions should be 1/2 for each player. The increase in Row’s Top/Left payoff from 72 to 360 would make Top a more attractive choice for a wide range of beliefs, i.e. Row will choose Top unless Column is almost sure to choose Right. Intuitively, one would expect that this change would move Row’s choice proportion up from the 1/2 level that is observed in the balanced game. This intuition is apparent in the choice data for an experiment done with Veconlab software, where the payoffs were in pennies, and with random matching. Each of the three sessions involved 10-12 players, with 25 periods of random matching. The proportion of Top choices is 67% for the “360 treatment” game in Table 11.2.

Table 11.2. An Asymmetric Matching Pennies Game
(Row’s payoff, Column’s payoff)

<table>
<thead>
<tr>
<th></th>
<th>Column Player:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Left</td>
</tr>
<tr>
<td><strong>Top</strong></td>
<td>360, 36 ⇒</td>
</tr>
<tr>
<td><strong>Bottom</strong></td>
<td>36, 72 ↑</td>
</tr>
</tbody>
</table>

This intuitive “own payoff effect” of increasing Row’s Top/Left payoff is consistent with the Nash equilibrium prediction. First, notice that there is no equilibrium in non-random strategies, as can be seen from the arrows in Table 11.2, which go in a counter-clockwise circle. To derive the mixed equilibrium
prediction, let \( p \) denote Row’s beliefs about the probability of Right, so \( 1 - p \) is the probability of Left. Thus Row’s expected payoffs are:

\[
\text{Row’s Expected Payoff for Top} = 360(1-p) + 36p = 360 - 324p.
\]

\[
\text{Row’s Expected Payoff for Bottom} = 36(1-p) + 72p = 36 + 36p.
\]

It follows that the difference in these expected payoffs is: \((360 - 324p) - (36 + 36p)\), or equivalently, \(324 - 360p\). Row is indifferent if the expected payoff difference is zero, i.e. if \( p = 324/360 = 0.9 \). Therefore, Row’s best response line stays at the top of the left panel in Figure 11.3 as long as the probability of Right is less than 0.9. The striking thing about this figure is that Column’s best response line has not changed from the symmetric case; it rises from the Bottom/Left corner and crosses over when the probability of Top is 1/2. This is because Column’s payoffs are exactly reflected (36 and 72 on the Left side, 72 and 36 on the right side). In other words, the only way that Column would be willing to randomize is if Row chooses Top and Bottom with equal probability.

In order to produce an imbalance that makes Top less attractive, we reduce Row’s Top/Left payoff in Table 11.1 from 72 to 40. Thus Row’s expected payoff for Top is: \(40(1-p) + 36p = 40 - 4p\), and the expected payoff for Bottom is: \(36(1-p) + 72p = 36 + 36p\). These are equal when \( p = 4/40 \), or 0.1. What has happened in the left panel of Figure 11.3 is that the reduction in Row’s Top/Left payoff has pushed Row’s best response line to the bottom of the figure, unless
Column is expected to play Right with a probability that is greater than 0.1. This downward shift in Row’s best response line is shown on the left side of Figure 11.4. The result is that the best response lines intersect where the probability of Top is 1/2, which was the same prediction obtained from the left sides of Figures 11.2 and 11.3. Thus a change in Row’s Top/Left payoff from 36 to 72 to 360 does not change the Nash equilibrium prediction for the probability of Top. The mathematical reason for the result is that Column’s payoffs do not change, and Row must choose each decision with equal probability or Column would not want to choose randomly.

These invariance predictions are not borne out in the data from the Veconlab experiments reported here. Each of the three sessions involved 25 periods for each treatment, with the order of treatments being alternated. The percentage of Top choices increased from 24% to 74% when the Row’s Top/Left payoff was increased from 40 to 360. The Column players reacted to this change by choosing Right only 36% of the time in the 40 treatment, and 67% of the time in the 360 treatment.

The qualitative effect of Row’s own-payoff effect is captured by a noisy-response model where the sharp-cornered best response functions on the left sides of Figures 11.3 and 11.4 are rounded off, as shown on the right sides of the figures. Notice that the intersection of the curved lines implies that Row’s proportion of Top choices will be below 1/2 in the 40 treatment and above 1/2 in the 360 treatment, and the predicted change in the proportion of Right decisions...
will not be as extreme as the movement from 0.1 to 0.9 implied by the Nash prediction. The actual data averages are shown by the black dots on the right sides of Figures 11.3 and 11.4. These data averages exhibit the strong own-payoff effects that are predicted by the curved best-response lines, and the prediction is fairly accurate, especially for the 40 treatment.

The quantitative accuracy of these predictions is affected by the amount of curvature that is put into the noisy response functions, and is ultimately a matter of estimation. Standard estimation techniques are based on writing down a mathematical function with a “noise parameter” that determines the amount of curvature and then choosing the parameter that provides the best fit. The nature of such a function is discussed next.

IV. Probabilistic Choice

Anyone who has taken an introductory psychology course will remember the little stimulus-response diagrams. Stochastic or noisy response models were developed after researchers noticed that responses could not always be predicted with certainty. The work of a mathematical psychologist, Duncan Luce (1959), suggested a way to model noisy choices, i.e. by assuming that response probabilities are increasing functions of the strength of the stimulus. For example, suppose that a person must judge which of two very faint sounds is loudest. The probability associated with one sound should be an increasing function of the decibel level of that sound. Since probabilities must sum to 1, the choice probability associated with one sound should also be a decreasing function of the intensity of the other sound.

In economics, the stimulus intensity associated with a given response (decision) might be thought of as the expected payoff of that decision. Suppose that there are two decisions, $D_1$ and $D_2$, with expected payoffs that we will represent by $E_1$ and $E_2$. For example, the decisions could be Row’s choice of Top or Bottom, and the expected payoffs would be calculated using the equations given earlier, e.g. $E_1 = 360(1-p) + 36p$, where $p$ represents the probability which the Row player thinks Column will choose Right. Instead of using the mathematical expressions for these expected payoffs, we will just use the shorthand notation, $\pi_1$ and $\pi_2$, which is both simpler and more general since it applies to any two decisions. (Think of $\pi$, the Greek letter pronounced like “pie,” as shorthand for payoff.)

Using Luce’s suggestion, the next step is to find an increasing function, i.e. one with a graph that “goes uphill.” (Mathematically speaking, a function, $f$, is increasing if a $f(\pi_1) > f(\pi_2)$ whenever $\pi_1 > \pi_2$, but the intuitive idea is that the slope in a graph is from lower-left to upper-right.) Several examples of increasing functions are the linear function: $f(\pi_1) = \pi_1$, or the exponential function, $f(\pi_1) = \pi_1 e^{\pi_1}$.
exp(\(\pi_1\)). The linear function obviously has an uphill slope; its graph is just the 45-degree line. The exponential function has a curved shape, like a hill that keeps getting steeper, like a snow-capped Mt. Fuji in a Japanese woodblock print.

Once we have found an increasing function, we might be tempted to assume that the probability of the decision is determined by the function itself: \(\Pr(D_1) = f(\pi_1)\) and \(\Pr(D_2) = f(\pi_2)\). The problem with this approach is that it does not ensure that the two probabilities sum to 1. This is easily fixed by a simple trick with a fancy name: “normalization.” Just divide each function by the sum of the functions:

\[
\frac{f(\pi_1)}{f(\pi_1) + f(\pi_2)} \quad \text{and} \quad \frac{f(\pi_2)}{f(\pi_1) + f(\pi_2)}.
\]

If you are feeling a little unsure, try adding the two ratios in (11.1) to show that \(\Pr(D_1) + \Pr(D_2) = 1\). Now let’s see what this gives us for the linear case:

\[
\Pr(D_1) = \frac{\pi_1}{\pi_1 + \pi_2} \quad \text{and} \quad \Pr(D_2) = \frac{\pi_2}{\pi_1 + \pi_2}.
\]

Suppose \(\pi_1 = \pi_2 = 1\). Then each of the probabilities in (11.1) will equal \(1/(1+1) = 1/2\). This result holds as long as the expected payoffs are equal, even if they are not both equal to 1. This makes sense; if each decision is equally profitable, then there is no reason to prefer one over the other, and the choice probabilities should be 1/2 each. Next notice that if \(\pi_1 = 2\) and \(\pi_2 = 1\), the probability of choosing decision \(D_1\) is higher (2/3), and this will be the case whenever \(\pi_1 > \pi_2\).

One potential limitation to the usefulness of the payoff ratios in (11.2) is that expected payoffs may be negative if losses are possible. This problem can be avoided if we choose a function in (11.1) that cannot have negative values, i.e. \(f(\pi) > 0\) even if \(\pi < 0\). This non-negativity is characteristic of the exponential function, which can be used in (11.1) to obtain:

\[
\Pr(D_1) = \frac{\exp(\pi_1)}{\exp(\pi_1) + \exp(\pi_2)} \quad \text{and} \quad \Pr(D_2) = \frac{\exp(\pi_2)}{\exp(\pi_1) + \exp(\pi_2)}.
\]

This avoids the possibility of negative probabilities, and all of the other useful properties of (11.2) are preserved. The probabilities in (11.3) will sum to one, they will be equal when the expected payoffs are equal, and the decision with the higher expected payoff will have a higher choice probability. The probabilistic choice model that is based on exponential functions is known as the logit model.
The curved lines in the right parts of the figures in this chapter were constructed using logit choice functions, but not quite the ones in equation (11.3). It is true that (11.3) applied to the expected payoffs for the matching pennies games will produce curved response functions, but the lines will not have as much curvature as those in the figures in this chapter. The lines drawn with equation (11.1) have corners that are too sharp to explain the “own payoff effects” that we are seeing in the matching pennies games. Just as we could make it harder to distinguish between the width of two pins by making them each half as thick, we can add more noise or randomness into the choice probabilities in (11.3) by reducing all expected payoffs by a half or more. Intuitively speaking, dividing all expected payoffs by 100 may inject more randomness, since dollars become pennies, and non-monetary factors (boredom, indifference, playfulness) may have more influence. The right panels of the figures in this chapter were obtained by using the logit model with all payoffs being divided by 10:

\[(11.4) \quad \Pr(D_1) = \frac{\exp(\pi_1/10)}{\exp(\pi_1/10) + \exp(\pi_2/10)}, \quad \Pr(D_2) = \frac{\exp(\pi_2/10)}{\exp(\pi_1/10) + \exp(\pi_2/10)}.\]

At this point, you are probably wondering, why 10, why not 100? The degree to which payoffs are “diluted” by dividing by larger and larger numbers will determine the degree of curvature in the noisy response functions. Thus we can think of the number in the denominator of the expected payoff expressions as being an error parameter that determines the amount of randomness in the predicted behavior. The logit error parameter will be called \(\mu\), and it is used in the logit choice probabilities:

\[(11.5) \quad \Pr(D_1) = \frac{\exp(\pi_1/\mu)}{\exp(\pi_1/\mu) + \exp(\pi_2/\mu)}, \quad \Pr(D_2) = \frac{\exp(\pi_2/10)}{\exp(\pi_1/\mu) + \exp(\pi_2/\mu)}.\]

The logit form in (11.5) is flexible, since the degree of curvature is captured by a parameter that can be estimated. It also has the intuitive property that an increase in the payoffs will reduce the “noise,” i.e. will reduce the curvature of the noisy best response curves in the figures (see question 4).

**Extensions and Further Reading**

The use of probabilistic choice functions in the analysis of games was pioneered by McKelvey and Palfrey (1995), and the intersections of noisy best response lines in Figures 11.2-11.4 correspond to the predictions of their quantal response equilibrium. (A “quantal response” is essentially the same thing as a
Goeree, Holt, and Palfrey (2001) apply this framework to the analysis of a number of matching pennies games. Their analysis also includes the effects of risk aversion, which can explain the over-prediction of own-payoff effects for high payoffs that is seen on the right side of Figure 11.3. Risk aversion introduces diminishing marginal utility that reduces the attractiveness of the large payoff for the row player, and this shifts that person's best response line down so that the intersection is closer to the data average point.

All of the games considered in this chapter involved two decisions, but the same principles can be applied to games with more decisions (see Goeree and Holt, 1999; Capra, Goeree, Gomez, and Holt, 1999, 2001, which will be discussed in the next two chapters).

The focus in this chapter has been on games, but probabilistic choice functions can be applied to simple decision problems. For example, recall that a risk neutral person would make exactly 4 safe choices in the lottery choice menu presented in Chapter 4, and a person with “square root utility” would make 6 safe choices. These predictions produce lines with sharp corners, e.g. the dashed line in Figure 4.2, which looks a lot like the curved lines in the figures in this chapter. The actual data averages shown in that figure have smoothed corners that would result from a probabilistic choice function, and Holt and Laury (2001) did find that such a function provided a good explanation of the actual pattern of choice proportions.

Questions

1. In a matching pennies game played with pennies, one person loses a penny and the other wins a penny, so the payoffs are 1 and −1. Show that there is a simple way to transform the game in Table 11.1 (with payoffs of 40 and 80) into this form. (Hint: first divide all payoffs by 20, and then subtract a constant from all payoffs. This would be equivalent to paying in 20-cent coins if they existed, and charging an entry fee to play.)

2. Consider a soccer penalty kick situation where the kicker is equally skillful at kicking to either side, but the goalie is better diving to one side. In particular, the kicker will always score if the goalie dives away from the kick. If the goalie dives to the side of the kick, the kick is always blocked on the goalie’s right but is only blocked with probability one half on the goalie’s left. Represent this as a simple game, in which the goalie earns a payoff of +1 for each blocked kick and −1 for each goal, and the kicker earns −1 for each blocked kick and +1 for each goal. A 0.5 chance of either outcome results in an expected payoff of 0. Determine the equilibrium probabilities used in the Nash equilibrium.

3. Show that the two choice probabilities in (11.5) sum to 1.
4. Show that doubling all payoffs, i.e. both $\pi_1$ and $\pi_2$, has the same effect as reducing the noise parameter $\mu$ in (11.5) by half.

5. Show that multiplying all payoffs in (11.2) by 2 will not affect choice probabilities.

6. Show that adding a constant amount, say $x$, to all payoffs in (11.5) will have no effect on the choice probabilities. Hint: Use the fact that an exponential function of the sum is the product of the exponential functions of the two components of the product: $\exp(\pi+x) = \exp(\pi)\exp(x)$. 

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Chapter 12. The Traveler’s Dilemma

Each player’s best decision in a (single play) prisoner’s dilemma is to defect, regardless of what the other person is expected to do. The dilemma is that both could be better off if they resist these incentives and cooperate. The “traveler’s dilemma” is a game with a richer set of decisions; each person must claim a money amount, and the payment is the minimum of the claims, plus a (possibly small) payment incentive to be the low claimant. Both would be better off making identical high claims, but each has an incentive to “undercut” the other to obtain the reward for having the lower claim. This game is similar to a prisoner’s dilemma in that the unique equilibrium involves payoffs that are lower than can be achieved with cooperation. But here there is a more interesting dimension, since the optimal decision in the traveler’s dilemma does depend on what the other person is expected to do. Thus the game is more sensitive to interactions of imprecise beliefs and small variations in decisions. These interactions can cause data patterns to be quite far from Nash predictions, as will be illustrated with a set of iterated spreadsheet calculations. This is one of my favorite games! It can be played with the instructions in the Appendix or with the Vecon program TG.

I. A Vacation with An Unhappy Ending?

Once upon a time, two travelers were returning from a tropical vacation where they had purchased identical antiques that were packed in identical suitcases. When both bags were lost in transit, the passengers were asked to produce receipts, an impossible task since the antiques were purchased with cash. The airline representative calmly informed the travelers that they should go into separate rooms and fill out claim sheets for the contents of the suitcases. He assured them that both claims would be honored if a) they are equal, and b) they are no greater than the liability limit of $200.00 per bag in the absence of proof of purchase. There was also a minimum allowed claim of $80.00 to cover the cost of the luggage and inconvenience. One of the travelers expressed some frustration, since the value of the suitcase and antique combined was somewhat above $200.00. The other traveler, who was even more pessimistic, asked what would happen if the claims were to differ. To this, the reply was: “If the claims are unequal, then we will assume that the higher claim is unjustly inflated, and we will reimburse you both at an amount that equals the minimum of the two claims. In addition, there will be a $5.00 penalty assessed against the higher claim and a $5.00 reward added to the compensation of the lower claimant.” The dilemma was that each could obtain reasonable compensation if they made matching high claims of $200, and resisted the temptation to come down to $199 to capture the
$5 reward for being low. If one person expects the other one to ask for $199, the best response is $198, but what if the other person is thinking in similar thoughts? This line of reasoning leads to an unfortunate possibility, that a cycle of anticipated moves and counter-moves may cause each traveler to put in very low claims. Unfortunately, there was no way for the two travelers to communicate between rooms. The unhappy ending has both making low claims, and the alternative happy ending involves two high claims. The actual outcome is an empirical question that we will consider below.

This game was introduced by Basu (1994), who viewed it as a dilemma for the game theorist. With a very low penalty rate, there is little risk in making a high claim, and yet each person has an incentive to “undercut” any common claim level. For example, suppose that both are considering claims of $200; perhaps they whispered this to each other on the way to the separate rooms. Once alone, each might reason that a deviation to $199 would reduce the minimum by $1, which is more than made up for by the $5 reward for being low. In fact, there is no belief about the other’s claim that would make one want to claim the upper limit of $200. If one expects any lower claim, then it is better to undercut that lower claim as well. Reasoning in this manner, we can rule all common claims as candidates for equilibrium, except for the lowest possible claim of $80. Similar reasoning shows that no configuration with unequal claims can be an equilibrium (Question 1).

The unique Nash equilibrium for the traveler’s dilemma has another interesting property: it can be derived from an assumption that each person knows that the other is perfectly rational. Recall that there is no belief about the other’s claim that would justify a claim of $200. Since they each know that the other will never claim $200, then the upper bound has shifted to $199, and there is no belief that justifies a claim of $199. To see this, suppose that claims must be in integer dollar amounts and note that $199 is a best response to the other’s claim of $200 but it is not a best response to any lower claim. Since $200 will not be claimed by any rational person, it follows that $199 is not a best response to any belief about the other’s decision. Reasoning in this manner, we can rule out all successively lower claims except for the very lowest feasible claim. (This argument based on common knowledge of rationality is called “rationalizability,” and the minimum possible claim in this game is the unique rationalizable equilibrium. Economists often place juicy, persuasive adjectives in front of the word equilibrium, sometimes to no avail. This will turn out to be one of those times.)

The dilemma for the theorist is that the Nash equilibrium is not sensitive to the size of the penalty/reward level, as long as this level is larger than the smallest possible amount by which a claim can be reduced. For example, the unilateral incentive to deviate from any common claim in the traveler’s dilemma is not affected by if the penalty/reward rate is changed from $5 to $4. If both
were planning to choose a claim of $200, then one person’s deviation to $199 would reduce the minimum to $199, but would result in a reward of $4 for the deviator. Thus the person deviating would earn $199 + $4 instead of the $200 that would be obtained if they both claim $200. The same argument can be used to show that there is an incentive to undercut any common claim, whether the penalty/reward rate is $2 or $200. Thus the Nash equilibrium is not sensitive to this penalty/reward rate as long as it exceeds $1, whereas it one might expect observed claims to be responsive to large changes in this payoff parameter.

II. Data
The traveler’s dilemma was analyzed by Capra et al. (1999). This was an exciting experiment. We expected the penalty/reward parameter to have a strong effect on actual claim choices, even though the unique Nash prediction would be independent of changes in this parameter. The design involved two parts, A and B, with different penalty/reward parameters. Each session involved about 10-12 subjects, who were randomly paired at the start of each round, for a series of 10 rounds. The part A data averages for four of the treatments are plotted in Figure 12.1, where the horizontal axis shows the round number and the penalty/reward parameter is denoted by $R$.

![Figure 12.1. Data for the Traveler’s Dilemma (Capra et al. 1999)](image-url)
With a high penalty/reward parameter of $0.80, the claims average about $1.20 in round 1 and fall to levels approaching $0.80 in the final four rounds, as shown by the thick solid line at the bottom of the figure. The data for the $0.50 treatment start somewhat higher but also approach the Nash prediction in the final rounds. In contrast, the round 1 averages for the $0.10 and $0.05 treatments, plotted as dashed lines, start at about a dollar above the Nash prediction and actually rise slightly, moving away from the Nash prediction. The data for the intermediate treatments ($0.20 and $0.25) are not shown, but they stayed in the middle range ($1.00 to $0.50) below the dashed lines and above the solid lines, with some more variation and crossing over.

III. Learning and Experience

Notice that the most salient feature of the traveler’s dilemma data, the strong effect of the penalty/reward parameter, is not predicted by the Nash equilibrium. Some might dismiss this game on the grounds that it is somewhat artificial. There is some truth to this, although some standard economic games do involve payoffs that depend on the minimum price, as is the case with “Bertrand” price competition (Chapter 15) or the next section’s minimum-effort coordination game. An alternative perspective is that this game involves an intentionally abstract setting, which serves as a paradigm for some types of strategic interactions. The traveler’s dilemma is no more about lost luggage than the prisoner’s dilemma is about actual prisoners. If standard game theory cannot predict well in such simple situations, then some rethinking is overdue. At a minimum, it would be nice to have some idea of when the Nash equilibrium will be useful and when it will not. Even better would be a theoretical apparatus that explains both the convergence to Nash predictions in some treatments and the divergence in others. The rest of this chapter pertains to some possible approaches to this problem. We begin with an intuitive discussion of learning.

Behavior in an experiment with repeated random pairings may evolve from round to round as people learn what to expect. For example, consider the data in Table 12.1 from a classroom experiment conducted at the University of Virginia. Claims were required to be between 80 and 200 cents, and the penalty/reward parameter was 10 cents. There were 20 participants, who were divided into 10 pairs. Each 2-person team had a handheld wireless PDA with a touch-sensitive color screen that showed the HTML displays from the Veconlab software. It was a nice spring day, and this class was conducted outside on the “lawn,” with students being seated informally in a semi-circle. The table shows the decisions for five of the teams during the first four rounds. The round is listed on the left, and the average of all 10 claims is shown in the second column. The
remaining columns show the team’s own decisions, which are listed above the
decision of the other team for that round (shown in parentheses).

First consider the round 1 decision for “Stacy/Naomi,” who claimed 150. The
other team was lower, at 135, so the earnings for Stacy/Naomi were the
minimum, 135, minus the penalty of 10 cents, or 125. “SuzSio” began lower, at
100, and encountered a claim of 133 in round 1. This team then cut their claim to
95, and encountered an even higher claim of 191 in round 2. This caused them to
raise their claim to 125, and they finished round 10 with a claim of 160.

Table 12.1. Traveler’s Dilemma Data for a Classroom Experiment with R = 10
Key: Own Claim (Other’s Claim)

<table>
<thead>
<tr>
<th>Round</th>
<th>Average (12 claims)</th>
<th>SuzSio</th>
<th>K Squared</th>
<th>Kurt/Bruce</th>
<th>JessEd</th>
<th>Stacy/Naomi</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>137</td>
<td>100 (133)</td>
<td>080 (195)</td>
<td>139 (140)</td>
<td>133 (100)</td>
<td>150 (135)</td>
</tr>
<tr>
<td>2</td>
<td>131</td>
<td>095 (191)</td>
<td>098 (117)</td>
<td>135 (140)</td>
<td>80 (130)</td>
<td>127 (200)</td>
</tr>
<tr>
<td>3</td>
<td>144</td>
<td>125 (135)</td>
<td>096 (117)</td>
<td>135 (100)</td>
<td>199 (199)</td>
<td>134 (200)</td>
</tr>
<tr>
<td>4</td>
<td>142</td>
<td>115 (125)</td>
<td>130 (100)</td>
<td>125 (115)</td>
<td>198 (115)</td>
<td>150 (1.34)</td>
</tr>
</tbody>
</table>

The team “K Squared” (Katie and Kari) began round 1 with a decision of
80 cents, which is the Nash equilibrium. They had the lower claim and earned the
minimum, 80, plus the 10 cents for being low. After observing the other’s claim
of 195 for that round, they raised their claim to 98 in round 2, and eventually to
120 in round 10. The point of this example is that the best decision in a game is
not necessarily the equilibrium decision; it makes sense to respond to and
anticipate the way that the others are actually playing. Here the Nash equilibrium
decision of 80 played in the first four periods would have earned 90 in each
round, for a total of 360. “K Squared” actually earned 409 by adapting to
observed behavior and taking more risk. A claim of 200 in all rounds would have
been even more profitable (question 4).

Teams who were low relative to the other’s claim tended to raise their
claims, and teams that were high tended to lower them. This qualitative
adjustment rule, however, does not explain why “SuzSio’s” claim was lowered
even after it had the lower claim in round 1. This reduction seems to be in
anticipation of the possibility that other claims might fall. In the final round, the
claims ranged from 110 to 200, with an average of 146 and a lot of dispersion.
One way to describe the outcome is that people have different beliefs based on different experiences, but by round 10 most expect claims to be about 150 on average, with a lot of variability. If claims had converged to a narrow band, say with all at 150, then the “undercutting” logic of game theory might have caused them to decline as all seek to get below 150. But the variability in claims did not go away, perhaps because people had different experiences and different reactions to those experiences. This variability made it harder to figure out the best response to others’ claims. In fact, claims did not diminish over time; if anything there was a slight upward trend. The highest earnings were obtained by the “JessEd” team, which had a relatively high average claim of 177.

The part A treatment for this session was followed by 10 more rounds with a higher payoff parameter (50 cents). This created a strong incentive to be the low claimant, and claims did decline to the Nash equilibrium level of 80 after the first several rounds. Thus we see that convergence to a Nash equilibrium seems to depend on the magnitudes of incentives, not just on whether one decision is slightly better than another. An analysis that is sensitive to the magnitudes of payoff differences will be considered next. The goal is to develop a theoretical explanation of the observed convergence to the Nash prediction in the high-R treatments and the divergence in the low-R treatments.

IV. A Computational Model of Noisy Behavior

Recall from Chapter 11 that stochastic response functions can be used to capture the idea that the magnitudes of payoff differences matter. The form of these functions is based on the intuitive idea that a person is much more likely to be able to determine which of two sounds is louder when the decibel levels are not close together. In these games, the expected payoff is the stimulus analogous to the decibel level, so let’s begin by calculating expected payoffs for each decision. To keep the calculations from becoming tedious, suppose that there are only 13 possible decisions in even ten-cent amounts: 80, 90, 100, … 200. A person’s beliefs will be represented by 13 probabilities that sum to 1.

These belief probabilities can be used to calculate expected payoffs for each decision, and the logit functions introduced in the previous chapter can then be used to determine the choice probabilities that result from the initial beliefs. The intuitive idea is that a decision is more likely if its expected payoff is higher. In particular, choice probabilities are assumed to be increasing (exponential) functions of expected payoffs, normalized so that all probabilities sum to 1.

Some notation will be useful for the calculation of expected payoffs. Let $P_i$ be the probability associated with claim $i$, so beliefs are characterized by $P_{80}$, $P_{90}$, …, $P_{200}$. First consider a claim of 80, which will tie with probability $P_{80}$ and will be low with probability $1 - P_{80}$. The expected payoff is:
expected payoff for 80 = 80P_{80} + (80 + R)(1 - P_{80}).

Notice that the first term covers the case of a tie and the second covers the case of having the low claim. For a claim of 90, we have to consider the chance of having the higher claim and incurring the penalty of $R$, so the expected payoff is:

expected payoff for 90 = 90P_{90} + (90 + R)(1 - P_{80} - P_{90}) + (80 - R)P_{80}.

As before, the first term is for the possibility of a tie at 90, and the second is for the possibility that the other claim is above 90, which occurs with probability $1 - P_{80} - P_{90}$. The third term now reflects the possibility of having the higher claim.

The payoff structure can be clarified by considering a higher claim, say 150.

$$
\text{expected payoff for 150} = 150P_{150} + (150 + R)[1 - P_{80} - P_{90} - ... - P_{150}] + (80 - R)P_{80} + (90 - R)P_{90} + ... (140 - R)P_{140}.
$$

In order from top to bottom, the parts on the right side of (12.1) correspond to the cases of a tie, of having the lower claim, and of having the higher claim.

This section shows you how to do these calculations in an Excel spreadsheet that makes it possible to repeat the calculations iteratively in order to obtain predictions for how people might actually behave in a traveler’s dilemma game when we do not assume perfect rationality. The logic of this spreadsheet is to begin with a column of probabilities for each claim (80, 90, ... 200). Given these probabilities, the spreadsheet will calculate a column of expected payoffs, one for each claim. Let the expected payoff for claim $i$ be denoted by $\pi_i^e$, where $i = 80, 90, 100, \text{etc.}$ Then there is a column of exponential functions of expected payoffs, $\exp(\pi_i^e/\mu)$, where the expected payoffs have been divided by an error parameter $\mu$. We want choice probabilities to be increasing in expected payoffs, but these exponential functions cannot be used as probabilities unless they are normalized to ensure that they sum to 1. Thus the column of exponential functions is summed to get the normalizing element in the denominator of the logit probability formula: $\exp(\pi_i^e/\mu)/\Sigma_i(\exp(\pi_i^e/\mu))$.

The best way to read this section is to open up a spreadsheet. The instructions will be provided for Excel, but analogous instructions would work in other spreadsheet programs. Table 12.2 is laid out like a spreadsheet, with columns labeled A, B, ..., and rows labeled 1, 2, ...
Table 12.2. Excel Spreadsheet for Traveler’s Dilemma Logit Responses

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
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<td>6</td>
<td>X</td>
<td>X–R</td>
<td>X+R</td>
<td>P</td>
<td>PX</td>
<td>F(P)</td>
<td>P(X–R)</td>
<td>π</td>
<td>Exp(π/μ)</td>
<td>P</td>
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<td>1096</td>
<td>0.067</td>
</tr>
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<td>170</td>
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<td>70</td>
<td>1096</td>
<td>0.067</td>
</tr>
<tr>
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<td>16140</td>
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</tbody>
</table>

Table 12.3. Column Key for Spreadsheet in Table 12.2 (formulas for row 8 should be copied down to row 20)

<table>
<thead>
<tr>
<th>Column</th>
<th>Variable</th>
<th>Notation</th>
<th>Formula for Row 8</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Claim</td>
<td>X</td>
<td>$A7 + 10</td>
</tr>
<tr>
<td>B</td>
<td>Payoff if Lower</td>
<td>X–R</td>
<td>$A8 – $B$2</td>
</tr>
<tr>
<td>C</td>
<td>Payoff if Higher</td>
<td>X+R</td>
<td>$A8 + $B$2</td>
</tr>
<tr>
<td>D</td>
<td>Probability</td>
<td>P</td>
<td>1</td>
</tr>
<tr>
<td>E</td>
<td>Product</td>
<td>PX</td>
<td>D8*$A8</td>
</tr>
<tr>
<td>F</td>
<td>Cumulative P</td>
<td>F(P)</td>
<td>F7+D8</td>
</tr>
<tr>
<td>G</td>
<td>Product 2</td>
<td>P(X–R)</td>
<td>D8*$B8</td>
</tr>
<tr>
<td>H</td>
<td>Cumulative Product 2</td>
<td></td>
<td>H7 + G8</td>
</tr>
<tr>
<td>I</td>
<td>Expected Payoff</td>
<td>πe</td>
<td>E8 + H7 + (1–F8)*$C8</td>
</tr>
<tr>
<td>J</td>
<td>Exponential of payoff</td>
<td>exp(π/μ)</td>
<td>= exp(I8*$B$1)</td>
</tr>
<tr>
<td>Cell J21</td>
<td>Sum of exponentials</td>
<td>Σ(exp(π/μ))</td>
<td>= sum(J8..J20)</td>
</tr>
<tr>
<td>K</td>
<td>Probability</td>
<td>exp(π/μ)/Σ(exp(π/μ))</td>
<td>= J8/J$21</td>
</tr>
</tbody>
</table>
Step 1. …. It is convenient to have the error parameter, \( \mu \), be at a focal location in the upper left corner, along with the penalty/reward parameter. Put a value of 10 for the error parameter in cell B1, and put 10 for the penalty/reward parameter in cell B2. These numbers can later be changed to experiment with different amounts of noise for different treatments.

Step 2. Put a 70 in cell A7, so that the formula in A8 can be: \( \text{=sum(A7 + 10)} \), which will yield a value of 80. Then this formula can be copied to cells A9 to A20 by clicking on the lower right corner of the A8 box and dragging it down. (You could later go back and expand the table to allow for all penny amounts from 80 to 200, but the smaller table will do for now.)

Step 3. Before beginning to fill in the other cells, put values of 0 in cells F7 and H7; these zeros are used to start cumulative sums, in a manner that will be explained below.

Step 4. Notice that the formula that you used in step 2 is shown at the top of the far right column of Table 12.3. The formulas for row 8 cells of other columns are also shown in the right column. Enter these in cells B8, C8, … K8, making sure to place the \$ symbols in the places indicated. The dollar symbol forces the reference to stay fixed even when the formula is copied to another location. For example, the references to the error parameter will be $B$1, with two $ signs, since both the row and location of the reference to this parameter must stay fixed. A reference to cell $B8$, however, would keep the column fixed at B and allow one to copy the formula from row 8 to other rows. Please use the $ symbols only where indicated, and not elsewhere.

Step 5. The numbers in column K will not make sense until you put the formula for the sum of exponentials into cell J21; this formula is given in the next-to-last row of Table 12.3. Add a formula to sum the probabilities and payoffs by copying the formula in cell J21 to cells D21, E21, and K21.

At this point, the numbers in your spreadsheet should match those in Table 12.2. The initial probability column reflected a belief that the other person would choose 80 with certainty. Therefore the expected payoff in column I is 80 if one matches this claim. Given these beliefs, any higher claim will result in a 10-cent penalty, and the payoff will be other’s claim of 80, which is the minimum, minus 10, or 70 as shown in column I. This column will provide expected payoffs for each decision as the initial belief column D is changed, so let’s look at the structure of the formula in column I:
expected payoff for claim of 80 = E8
(12.2) + (1−F8)*C8 + H7.

This formula has three parts that correspond to the three parts of equation (12.1), i.e. depending on whether the other’s claim is equal to, higher, or lower than one’s own claim. We will discuss these three elements in turn.

Case of a Tie: In equation (12.2), the first term, E8, was calculated as the probability in D8 that the other chooses 80 times the claim itself in A8. Thus this first term covers the case of a tie.

Case of Having the Lower Claim: The second term in (12.2) involves C8, which is the claim plus the reward, R, so this term pertains to the case where one’s claim is the lower one. The (1−F8) term is the probability that the other’s claim is higher, so F8 is the probability that the other’s claim is less than or equal to one’s own claim of 80. Since there is no chance that the other is less than 80, F8 is calculated as F7, which has been set to 0, plus the probability that the other’s claim is exactly 80, i.e. C8. As this formula is copied down the column, we get a sum of probabilities, which can be thought of as the cumulative “less-than-or-equal-to” probabilities. Retracing our steps, one minus the less-than-or-equal-to probabilities in column F will be the probability that the other’s claim is higher, so the second term in (12.1) has the (1−F8) being multiplied by the claim plus the reward.

Case of Having the Higher Claim: Here we have to consider each of the claims that are lower than one’s own claim. Column B has claims with the penalty subtracted, and these payoffs are multiplied by the probability of that claim to yield the numbers in column G. Then column H calculates a cumulative “less-than-or-equal-to” sum of the elements in G. This sum is necessary since, for any given claim, there may be many lower claims that the other can make, with associated payoffs that must be multiplied by probabilities and summed to get an expected payoff.

Now that the expected payoffs are calculated in column I, they are put into exponential functions in column J after dividing by the error parameter in cell B2. These are then summed in cell J21, and the exponentials are divided by the sum to get the choice probabilities in column K. For example, in Table 12.2 the probability associated with a claim of 80 is about 0.184, as shown in cell K8. The other probabilities are about a third as high. These other probabilities are all
equal, since the other person is expected to choose a claim of 80 for sure, and therefore all higher claims lead to earnings of 70, as compared with the 80 that could be earned by matching the other’s expected claim. This reduction in expected payoff is not so severe as to prevent deviations from happening, at least for the error parameter of 10 in cell B1 that is being used here. Try a lower error parameter, say 5, to be sure that it results in a higher probability for the claim of 80, with less chance of an “error” in terms of making a claim that is higher than 80.

Next consider cell E21, which shows the expected value of the claim for the initial beliefs. If you still have a probability of 1 in cell D8 for the lowest claim, then the number in E21 should be 80. As you change the probabilities in column D (making sure that they sum to 1), the expected claim in E21 will change.

Finally, we can use the choice probabilities in column K to calculate the resulting expected claims. To make this calculation, we essentially need to get the formula in E21 over to the right of the K column. At this point, you should save your spreadsheet (in case something goes wrong) and then perform the final two steps:

Step 6. Highlight the shaded block of cells in Table 12.2, i.e. from E6 to K21. Then copy and paste this block with the cursor in cell L6, which will essentially replicate columns E to K, putting them in columns L to R. Now you should see that the average claim in cell L21 is about 133, way above the average claim on 80 based on the initial beliefs.

Step 7. The next step is to perform another iteration. You could mark the shaded block in Table 12.2 again (or skip this step if it is still on the clipboard) and then place the cursor in cell S6 and copy the block. The new average claim in cell S21 should be 166. In two more iterations this average will converge to 178 (in cell AG21), and this average will not change in subsequent iterations.

At this point, there will be a vector of probabilities in column AG with the following interesting property: if these represent initial beliefs, then the stochastic best responses to these beliefs will yield essentially the same probabilities, i.e. the beliefs will be confirmed. This is the notion of equilibrium that was introduced in chapter 11. Even more interesting is the fact that the level of convergence, 178, is approximately equal to the average claim for the $R = 10$ treatment in Figure 12.1. Now try changing the payoff parameter from 10 to 50 to be sure that the average claims converge to 50, and check to be sure that these convergence levels are not too sensitive to changes in initial beliefs, e.g. if you put a 1 in the bottom (A20) element of column D instead of in the A8 element. In this sense, the model of
logit stochastic responses can explain why behavior converges to the Nash prediction in some treatments (50 and 80), and why data go the other side of the set of feasible decisions in other treatments (5 and 10).

In equilibrium, beliefs are confirmed, and the result is called a logit equilibrium, which was used by Capra et al. (1999) to evaluate data from the experiment. Our spreadsheet calculations in this section have been based on an error rate of 10, which is close to the value of 8.3 that was estimated from the actual data (using maximum likelihood techniques), with a standard error of 0.5.

V. Extensions and Further Reading

The experiment just discussed involved repeated random matching, so that people can learn from experience, even though people are not interacting with the same person in each round. Goeree and Holt (2001) provide experimental evidence for traveler’s dilemma games played only once. There is no opportunity to learn from past experience in a one-shot game; each person must base their claim decision on introspection about what the other is likely to do, about what the other thinks they will do, etc. The average claims are strongly influenced by the size of the penalty/reward, even though changes in this parameter have no effect on the unique Nash equilibrium. Of course, it is not reasonable to expect data to conform to a Nash equilibrium in a one-shot game with no past experience, since this equilibrium (in pure strategies) implies that the other’s claim is somehow known. The development of models of introspection is an important and relatively poorly understood topic in game theory.

There are several other games with a similar payoff that depends on the minimum of all decisions. One such game, discussed in the next chapter, has the “weakest-link” property that the output is determined by the minimum of the individual effort levels. Similarly, the shopping behavior of informed consumers may make firms’ profits sensitive to whether or not their price is the minimum in the market. Capra et al. (2001) provide experimental data for a price competition game with “meet-or-release clauses,” which release the buyer from the contract if a lower price offer is found and the original seller refuses to match. If all consumers are informed and all sellers are producing the same product, then each would rather match the other instead of losing all business as informed consumers switch. A unilateral price cut, may, however pick up some business for a group of uninformed consumers. If one firm sets a lower price than the other, then it will obtain the larger market share, and the high-price firm will have to match the other’s price to get any sales at all. This is like the traveler’s dilemma in that earnings are determined by the minimum price, with a penalty for having a higher price. In particular, each firm earns an amount that equals the minimum price times their sales quantity, but the firm that had the lower price initially will have the larger market share by virtue of picking up sales to the informed shoppers.
The Capra et al. (2001) price-competition game also has a unique Nash equilibrium price at marginal cost, since at any higher price there is a unilateral incentive to cut price by a very small amount and pick up the informed shoppers. The Nash equilibrium is independent of the number of informed buyers who respond to even small price differences. Despite this independence property, it is intuitively plausible that a large fraction of buyers who are uninformed about price differences would provide sellers with some power to raise prices. This intuition was confirmed by the results of the experiments reported by the authors. As with the traveler’s dilemma, data averages were strongly affected by a parameter that determines the payoff differential for being low, even though this parameter does not affect the unique Nash equilibrium at the lowest possible decision.

Questions

1. Show that unequal claims cannot constitute a Nash equilibrium in a traveler’s dilemma. Hint: Which person would have a unilateral incentive to deviate, the high claimant or the low claimant?

2. What is the Nash equilibrium for the traveler’s dilemma game where there is a $5 penalty and a $5 reward, and claims must be between $50 and $50? (A negative claim means that the traveler pays the airline, not the reverse.)

3. Consider a traveler’s dilemma with $N$ players who each lose identical items, and the airline requests claim forms to be filled out with the understanding that claims will be between $80$ and $200$. If all claims are not equal, there is a $5$ penalty if one’s claim is not the lowest and a $5$ reward rate for the person with the lowest claim. Speculate on what the effect on average claims of increasing $N$ from 2 to 4 players in a setup with repeated random matchings.

4. Calculate the what the earnings would have been for the “K square” team if they had chosen a claim of 200 in each of the first four rounds of the game summarized in table 12.1.

5. Use the spreadsheet constructed in section IV to show that the choices converge to the Nash level of 80 when the error rate is reduced to 1 for the $R = 10$ treatment.
Chapter 13. Coordination Games

A major role of management is to coordinate decisions so that better outcomes can be achieved. The game considered in this chapter highlights the need for such external management, since otherwise players may get stuck in a situation where nobody exerts much effort because others are not expected to work hard either. In these games, the productivity of each person’s effort depends on that of others, and there can be multiple Nash equilibria, each at a different common effort level. Behavior is sensitive to factors like changes in the effort cost or the size of the group, even though these have no effect on the set of Nash equilibria. The experiment can be run with the Veconlab game CG.

I. “The Minimum Effort Game? That’s One I Can Play!”

Most productive processes involve specialized activities, where distinct individuals or teams assemble separate components that are later combined into a final product. If this product requires one of each of the components, then the number of units finally sold is sensitive to bottlenecks in production. For example, a marine products company that produces 100 hulls and 80 engines will only be able to market 80 boats. Thus there is a bottleneck caused by the division with the lowest output. Students (and professors too) have little trouble understanding the incentives of this type of “minimum-effort game,” as one student’s comment in the section title indicates.

The minimum-effort game was originally discussed by Rousseau in the context of a stag hunt, where a group of hunters form a large circle and wait for the stag to try to escape. The chances of killing the stag depend on the watchfulness and effort exerted by the encircling hunters. If the stag observes a hunter to be napping or hunting for smaller game instead, then the stag will attempt an escape through that sector. If the stag is able to judge the weakest link in the circle, then the chances of escape depend on the minimum of the hunters’ efforts. The other aspect of the payoffs is that effort is individually costly, e.g., in terms of giving up the chance of bagging a hare or taking a rest.

The game in Table 13.1 is a 2x2 version of a minimum effort game. First, consider the lower-left corner where each person has a low effort, and payoffs are 70 each. If the Row player increases effort, while the Column player maintains low efforts, then the relevant Row payoff is reduced to 60, as can be seen from the top-left box. Think of it this way, the unilateral increase in effort will not increase the minimum, but the extra cost reduces Row’s payoff form 70 to 60, so the cost of this extra “unit” of effort is 10.
Table 13.1. A Minimum Effort Game (Row’s payoff, Column’s payoff)

<table>
<thead>
<tr>
<th>Column Player:</th>
<th>Low Effort (1)</th>
<th>High Effort (2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>High Effort (2)</td>
<td>60, 70</td>
<td>80, 80</td>
</tr>
<tr>
<td>Low Effort (1)</td>
<td>70, 70</td>
<td>70, 60</td>
</tr>
</tbody>
</table>

Now suppose that the initial situation is a high effort for Row and a low effort for Column, as in the upper-left box. An increase in Column’s effort will raise the minimum, which raises Row payoff from 60 to 80. Thus a “unit” increase in the minimum effort will raise payoffs by 20, holding one’s own effort constant. These observations can be used to devise a mathematical formula for payoffs that will be useful in devising more complex games. Let the low effort be 1 and the high effort be 2, as indicated by the numbers in parentheses next to the row and column labels in the payoff table. Then the payoffs in this table are determined by the formula: 60 plus (20 times the minimum effort) minus (10 times one’s own effort). Let \( M \) denote the minimum effort and \( E \) denote one’s own effort, so that this formula is:

\[
\text{own payoff} = 60 + 20M - 10E.
\]

For example, the upper right payoffs are determined by noting that both efforts are 2, so the minimum \( M \) is 2, and the formula yields: 60 + (20)*2 – (10)*2 = 80.

The formula in (13.1) was used to construct the payoffs in Table 13.2, for the case of efforts that can range from 1 to 7. Notice that the four numbers in the bottom-left corner of the table correspond to the row payoffs in Table 13.1 above. With a larger number of possible effort levels, we see the dramatic nature of the potential gains from coordination on high-effort outcomes. At a common effort of 7, each person earns 130, which is almost twice the amount earned at the lowest effort level. Movements along the diagonal from the lower-left to the upper-right corner show the benefits from coordinated increases in effort; a one unit increase in the minimum effort raises payoff by 20, minus the cost of 10 for the increased effort, so each diagonal payoff is 10 larger than the one lower on the diagonal.

Besides the gains from coordination, there is an additional feature of Table 13.2 that is related to risk. When the row player chooses the lowest effort (in the bottom row), the payoff is 70 for sure, but the highest effort may yield payoffs that range from 10 to 130, depending on the column player’s choice. This is the
strategic dilemma in this coordination game: there is a large incentive to coordinate on high efforts, but the higher effort decisions are risky.

Table 13.2. The Van Huyck et al. (1990) Minimum Effort Game
With Row Player’s Payoffs Determined by the Minimum of Others’ Efforts

<table>
<thead>
<tr>
<th>Row’s Effort</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>10</td>
<td>30</td>
<td>50</td>
<td>70</td>
<td>90</td>
<td>110</td>
<td>130</td>
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<tr>
<td>6</td>
<td>20</td>
<td>40</td>
<td>60</td>
<td>80</td>
<td>100</td>
<td>120</td>
<td>120</td>
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<tr>
<td>5</td>
<td>30</td>
<td>50</td>
<td>70</td>
<td>90</td>
<td>110</td>
<td>110</td>
<td>110</td>
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<tr>
<td>4</td>
<td>40</td>
<td>60</td>
<td>80</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
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<tr>
<td>3</td>
<td>50</td>
<td>70</td>
<td>90</td>
<td>90</td>
<td>90</td>
<td>90</td>
<td>90</td>
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<tr>
<td>2</td>
<td>60</td>
<td>80</td>
<td>80</td>
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<td>80</td>
<td>80</td>
<td>80</td>
</tr>
<tr>
<td>1</td>
<td>70</td>
<td>70</td>
<td>70</td>
<td>70</td>
<td>70</td>
<td>70</td>
<td>70</td>
</tr>
</tbody>
</table>

An additional element of risk is introduced when more than two people are involved. Suppose that payoffs are still determined by the formula in (13.1), where \( M \) is the minimum of all players’ efforts. The result is still the payoff table 13.2, where the payoff numbers pertain to the row player as before, but where the columns denote the minimum of the other player’s efforts. For example, the payoffs are all 70 in the bottom row because Row’s effort of 1 is the minimum regardless of which column is determined by the minimum of the others’ efforts. Notice that the incorporation of larger numbers of players into this minimum-effort game does not alter the essential strategic dilemma, i.e. that high efforts involve high potential gains but more risk. At a deeper level, however, there is more risk with more players, since the minimum of a large number of independently selected efforts is likely to be small when there is some variation from one person to another. This is analogous to having a large number of hunters spread out in a large circle, which gives the stag more of a chance to find a sector where one of the hunters is absent or napping.

II. Nash Equilibria, Numbers Effects, and Experimental Evidence

These intuitive considerations (the gains from coordination and the risks of un-matched high efforts) are not factors in the structure of the Nash equilibria in this game. Consider Table 13.1, for example. If the other person is going to exert a low effort, the best response is a low effort that saves on effort cost. Thus the lower-left outcome, low efforts for each, is a Nash equilibrium. But if the other player is expected to choose a high effort, then the best response is a high
effort, since the gain of 20 for the increased minimum exceeds the cost of 10 for the additional unit of effort. Thus the high-effort outcome in the upper-right corner of Table 13.1 is also a Nash equilibrium. This is an important feature of the coordination game: there are multiple equilibria, with one that is preferred to the other(s).

The presence of multiple equilibria is a feature that differentiates this game from a prisoner’s dilemma, where all players may prefer the high-payoff outcome that results from cooperative behavior. This high-payoff outcome is not a Nash equilibrium in a prisoner’s dilemma since each person has a unilateral incentive to “defect.” The equilibrium structure for the game in Table 13.1 is not affected by adding additional players, each choosing between efforts of 1 and 2, and with the row player’s payoffs determined by the column that corresponds to the minimum of the others’ efforts. In this case, adding more players does not alter the fact that there are two equilibria (in non-random strategies): all choose low efforts or all choose high efforts. Just restricting attention to the Nash equilibria would mean ignoring the intuition that adding more players would seem to make the choice of a high effort riskier, since it is more likely that one of the others will choose low effort and pull the minimum down.

The problem of multiple equilibria is more dramatic with more possible efforts, since any common effort is a Nash equilibrium in Table 13.2. To see this, pick any column and notice that the row player’s payoffs are highest on the shaded diagonal. As before, this structure is independent of changes in the number of players, since such changes do not alter the payoff table. These considerations were the basis of an experiment conducted by Van Huyck et al. (1990), who used the payoffs from Table 13.2 for small groups (size 2) and large groups (size 14-16). The large groups played the same game ten consecutive times with the same group, and the lowest effort was announced after each round to enable all to calculate their payoffs (in pennies). Even though a majority of individuals selected high efforts of 6 or 7 in the first round, the minimum effort was no higher than 4 in the first round for any group. A minimum of 4 meant that the higher efforts were “wasted,” and effort reductions followed in subsequent rounds. The minimum fell to the lowest level of 1 in all groups, and almost all decisions in the final round were at the lowest level.

This experiment is important since previously it had been a common practice in theoretical analysis to assume that individuals could coordinate on the best Nash equilibrium when there was general agreement about which one is best, as is the case for Table 13.2. In contrast, the subjects in the experiment managed to end up in the equilibrium that is worst for all concerned. With groups of size 2, individuals were able to coordinate on the highest effort, except when pairings were randomly reconfigured in each round. With random matching, the outcomes were variable, with average efforts in the middle range. In any case, it is clear
that group size had a large impact on the outcomes, even though changes in the numbers of players had no effect on the set of equilibria.

The coordination failures for large groups captured the attention of macroeconomicists, who had long speculated about the possibility that whole economies could become mired in low-productivity states, where people do not engage in high levels of market activity because no one else does. The macroeconomic implications of coordination games are discussed in Bryant (1983), Cooper and John (1998), and Romer (1996), for example.

III. Effort-Cost Effects

Next consider what happens when the cost of effort is altered. For example, suppose that the effort cost of 10 used to construct Table 13.1 is raised to 19. In this case, a one unit increase in each person’s effort raises payoffs by 20 minus the cost of 19, so the payoffs in the upper-right box of the table are 71, only 1 higher than the payoffs in the lower-left box. Notice that this increase in effort cost did not change the fact that there are two Nash equilibria, and that both players prefer the high-effort equilibrium. However, simple intuition suggests that effort levels in this game may be affected by effort costs. From the row player’s perspective, the top row offers a possible gain of only one cent and a possible loss of 10 cents, as compared with the bottom row.

<table>
<thead>
<tr>
<th>Table 13.3. A Minimum-Effort Game With High Effort Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Row’s payoff, Column’s payoff)</td>
</tr>
<tr>
<td><strong>Column Player:</strong></td>
</tr>
<tr>
<td><strong>Low Effort (1)</strong></td>
</tr>
<tr>
<td><strong>Row Player:</strong></td>
</tr>
<tr>
<td>High Effort (2)</td>
</tr>
<tr>
<td>Low Effort (1)</td>
</tr>
</tbody>
</table>

Goeree and Holt (1999, 2001) report experiments in which the cost of effort is varied between treatments, using the payoff formula:

(13.2) \[ \text{payoff} = M - cE, \]

where \( M \) is the minimum of the efforts, \( E \) is one’s own effort, and \( c \) is a cost parameter that is varied between treatments. Decisions were restricted to be any amount between (and including) $1.10 and $1.70. As before, any common effort is a Nash equilibrium in this game, as long as the cost parameter, \( c \), is between 0
This is because a unilateral decrease in effort by one unit will reduce the minimum by 1, but it will only reduce the cost by an amount that is less than 1. Therefore, a unit decrease in effort will reduce one’s payoff by $1 - c$. Conversely, a unilateral increase in effort by one unit above some common level will not raise the minimum, but the payoff will fall by $c$. Even though deviations from a common effort are unprofitable when $c$ is greater than 0 and less than 1, the magnitude of $c$ determines the relative cost of “errors” in either direction. A large value of $c$, say 0.9, makes increases in effort more costly, and a small value of $c$ makes decreases more costly.

Figure 13.1 shows the results for sessions that consisted of 10-12 subjects who were randomly paired for a series of 10 rounds. There were three sessions with a low effort cost of 25 cents, and the averages by round for these sessions are plotted as thin dashed lines. The thick dashed line is the average over all three treatments. Similarly, the thin solid lines plot round-by-round averages for the three sessions with a high effort cost of 75 cents, and the thick solid line shows the average for this treatment.

Efforts in the first round average in the range from $1.35 to $1.50, with no separation between treatments. Such separation arises after several rounds, and
average efforts in the final round are $1.60 for the low-cost treatment, versus $1.25 for the high-cost treatment. Thus we see a strong cost effect, even though any common effort is a Nash equilibrium.

One session in each treatment seemed to approach the boundary, which raises the issue of whether behavior will “lock” on one of the extremes. This pattern was observed in a Vconlab classroom experiment in which efforts went to $1.70 by the 10th period. This kind of extreme behavior is not universal, however. Goeree and Holt (1999) report a pair of sessions that were run for 20 rounds. With an effort cost of 25 cents, the decisions converged to about $1.55, and with an effort cost of 75 cents the decisions leveled off at about $1.38. Both of these outcomes seem to fit the pattern seen in Figure 13.1.

IV. An Analysis of Noisy Behavior

The traveler’s dilemma and the coordination game have similar payoff structures, so it is straightforward to modify the spreadsheet in Table 12.2 so that it applies to the coordination game. Here are the steps:

Step 1. Save the traveler’s dilemma spreadsheet under a different name, e.g. cg.xls, before deleting all information to in column L and farther to the right.

Step 2. The coordination experiment has fewer possible decisions, when restricted to 10-cent increments: 110, 120, … 170. Therefore, delete all material in rows 7-10, and in rows 18-20, leaving row 21 as it is. You will have to enter the possible effort levels in column A: 110 in A11, 120 in A12, etc.

Step 3. There is no penalty/reward parameter in the coordination game, so enter a 0 in cell B2. Next put “C =” in cell A3, and enter a value of 0.75 in cell B3.

Step 4. The starting values for the cumulative column sums will have to be moved, so put values of 0 in cells F10 and H10.

Step 5. To set the initial probabilities, change the entry in cell D11 to 1, and leave the other entries in that column to be 0.

Step 6. Next we have to subtract the effort cost from the expected payoff formula in cell I11. The cost per unit effort in cell B3 must be multiplied by the effort in column A to calculate the total effort cost. Thus you should append the term –$B$3*$A11 to the end of the formula that is already in cell I11. Then this modified formula should be copied into rows 12 to 17 in this column.
Table 13.4. Excel Spreadsheet for Coordination Game Logit Responses

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>I</th>
<th>J</th>
<th>K</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>μ = 10</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>R = 0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>C = 0.75</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>7</td>
<td>X</td>
<td>X</td>
<td>P</td>
<td>PX</td>
<td>F((P))</td>
<td>PX</td>
<td>(π)</td>
<td>(\text{Exp}(\frac{(π)}{\μ}))</td>
<td>P</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>70</td>
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<td></td>
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<td>110</td>
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<td>9</td>
<td>80</td>
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<td>27.5</td>
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<tr>
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<td>0</td>
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<td>20</td>
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<td>11</td>
<td>100</td>
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<td></td>
<td>0</td>
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<td>110</td>
<td>12.5</td>
<td>3.49</td>
<td>0.128</td>
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<tr>
<td>12</td>
<td>110</td>
<td></td>
<td></td>
<td>0</td>
<td>110</td>
<td>110</td>
<td>5</td>
<td>1.64</td>
<td>0.056</td>
<td></td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>120</td>
<td></td>
<td></td>
<td>0</td>
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<td>-2.5</td>
<td>0.77</td>
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<tr>
<td>14</td>
<td>130</td>
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<td>16</td>
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<td>16140</td>
<td>1</td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

**Step 7.** At this point, the numbers should match those in Table 13.4, with a new vector of probabilities in column K. To perform the first iteration, mark the block from E6 to K21, copy, and place the cursor in cell L6 before pasting. This same procedure can be repeated 10 times, so that the average effort after 10 iterations will be in cell AG21.

The expected effort after one iteration, found in L21, is about 119. This rises to 125 by the fifth iteration. When the initial probabilities in column D are changed to put a probability of 1 for the highest effort and 0 for all other efforts, the iterations converge to 126 by the 10th iteration. When the cost of effort is reduced from 0.75 to 0.25, the average claims converge to the 155-156 range after several iterations. These average efforts are quite close to the levels for the two treatment averages in Figure 13.1 in round 10. As noted in the previous chapter, the convergence of iterated stochastic responses indicates an equilibrium in which a given set of beliefs about others’ effort levels will produce a matching distribution of effort choice probabilities. The resulting effort averages are quite good predictors of behavior in the minimum-effort experiment when the error rate
is set to 10. Even though any common effort is a Nash equilibrium, intuition suggests that lower effort costs may produce higher efforts. These spreadsheet calculations of the logit equilibrium are consistent with this intuition; they explain why efforts are inversely correlated with effort costs. A lower error rate will push effort levels to one or another of the extremes, i.e. to 110 or to 170 (question 4).

V. Extensions

Goeree and Holt (1999) also considered the effects of raising the number of players per group from 2 to 3, holding effort cost constant, which resulted in a sharp reduction of effort levels. Effort-cost effects were observed as well in games in which payoffs were determined by the median of the three efforts. Some coordination games may be played only once; Goeree and Holt (2001) report strong effort-cost effects in such games as well. A theoretical analysis of these effects (in the context of a logit equilibrium) can be found in Anderson, Goeree, and Holt (2001).

There is an interesting literature on factors that facilitate coordination on good outcomes in matrix games, e.g. Sefton (1999), Straub (1995), and Ochs (1995). In particular, notions of “risk dominance” and “potential” provide good predictions about behavior when there are multiple equilibria. Models of learning and evolution have also been widely used in the study of coordination games (see references in Ochs, 1995).

Questions

1. Consider the game in the table below, and find all Nash equilibria in pure strategies. Is this a coordination game?

   | Column Player: | Left (26%) | Middle (8%) | Non-Nash (68%) | Right (0%) |
   | Top (68%)     | 200, 50    | 0, 45       | 10, 30         | 20, −250   |
   | Bottom (32%)  | 0, −250    | 10, −100    | 30, 30         | 50, 40     |

2. The numbers under each decision label in the above table show the percentage of people who chose that strategy when the game was played.
only once (Goeree and Holt, 2001). Why is the column player’s most commonly used strategy labeled as “Non-Nash,” and why is it played so frequently?

3. (Advanced) Find the mixed-strategy equilibrium for the game shown in question 1. (Hint: In this equilibrium, column randomizes between Left and Middle. The probabilities associated with Non-Nash and Right are zero, so consider the truncated 2x2 game involving only Left and Middle for column and Top and Bottom for row. Finally, check to be sure that column is not tempted to deviate to either of the two strategies not used.)

4. Use the spreadsheet for the minimum-effort coordination game to fill in average efforts for the following Table. Use initial belief probabilities of 1/8 (or 0.125) for each of the elements in column D between rows 11 to 17. Then discuss the effects of the error parameter on these predictions.

<table>
<thead>
<tr>
<th></th>
<th>µ = 1</th>
<th>µ = 5</th>
<th>µ = 10</th>
<th>µ = 20</th>
</tr>
</thead>
<tbody>
<tr>
<td>High Cost (c = 75)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Low Cost (c = 25)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

5. Consider the game shown below, which gives the Column Player a safe decision, S. Row’s payoffs depend on a parameter $X$ that changes from one treatment to the other. Goeree and Holt (2001) report that the magnitude of $X$ affects the frequency with which the Row Player chooses Top, so the issue is whether this effect is predicted in theory. Find all Nash equilibria in pure and mixed strategies for $X = 0$, and for $X = 400$, under the assumption that each person is risk neutral.

<table>
<thead>
<tr>
<th></th>
<th>Column Player:</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Row Player:</td>
<td>L</td>
<td>R</td>
<td>S</td>
<td></td>
</tr>
<tr>
<td>Top</td>
<td>90, 90</td>
<td>0, 0</td>
<td>X, 40</td>
<td></td>
</tr>
<tr>
<td>Bottom</td>
<td>0, 0</td>
<td>180, 180</td>
<td>0, 40</td>
<td></td>
</tr>
</tbody>
</table>
Part IV. Market Experiments

The games in this part represent many of the standard market models that are used in economic theory. In the Cournot model, firms choose quantities independently, and the aggregate quantity determines the market price. This setup is widely used in economic theory, and it has the intuitive property that the equilibrium price outcomes are decreasing in the total number of sellers. The web game in Chapter 14 implements a setup with linear demand and constant cost, which permits an analysis of the effects of changing the number of competitors or the cost and demand parameters. The Cournot model may be appropriate when firms precommit to production decisions, but in many situations it may be more realistic to model firms as choosing prices independently. Chapter 15 pertains to the simplest model of price competition, the Bertrand model. The product is homogenous, so all sales go the low-priced seller. The game again pertains to a special case with linear demand and constant cost.

Consumers are simulated by the use of an aggregate market demand function in both the Cournot and Bertrand chapters. In contrast, some participants are assigned buyer roles in the double auction and posted offer games that are presented in Chapters 16 and 17 respectively. Buyer and seller activities are essentially symmetric in the double auction, except that buyers tend to bid the price up, and sellers tend to undercut each other’s prices. All bids and asks are public information, as is the transaction price that result when someone accepts the price proposed by someone on the other side of the market. This strategic symmetry is not present in the posted offer auction, where sellers post prices independently, and buyers are given a chance to purchase at the posted prices (further bargaining and discounting is not permitted). The posted offer resembles a retail market, with sellers producing “to order” and selling at catalogue or “list” prices. In contrast, the double auction more closely resembles a competitive “open outcry” market.

The final chapter in this part presents a simplified model of entry and exit, where sellers make entry decisions independently. Entrants’ earnings are a decreasing function of the number of firms that enter, and the alternative payoff for non-entrants is fixed. The Nash equilibrium in this situation may involve randomized strategies.
Chapter 14. Cournot Markets

This market game puts participants in the role of being sellers who choose output quantities independently. The setup allows one to evaluate the predictions of the Cournot (Nash) equilibrium in the context of quadratic payoff functions that result for linear demand and constant cost functions. The Veconlab program “CR” provides an easy way to run enough periods to get some kind of convergence under alternative treatment conditions (e.g. changes in the numbers of sellers per market, in the cost or demand parameters). The repeated nature of most oligopoly markets makes is preferable to use fixed (non-random) matchings, which may facilitate some tacit collusion with small groups.

I. Introduction

The main goal of a theory of industrial organization is to explain the relationship between the aspects of market structure and institutions and the resulting price and efficiency outcomes of market activity. The structural characteristics of a market include factors such as cost and demand conditions, the sizes and numbers of competitors, and the ease of new entry. The institutional characteristics of a market are determined by the rules and accepted practices that govern the competition. Sometimes the rules are specified by an organized trading exchange, like a futures market. In other cases, standard practices have evolved over time and have come to be accepted. For example, firms may mail out catalogue prices at fixed intervals, and then produce to meet the demand that results from these posted prices. This chapter considers a different market institution in which firms choose production quantities independently, and then prices adjust to clear the market. Regardless of how the underlying pricing and production process is modeled, the basic paradigm in industrial organization has these institutional and structural conditions determining economic outcomes, i.e. price, quantity, efficiency, etc. The quantity-competition model to be presented in this chapter is widely used, in part because it provides intuitive predictions about the effects of increased competition on these outcomes.

The model dates to Augustin Cournot (1838), who explained the setup with an example of villagers who go to a spring to carry back water to be sold in the village square. If selling spring water seems a little far-fetched, try to imagine what it would be like to carry all of the water you use some distance to your house. Each seller would decide how much water to bring to market, knowing that a shortage would enable them to use competition among buyers to raise price. Alternatively, an excess supply of water would tend to drive prices down as sellers would cut price to move unwanted inventory. Thus Cournot postulated
that the market price of water would be a decreasing function of the total quantity brought to market by all sellers combined.

Cournot, A. (1938)
Chapter 15. Bertrand Competition

Sellers in this market game choose prices simultaneously, and all sales are made to the firm with the lowest price. Although tacit collusion is possible with small groups, the game can be used to illustrate the harsh nature of price competition in a market with a homogeneous product. The Veconlab program “BR” allows you to change the treatment parameters to investigate the effects of things like the number of sellers and the shapes of the cost and demand functions.

Chapter 16. Double Auction Markets

Traders in a double auction can see all transactions prices and the current bid/ask spread, as is the case with trading on the New York Stock Exchange. The double auction is an extremely competitive institution, given the temptation for traders to improve their offers over time in order to make trades at the margin. The discussion focuses on the nature of price and efficiency outcomes when there are unanticipated demand shifts, and when there is an asymmetry in concentration, which provides sellers with “market power.” The game is can be implemented “by hand” with the instructions in the appendix, or with the Veconlab program DA, which allows you to specify the numbers of buyers, sellers, buyer capacities, seller capacities, periods and whether values and costs are customized or randomly generated from selected intervals.

Chapter 17. Posted-Offer Auction Markets

A posted-offer auction resembles a retail market in which sellers post prices on a take-it-or-leave-it basis. Buyers are then selected randomly and are given the chance to make purchases. This chapter focuses on the price and efficiency effects of seller market power and unanticipated demand shifts, in comparison with the double auction. A posted-offer auction can be run by writing posted prices on the blackboard and dice throws to select buyers, or by using the Veconlab game PO.
Chapter 18. Market Entry and Magic

In market entry games, participants decide independently whether or not to enter a market for which the earnings per entrant are a decreasing function of the number of entrants. The outside option earnings from not entering are fixed. There are typically many Nash equilibria, including one involving randomized strategies. These games are different from the coordination games discussed previously, since there are not equilibria that are better (or worse) for all participants. Kahneman once remarked on the tendency for payoffs to be equalized for the two decisions: “To a psychologist, it looks like magic.” This impression should change for those who have participated in the hand-run or Veconlab versions of this game, which should be run prior to the class discussion.
Part V. Auctions

Auctions can be very useful in the sale of perishable commodities like fish and flowers. The public nature of most auctions is also a desirable feature when equal treatment and “above-board” negotiations are important, as in the public procurement of milk, highway construction, etc. These days, internet auctions can be particularly useful for a seller of a rare or specialty item who needs to connect with geographically dispersed buyers. And auctions are being increasingly used to sell bandwidth licenses, as an alternative to administrative or “beauty contest” allocations, which can generate wasteful lobbying efforts. (Such lobbying is considered in the chapter on rent seeking in the Part VII below.)

Participants of all ages typically find auction games to be exciting, given the win/lose nature of the competition. The two main classes of models are those where bidders know their own private values, and those where the underlying common value of the prize is not known. Private values may differ from person to person, even when the characteristics of the prize are perfectly known. For example, two prospective house buyers may have different family sizes or numbers of vehicles, and therefore, the same square footage may be of much less useful to one than to another, depending on how it is configured into common areas, bedrooms, and parking. An example of a common value auction is the bidding for an oil lease, where each bidder makes an independent geological study of the likely recovery rates for the tract of land being leased. Winning in such an auction can be stressful if it turns out that the bidder overestimated the prize value, a situation known as the “winner’s curse.”

Private value auctions are introduced in Chapter 19, where the focus is on comparisons of bidding strategies with Nash equilibrium predictions for different numbers of bidders. There is also a discussion of several interesting variations: 1) an “all-pay” provision that forces all bidders to pay their own bids, whether or not they win, 2) a “Santa Clause” provision that returns a fraction of auction revenues to the bidders (distributed equally), and 3) a “second-price” rule that only requires the high bidder to pay the second highest bid price.

Common value auctions are considered in Chapter 21, where the focus is on the effects of the numbers of bidders on the winner’s curse. The buyer’s curse, discussed in Chapter 20, is a similar phenomenon that arises with a bidder seeking to purchase a firm of unknown profitability from the current owner. In each case, the curse effect arises because bidders may not realize that having a successful bid is an event that conveys useful information about others’ valuations or estimates of value.

The internet has opened up many exciting possibilities for introducing new auction designs, and laboratory experiments can be used to “testbed” possible
procedures prior to the final selection of the auction rules. The Georgia Irrigation Auction described in Chapter 22 was largely designed and run by experimental economists (Cummings, Holt, and Laury, 2001). It permitted the Georgia Environmental Protection Department to use about five million dollars to compensate some farmers for not using their irrigation permits during a severe draught year. The Vecon webgame captures some (but not all) features of the auction that was run in March 2001.

The irrigation reduction auction is an example of a multi-unit auction, which raises a number of interesting possibilities. One possibility is to let winning bidders be paid their per-acre bid amounts, which means that some farmers will be receiving compensation rates that are higher than those received by others. This is called a “discriminatory” auction. An alternative would have been to will pay all at the per-acre bid rate of the marginal bidder, i.e. the lowest bid that was not accepted. This is called a “uniform price auction.” Some of these and other variations are also discussed in Chapter 22.
Chapter 19. Private Value Auctions

Internet newsgroups and online trading sites offer a fascinating glimpse into the various ways that collectables can be sold at auction. Some people just post a price, and others announce a time period in which bids will be entertained, with the sale going to the person with the highest bid at the time of the close. These bids can be collected by email and held as “sealed bids” until the close, or the highest standing bid at any given moment can be announced in an ascending bid auction. Trade is motivated by differences in individual values, e.g. as some people wish to complete a collection and others wish to get rid of duplicates. This chapter pertains to the case where individual valuations differ randomly, with each person knowing only their own “private” value. The standard model of a private-value auction is one where bidders’ values are independent draws of a uniform distribution. For example, one could throw a 10-sided die twice, with the first throw determining the “tens” digit and the second throw determining the “ones” digit. The chapter begins with the simplest case, where the bidder receives a value and must bid against a simulated opponent, whose bids are in turn determined by a draw from a uniform distribution. Next, we consider the case where the other bidder is another participant. In each case, the slope of the bid/value relationship is compared with the Nash prediction. These auctions can be done by hand with dice (see the Appendix), or with the Veconlab game PV. The webgame allows options that include an “all-pay” rule, a “second price auction,” and a “Santa Clause” provision that returns a fraction of auction revenue to the bidders.

I. Introduction

The rapid development of e-commerce has opened up opportunities for creating new markets that coordinate buyers and sellers at diverse locations. The vast increase in the numbers of potential traders online also makes it possible for relatively thick markets to develop, even for highly specialized commodities and collectables. Many of these markets are structured as auctions, since an auction permits bids and asks to be collected 24 hours a day over an extended period. Gains from trade in such markets arise because different individuals have different values for a particular commodity. There are, of course, many ways to run an auction, and different sets of trading rules may have different performance properties. Sellers, naturally, would be concerned with selecting the type of auction that will enhance sales revenue. From an economist’s perspective, there is interest in finding auctions that promote the efficient allocation of items to those who value them the most. If an auction fails to find the highest-value buyer,
this may be corrected by trading in an “after market,” but such trading itself entails transactions costs, which may be five per cent of the value of the item, and even more for low-value items where shipping and handling costs are significant.

Economists have typically relied on theoretical analysis to evaluate efficiency properties of alternative sets of auction rules. The seminal work on auction theory can be found in a 1962 *Journal of Finance* paper by William Vickrey, who later received a Nobel Prize in economics. Prior to Vickrey, an analysis of auctions would likely be based on a Bertrand-type model in which prices are driven up to be essentially equal to the resale value of the item. It was Vickrey’s insight that different people are likely to have different values for the same item, and he devised a mathematical model of competition in this context. The model is one where there is a probability distribution of values in the population, and the bidders are drawn at random from this population. For example, suppose that a buyer’s value for a car with a large passenger area and low gas mileage is inversely related to how long they have to drive to work. There is a distribution of commuting distances in the population, so there will be a distribution of individual valuations for the car. Each person knows their own needs, and hence their own valuation, but they do not know for sure what other bidders’ values are.

The mathematical model in its simplest form is one where the population distribution of values is uniform on an interval. The uniform distribution, which was so useful in simplifying the analysis of sequential search in chapter 8, is also convenient here. Before discussing the Vickrey model, it is useful to begin with an overview of different types of auctions.

II. Auctions: Up, Down, and the “Little Magical Elf”

Suppose that you are a collector of cards from *Magic: The Gathering*. These cards are associated with a popular game in which the contestants play the role of wizards who duel with spells from cards in a deck. The cards are sold in random assortments, so it is natural for collectors to find themselves with some redundant cards. In addition, rare cards are more valuable, and some out-of-print cards sell for hundreds of dollars. Let’s consider the thought process that may occur to you as you contemplate a selling strategy. Suppose that you log into the newsgroup site and offer a bundle of cards in exchange for a bundle that may be proposed by someone else. You do receive some responses, but the bundles offered in exchange contain cards that you do not desire, so you decide to post a price for each of your cards. Several people seek to buy at your posted price, but suppose that someone makes a price offer that is a little above your initial price in anticipation of excess demand. This causes you to suspect that others are willing to pay more as well, so you post another note inviting bids on the cards, with a one-week deadline. The first bid you receive on a particularly nice card is $40,
and then a second bid comes in at $45. You wonder whether the first bidder would be willing to go up a bit, say to $55. If you go ahead and post the highest current bid on the card each time a new high bid is received, then you are essentially conducting an English auction with ascending bids. On the other hand, if you do not announce the bids and sell to the high bidder at closing time, then you will have conducted a first-price sealed-bid auction. David Lucking-Reiley (1999) reports that the most commonly used auction method for Magic cards is the English auction, although some first-price auctions are observed.

Lucking-Reiley also found a case where these cards were sold in a descending-bid Dutch auction. Here the price begins high and is lowered until someone agrees to the current proposed price. This type of descending-bid auction is used in Holland to sell flowers. Each auction room contains a several clocks, marked with prices instead of hours. Carts of flowers are rolled into the auction room on tracks in rapid succession. When a particular cart is “on deck,” the quality grade and grower information are flashed on an electronic screen. Then the hand of clock falls over the price scale, from higher to lower prices, until the first bidder presses a button to indicate acceptance. This process proceeds quickly, which results in a high sales volume that is largely complete in the by late morning. The auction house is located next to the Amsterdam airport, so that flowers can be shipped by air to distant locations like New York and Tokyo.

If you know your own private value for the commodity being auctioned, then the Dutch auction is like the sealed-bid auction. This argument, due to Vickrey (1961) is based on the fact that you learn nothing of relevance as the clock hand falls in a Dutch auction. Consequently, you might as well just choose your stopping point in advance, just as you would select a bid to submit in a sealed-bid auction. These two auction methods also share the property that the winning bidders pay a price that equals their own bid. In each case, a higher bid will raise the chances of winning, but the higher bid lowers the value of winning. In both auctions, it is never optimal to bid an amount that equals the full amount that you are willing to pay, since in this case you would be indifferent between winning and not winning. To summarize, the descending-bid Dutch auction is strategically equivalent to the first-price, sealed-bid auction in the case of known private prize values.

This equivalence raises the issue of whether there is a type of sealed-bid auction that is equivalent to the ascending-bid English auction. With a known prize value, the best strategy in an English auction is to stay in the bidding until the price just reaches your own value. For example, suppose that one person’s value is $50, another’s is $40, and a third person’s value is $30. At a price of $20, all three are interested. When the auctioneer raises the price to $31, the third bidder drops out, but the first two continue to nod as the auctioneer raises the bid amount. At $41, however, the second bidder declines to nod. The first bidder,
who is willing to pay more, will agree to $41 but should feel no pressure to express an interest at a higher price since nobody else will speak up. After the usual “going once, going twice…” warning, the prize will be sold to the first bidder at a price of $41. Notice that the person with the highest value purchases the item, but only pays an amount that is approximately equal to the second highest value.

This observation that the bidding in an English auction stops at the second-highest value led Vickrey to devise a second-price sealed-bid auction. As in any sealed bid auction, the seller collects sealed bids and sells the item to the person with the highest bid. The winning bidder, however, only has to pay the second-highest bid. Vickrey noted that the optimal strategy in this auction is to bid an amount that just equals one’s own value. To see why this is optimal, suppose that your value is $10. If you bid $10 in a second-price auction, then you will only win when all other bids are lower than your own. If you decide instead to raise your bid to $12, you increase your chances of winning, but the increase is only in those cases where the second highest bid is above $10, causing you to lose money on the “win.” For example, if you bid $12 and the next bid is $11, you pay $11 for an item that is only worth $10 to you. Thus it is never optimal to bid above value in this type of auction. Next consider a reduction to a bid, say to $8. If the second highest bid were below $8, then you would win anyway and pay the second with or without the bid reduction. But if the second bid were above $8, say at $9, then your bid reduction would cause you to lose the auction in a case where you would have won profitably. In summary, the best bid is your own value in a second-price auction. If everybody does, in fact, bid at value, then the high-value person will win, and will pay an amount that equals the second highest value. But this is exactly what happens in an English auction where the bidding stops at the second-highest value. Thus the second-price auction is, in theory, equivalent to the English auction.

Lucking-Reiley (2000) points out that stamp collectors have long used Vickrey-like auctions as a way of including bidders who cannot attend an auction. For example, if two distant bidders mail in bids of $30 and $40, then the bidding would start at $31. If nobody entered a higher bid, then the person with the higher bid would purchase at $31. If somebody else agrees to that price, the auctioneer would raise the price to $32. In fact, the auctioneer would continue to “go one up” on any bidder present until the higher mail-in bid of $40 is reached. This mixture of an English auction and a sealed-bid second price auction was achieved by allowing “proxy bidding,” since it is the auctioneer who is entering bids based on the limit prices submitted by mail. It is a natural extension to entertain only mail-in bids and to simulate the English auction by awarding the prize to the high bidder at the second bid. Lucking-Reiley (2000) found records of a pure second-price stamp auction held in Northampton, Massachusetts in 1893. He also notes
that the most popular online auction, eBay, allows proxy bidding, which is explained: “Instead of having everyone sit at their computers for days on end waiting for an auction to end, we do it a little differently. Everyone has a little magical elf (aka proxy) to bid for them and all you need to do is tell your elf the most that you want to spend, and he’ll sit there and outbid the others for you, until his limit is reached.”

III. Bidding Against a Uniform Distribution

This section describes an experiment conducted by Holt and Sherman (2000) in which bidders received private values, and others’ bids were simulated by the throw of ten-sided dice. This experiment lets one begin to study the tradeoffs involved in optimal bidding without having to do a full game-theoretic analysis of how others’ bids are actually determined in a market with real (non-simulated) bidders. At the start of each round, the experimenter would go to each person’s desk and throw a ten-sided three times to determine a random number between $0.00 and $9.99. This would be the person’s private value for the prize being auctioned. Since each penny amount in this interval is equally likely, the population distribution of values in this setup is uniform. After finding out their value, each person would select a bid knowing that the “other person’s” bid would be randomly determined be three throws of the ten-sided die. If the randomly determined “other person’s bid” turns out to be lower than the bidder’s own bid, then the bidder would earn the difference between the value and the bid selected. If the other’s bid were higher, then the bidder would earn nothing.

Suppose that the first three throws of the die determine a value that will be denoted by \( v \), where \( v \) is now some known dollar amount between $0.00 and $9.99. The only way to win money is to bid below this value, but how much lower? The strategic dilemma in an auction of this type is that a higher bid will increase the chances of winning, but the value of winning with a higher bid is diminished because of the higher price that must be paid. Optimal bidding involves finding the right point in this tradeoff, given one’s willingness to tolerate risk, which can be considerable since the low bidder in the auction earns nothing.

This strategic tradeoff can be understood better by considering the bidder’s expected payoff under a simplifying assumption of risk neutrality. This expected payoff consists of two parts: the probability of winning and the payoff conditional on winning. A person with a value of \( v \) who wins with a bid of \( b \) will have to pay that bid amount, and hence will earn \( v - b \). Thus the expected payoff is the product of the winner’s earnings and the probability of winning:

\[
\text{Expected Payoff} = (v - b) \Pr(\text{winning with } b).
\]
The probability of winning with a bid of \( b \) is just the probability that this bid is above the simulated other bid, i.e. above the result of the throws of the 10-sided die. The other bid is equally likely to be any penny amount: $0.00, $0.01, \ldots $9.99. For simplicity, we will ignore ties and assume that the other person will win in the event of a tie. Then a bid of 0 would win with probability 0, a bid of $10.00 would win with probability 1. This suggests that the probability of winning is \( b/10 \), which is 0 for a bid of 0 and 1 for a bid of 10. For a bid of $5, the probability of winning is exactly 1/2 according to this formula, which is correct since there are 500 ways that the other bid is below $5 ($0.00, $0.01, \ldots $4.99), and there are 500 ways that the bid is greater than or equal to $5 ($5.00, $5.01, \ldots $9.99). Using this formula \( (b/10) \) for the probability of winning, the expression for the bidder’s expected payoff in (19.1) can be expressed:

\[
(19.2) \quad \text{Expected Payoff} = (v - b)(b/10) = vb/10 - b^2/10.
\]

This expected payoff exhibits the strategic dilemma discussed earlier. The payoff conditional on winning, \( v - b \), is decreasing in the bid amount, but the probability of winning, \( b/10 \), is increasing in \( b \). The optimal bid involves finding the right balance between these two good things, high payoff and high probability of winning.

The bidder knows the value, \( v \), at the time of bidding, so the function on the right side of (19.2) can be graphed to find the highest point, as in Figure 19.1 for the case of \( v = 8 \). The bid, \( b \), is on the horizontal axis, the function starts with a height of 0 when \( b = 0 \) since a 0 bid has no chance of winning. This at the other end, a bid that equals the value \( v \) will also yield a 0 expected payoff, since the payoff for bidding the full value of the prize is 0 regardless of whether one wins or loses. In between these two points, the expected payoff function shows a hill-shaped graph, which rises and then falls as one moves to higher bids (from left to right).
One way to find the best bid is to use (19.2) to set up a spreadsheet to calculate the expected payoff for each possible bid and find the best one (question 1). For example, if \( v = \$4 \), then the expected payoffs are: \$0 for a bid of \$0, \$0.30 for a bid of \$1, \$0.40 for a bid of \$2, \$0.30 for a bid of \$3, and \$0.00 for a bid of \$4. Filling in the payoffs for all possible bids in penny increments would confirm that the best bid is \$2\) when one’s value is \$4. Similarly, it is straightforward to show that the optimal bid is \$2.50\) when one’s value is \$5. These calculations suggest that the best strategy (for a risk-neutral person) is to bid one half of one’s value.

The graphical intuition behind bidding half of value is shown in Figure 19.1. When the value is \$8\), the expected payoff function starts at the origin of the graph, rises, and falls back to \$0\) when one’s bid is equal to the value of \$8\). The expected payoff function is quadratic, and it forms a hill that is symmetric around the highest point. The symmetry is consistent with the fact that the maximum is located at \$4\), halfway between \$0\) and the value of \$8\).

At the point where the function is flat, the slope of a dashed tangent line is zero, and the tangency point is directly above a bid of \$4\) on the horizontal axis. This point could be found graphically for any specific private value. Alternatively, we can use calculus to derive a formula that applies to all possible values of \( v \). (The rest of this paragraph can be skipped by those who are already familiar with calculus.) A reader who is not familiar with calculus should go ahead and read the discussion that follows; the only thing you will need besides a little intuition is a couple of rules for calculating derivatives (finding the slopes of tangent lines). First consider a linear function of \( b \), say \( 4b \). This is a straight line with a slope of 4, so the derivative is 4. This rule generalizes to any linear function with a constant slope of \( k \): the derivative of \( kb \) with respect to \( b \) is just \( k \).
The second rule that will be used is that the derivative of a quadratic function like $b^2$ is linear: $2b$. This formula is easily modified to allow for multiplicative constants, e.g. the derivative of $3b^2$ is $3(2b)$, or the derivative of $-b^2$ is $-2b$.

The expected payoff in equation (19.2) consists of two terms. The first one, can be written as $(v/10)b$, which is a linear function of $b$ with a slope of $v/10$. Therefore the derivative of the expected payoff will have a $v/10$ term in it, as can be seen on the right side of (19.3):

\[(19.3) \quad \text{Derivative of Expected Payoff} = \frac{v}{10} - \frac{2b}{10}.\]

The second term in the expected payoff expression (19.2) is $-b^2/10$, and the derivative of this term is $-2b/10$ because the derivative of $b^2$ is $2b$. To summarize, the derivative on the right side of (19.3) is the sum of two terms, each of which is the derivative of the corresponding term in the expected payoff in (19.2).

The optimal bid is the value of $b$ for which the slope of a tangent line is 0, so the next step is to equate the derivative in (19.3) to 0:

\[(19.4) \quad \frac{v}{10} - \frac{2b}{10} = 0.\]

This equation is linear in $b$, and can be solved to obtain the optimal bidding strategy:

\[(19.5) \quad b^* = \frac{v}{2} \quad \text{(optimal bid for risk neutral person)}.\]

The calculus method is general in the sense that it yields the optimal bid for all possible values of $v$, whereas the graphical and numerical methods had to be done separately for each value of $v$ being considered.

To summarize, the predicted bid is a linear function of value, with a slope of 0.5. The actual bid data in the Holt and Sherman experiment formed a scatter plot with an approximately linear shape, but most bids were above the half-value prediction. A linear regression yielded the estimate:

\[(19.6) \quad b = 0.14 + 0.667v \quad (R^2 = 0.91),\]
where the intercept of 14 cents, with a standard error of 0.61, was not significantly different from 0. The slope, with a standard error of 0.017, was significantly different from 1/2.

By bidding above one half of value, bidders are obtaining a higher chance of winning, but a lower payoff if they win. A willingness to take less money in order to reduce the risk of losing and getting a zero payoff may be due to risk aversion, as discussed in the next section. There could, of course, be other explanations for the over-bidding, but the setup with a simulated other bidder does permit us to rule out some possibilities. Since the other bidder was just a roll of the dice, the over-bidding cannot be due to issues of equity, fairness, or rivalistic desires to win or reduce the other’s earnings.

The experiment described in this section with simulated “other bids” is essentially an individual decision problem. An analogous game can be set up by providing each of two bidders with randomly determined private values drawn from a distribution that is uniform from $0 to $10. As before, a high bid results in earnings of the difference between the person’s private value and the person’s own bid. A low bid results in earnings of 0. This is known as a first-price auction since the high bidder has to pay the highest (first) price.

Under risk neutrality, the Nash equilibrium for this game is to bid one half of value. The proof of this claim is essentially an application of the analysis given above, where the distribution of the other’s bids is uniform on a range from 0 to $10. Now suppose that one person is bidding half of value. Since the values are uniformly distributed, the bid will be uniformly distributed from 0 to a level of $5, which is one half of the maximum value. If this person’s bids are uniformly distributed from 0 to $5, then a bid of 0 will not win, a bid of $5 will win with a probability that is essentially 1, and a bid of $2.50 will win with probability 1/2. So the probability of winning with a bid of $ is $b/5$. Thus the expected-payoff function is given in equation (19.2) if the 10 in each denominator is replaced by a 5. Equations (19.3) and (19.4) are changed similarly. Then multiplying both sides of the revised (19.4) by 5 yields (19.5).

IV. Bidding Against a Uniform Distribution with Risk Aversion (optional)

The analysis in this next section shows that a simple model of risk aversion can explain the general pattern of overbidding that is implied by the regression equation (19.6). The analysis uses simple calculus, i.e. the derivative of a “power function” like \( kx^s \), where \( k \) is a constant and the variable \( x \) is raised to the power \( s \). The derivative with respect to \( x \) of this “power function” is a new function where the power has been reduced by 1 and the original power enters multiplicatively. Thus the derivative of \( kx^s \) is \( ksx^{s-1} \), which is called the “power-function rule” of differentiation. A second rule that will be used is that the derivative of a product of two functions is the first function times the
derivative of the second, plus the derivative of the first function times the second. This is analogous to calculating the change in room size (the product of length and width) as the length times the change in width plus the change in length times the width. This “product rule” is accurate for “small” changes. Using the power-function and the product rules, those not familiar with calculus should be able to follow the arguments given below, but if you have trouble, skip to the discussion that is just below the generalized bidding rule in (19.11).

The most convenient way to model risk aversion in an auction is to assume that utility is a nonlinear function, i.e. that the utility of a money amount \( v-b \) is a power function \( (v-b)^{1-r} \) for \( 0 \leq r < 1 \). When \( r = 0 \), this function is linear, which corresponds to the case of risk neutrality. If \( r = 1/2 \), then the power \( 1-r \) is also \( 1/2 \) so the utility function is the square root function. A higher value of \( r \) corresponds to more risk aversion. Now the expected payoff function in (19.2) must be replaced with an expected utility function, which is the utility of the payoff times the probability of winning:

\[
(19.7) \quad \text{Expected Utility} = (v-b)^{1-r} \left( \frac{b}{10} \right).
\]

As before, the optimal bid is found by equating the derivative of this function to zero. The expected utility on the right side of (19.7) is a product of two functions of \( b \), so we use the power-function rule to obtain the derivative (first function times the derivative of the second plus the derivative of the first times the second function). The derivative of \( b/10 \) is \( 1/10 \), so the first function times the derivative of the second is the first term in (19.8). Next, note that the power function rule implies that the derivative of \( (v-b)^{1-r} \) is \( -(1-r)(v-b)^{-r} \), which appears in the second term in (19.8).

\[
(19.8) \quad (v-b)^{1-r} \left( \frac{1}{10} \right) - (1-r)(v-b)^{-r} \left( \frac{b}{10} \right).
\]

This derivative can be rewritten by putting parts common to each term in the parentheses, as shown on the left side of (19.9):

\[
(19.9) \quad (v-b)^{1-r} \left( \frac{1}{10} \right) - (1-r)b \left( \frac{(v-b)^{-r}}{10} \right) = 0.
\]

Multiplying both sides by \( (v-b)^{r}/10 \), one obtains:
\[ v - b - (1-r)b = 0. \]

This equation is linear in \( b \), and can be solved to obtain the optimal bidding strategy:

\[ b^* = \frac{v}{2 - r} \] (optimal bid with risk aversion).

This bidding rule reduces to the optimal bidding rule for risk neutrality (bidding half of value) when \( r = 0 \). Increases in \( r \) will raise the bids. The bids will be two-thirds of value, as implied in the regression equation (19.6), when \( r = 1/2 \).

V. Bidding Behavior in a Two-Person, First-Price Auction

Recall that the predicted bidding strategy is linear, with a slope of 1/2 when bidders are risk neutral. Bids exceeded the \( v/2 \) line in experiment with simulated other bids, and the same pattern emerges when the other bidder is a subject in the experiment. Figure 19.2 shows the bid/value combinations for 10 rounds of a classroom experiment done with the Veconlab software. There were 10 bidding teams, each composed of 1-2 students on a networked PC. The teams were randomly matched in a series of rounds. The bidding pattern is approximately linear, except for a leveling off with high values (a pattern that has also been noted by Dorsey and Razzolini, (2002). Only a small fraction of the bids are at or below the Nash prediction of \( v/2 \), which is graphed as the solid line. In fact, the majority of bids are above the dashed line with slope of .667 obtained from regression for the case discussed in the previous section; the bid-to-value ratio averaged over all bids in all rounds is about 0.7. This pattern of overbidding relative to the Nash prediction is typical, and the most commonly mentioned explanation is risk aversion.
An analysis of risk aversion for this game parallels the previous section’s analysis of bidding against a simulated bid that is uniform on the interval from 0 to 10 dollars. Suppose that the equilibrium bidding strategy with risk aversion is linear: \( b = \beta v \), where \( 0 < \beta < 1 \). Then the lowest bid will be 0, corresponding to a value of 0, and the highest bid will be \( 10\beta \), corresponding to a value of 10. The distribution of bids is represented in Figure 19.3. For any particular value of \( \beta \), the bids will be uniformly distributed from 0 to \( 10\beta \), as indicated by the dashed line with a constant height representing a constant probability for each possible bid. The figure is drawn for the case of \( \beta = 0.6 \), so bids are uniform from $0 to $6. A bid of 0 will never win, a bid of \( 10\beta = 6 \) will win with probability 1, and an intermediate bid will win with probability of \( b/6 \). In the general case, a bid of \( b \) will win with probability \( b/10\beta \).
In a two-person auction when the other’s bid is uniform from 0 to $10\beta$, the probability of winning in the expected payoff function will be $b/10\beta$ instead of the ratio $b/10$ that was used in section III. The rest of the analysis of optimal bidding in that section is unchanged, with the occurrences of 10 replaced by $10\beta$, which cancels out of the denominator of equation (19.9) just as the 10 did. The resulting equilibrium bid is given in equation (19.11) as before. This bidding strategy is again linear, with a slope that is greater than one half when there is risk aversion ($r > 0$). Recall that a risk aversion coefficient of $r = 1/2$ will yield a bid line with slope $2/3$, so the bids in Figure 19.2 are roughly consistent with risk aversion of at least $1/2$.

V. Extensions and Further Reading


Up to now, we have only considered two-person auctions. Auctions with more bidders are more competitive, which will cause bids to be closer to values. The formula in (19.5) can be generalized to the case of $N$ risk-neutral bidders drawing from the same uniform private value distribution:

\[ b^* = \frac{(N-1)v}{N} \] (optimal bid with risk neutrality),

with a further upward adjustment for the case of risk aversion. Notice that (19.12) specifies bidding half of value when $N = 2$, and that bids rise as a proportion of value for larger group sizes. As the number of bidders becomes very large, the ratio, $(N-1)/N$, becomes closer and closer to 1, and bids converge to values. This increase in competition causes expected payoffs to converge to zero as bid/value differences shrink.

The Vecon software permits a number of other interesting variations. One option is to require all bidders to pay their own bids, with the prize still going to the highest bid. For example, if a bidder with a value of $4$ bids $1$ and if a second bidder with a value of $9$ bids $5$, then the first earns $-1$ and the second
earns $9 − $5 = $4. In this strategic setting, bidders with low values (and low chances of winning) will reduce their bids towards 0, to avoid having to pay in the event of a loss. Those with a relatively high value will risk a higher bid, so that the bidding strategy will have a curved shape, only rising significantly above 0 at very high values. This curved pattern can be seen in the bids in Figure 19.4, which were observed in a research experiment conducted at the University of Virginia (with all payments made in cash).

This “all-pay” auction is sometimes used to model lobbying competition or a patent race, where the contender with the highest effort will win, but even the losers incur the costs associated with their efforts.

A second option provides for returning a fraction of the auction revenue to all bidders, divided equally among the bidders. This is analogous to using the revenue from the sale of prize to reduce taxes for all bidders. Such revenue rebates tend to raise the bids.

**Questions**

1. This question lets you set up a simple spreadsheet to calculate the optimal bid. Begin by putting the text “V =” in cell A1 and any numerical value, e.g. 8, in cell B1. Then put the possible bids in the A column in fifty-cent increments: 0 in A3, 0.5 in A4, 1 in A5, etc. (You can use penny amounts if you prefer.) The expected payoff formula in column B, beginning in cell B3, should contain a formula with terms involving $B$ (the prize...
value) and $A3 (the bid). Use the expected payoff formula in equation (19.2) to complete this formula, which is then copied down to the lower cells in column B. Verify the expected payoff numbers for a value of $4 that were reported in section II. By looking at the expected payoffs in column B, one can determine the bid with the highest expected payoff. If the value of 8 is in cell B1, then the optimal bid should be 4. Then change the value in cell B1 to 5 and show that the optimal bid is $2.50.

2. Write out the revised versions of equations (19.2), (19.3), (19.4), and (19.5) for the two-person auction assuming risk neutrality.

3. Write out the revised versions of equations (19.2), (19.3), (19.4), and (19.5) for the two-person auction under risk aversion with a power-function utility and parameter $r$. 
Chapter 20. The Buyer’s Curse

This chapter pertains to a situation in which a buyer cannot directly observe the underlying value of some object. A buyer’s bid will only be accepted if it is higher than the value of the object to the current owner. The danger is that a purchase is more likely to be made precisely when the owner’s value is low, which may lead to a loss for the buyer. The tendency to purchase at a loss in such situations is called the “buyer’s curse.” The Veconlab game “BC” sets up this situation, or alternative, the instructions in the appendix can be used with 10-sided dice.
Chapter 21. The Winner’s Curse

In many bidding situations, the value of the prize would be approximately the same for all bidders, although none of them can assess exactly what this “common value” will turn out to be. In such cases, each bidder may obtain an estimate of the prize value, and those with higher estimates are more likely to make higher bids. As a result, the winning bidder may overestimate the value of the prize and end up paying more than it is worth. This “winner’s curse” is analogous to the buyer’s curse discussed in the previous chapter. An auction where each bidder has partial information about an unknown prize value can be run using 10-sided dice to determine common value elements, as explained in the Appendix. Alternatively, it can be run with the Veconlab game “CV,” which has default settings that permit an evaluation of increases in the number of bidders, which tends to produce a more severe winner’s curse.

I. “I Won the Auction But I Wish I Hadn’t”

Once an economist asked several flooring companies to make bids on replacing some kitchen floor tile. Each bidder would estimate the amounts of materials needed and the time required to remove the old tile and install the floor around the various odd-shaped doorways and pantry area. One of the bids was somewhat lower than the others. The economist expected him to be happy when he was informed that the job was his. Instead, he exhibited considerable anxiety about whether he had miscalculated the cost, although in the end he did not withdraw the bid. This story illustrates the fact that winning an auction can be an informative event, or equivalently, the fact that you win produces new information about the unknown value of the prize. A rational bidder should anticipate this information in making the bid originally. This is a subtle strategic consideration that is almost surely learned by (unhappy) experience.

The possibility of paying too much for an object of unknown value is particularly dangerous for one-time auctions in which bidders are not able to learn from experience. Suppose that two partners in an insurance business work out of separate offices and sell separate types of insurance, e.g. business insurance from one office and life from the other. Each partner can observe the other’s “bottom-line” earnings, but cannot determine whether the other one is really working. One of the partners is a single mother who works long hours, with good results despite a low earnings-per-hour ratio. The other one, who enjoys somewhat of a market lucky niche, is able to obtain good earnings levels while spending a large fraction of each day “socializing” on the internet. After several years of happy partnership, each decides to try to buy the whole business from the stockholders,
who are current and former partners. When they make their bids, the one with the
profitable niche market is likely to think the other office is more as profitable as
the person’s own office, and hence to overestimate the value of the other office.
As a result, this partner could end up acquiring the business for a price that
exceeds its value.

The tendency for winners’ value estimates to be biased has long been
known in the oil industry, where drilling companies must make rather imperfect
estimates of the likely amounts of oil that can be recovered on a given tract of
land being leased. For example, Robert Wilson, a professor in the Stanford
Business School, was once consulting with an oil company that was considering
bidding at less than half of the estimated lease value. When he inquired about the
possibility of a higher bid, he was told that firms that bid this aggressively on such
a lease are no longer in this business (source: personal communication at a
conference on auction theory in the early 1980’s).

The intuition underlying the unprofitability of aggressive bidding is that
the firm with the highest bid in an auction is likely to have overestimated the lease
value. The resulting possibility of winning “at a loss” is more extreme when there
are many bidders, since the highest estimate out of a large number of estimates is
more likely to biased upward, even though each individual estimate is \textit{ex ante}
unbiased. For example, suppose that a single value estimate is unbiased. Then
the higher of two unbiased estimates will be biased upward. Analogously, the
highest of three unbiased estimates will be even more biased, and the highest of
100 estimates may show an extreme bias toward the largest possible estimation
error in the upward direction. Knowing this, a bidder in an auction with many
bidders should treat their own estimates as being inflated, which will likely turn
out to be true if they win, and the resulting bids may end up being low relative to
the estimates. This numbers effect can be particularly sinister, since the normal
strategic reaction to increased numbers of bidders is to bid higher, as in a private
value auction.

Wilson (1967, 1968) specified a model with this common-value structure
and showed that the Nash equilibrium with fully rational bidders involved some
downward adjustment of bids in anticipation of a winner’s-curse effect. Each
bidder should realize that their bid is only relevant for payoffs if it is the highest
bid, which means that they have the highest value estimate. So prior to making a
bid, the bidder should consider what they would want to bid knowing that all
other estimates are lower than theirs in the event that they win the auction. The \textit{a priori}
correction of this overestimation produces bids that will earn positive
payoffs on average.
II. A Simple Shoe-Box Model

Consider a simple situation where each of two bidders can essentially observe half of the value of the prize, as would be the case for two bidders who can drill test holes on different halves of a tract of land being leased for an oil well. Another example would be the case for the two insurance partners discussed in the previous section. In particular, let the observed value components or “signals” for bidders 1 and 2 be denoted by $v_1$ and $v_2$ respectively. The prize value is the average of the two signals:

$$\text{Prize Value} = \frac{v_1 + v_2}{2}.$$  

Both bidders know their own estimates, but they only know that the other person’s estimate is the realization of a random variable. In the experiment to be discussed, each person’s value estimate is drawn from a distribution that is uniform on the interval from $0$ to $10$. Bidder 1, for example, knows $v_1$ and that $v_2$ is equally likely to be any amount between $0$ and $10$.

Consider an example of a bidder in the first round of a common-value experiment with this setup. The bidder had a relatively high value signal of $8.69$, and submitted a bid of $5.03$ (presumably the three-cent increase above $5$ was intended to outguess anyone who might bid an even $5$). The other person’s signal was $0.60$, so the prize was only worth the average of $8.69$ and $0.60$, which is $4.64$. The bidder won this prize, but paid a price of $5.03$, which resulted in a loss. Three of the five winning bidders in that round ended up losing money, and about one out of five winners lost money in each of the remaining rounds.

The prize value function in (21.1) can be generalized for the case of a larger number of bidders with independent signals, by taking an average, i.e. dividing the sum of all signals by the number of bidders. In a separate classroom experiment, conducted with 12 bidders interacting on the Veconlab software, the sole winning bidder ended up losing money in three out of five rounds. As a result, aggregate earnings were zero or negative for most bidders. A typical case was that of bidder 9, who submitted the high bid of $6.10$ on a signal of $9.64$ in the fifth and final round. The average of all 12 signals was $5.45$, so this person lost 65 cents for the round. These results illustrate why a general finding that the winner’s curse effect is often more severe with large numbers of bidders.

Once an economist at the University of Virginia was asked to consult for a major telecommunications company that was planning to bid in the first major U.S. auction for radio wave bandwidth to be used for personal communications services. This company was a major player, but incredibly, it decided not to bid at the last minute. The company representative mentioned the danger of
overpayment for the licenses at auction. This story does illustrate a point: that players who have an option of earning zero with non-participation will never bid in a manner that yields negative expected earnings. The technical implication is that expected earnings in a Nash equilibrium for a game with an exit option cannot be negative. This raises the issue of how bidders rationally adjust their behavior to avoid losses in a Nash equilibrium, which is the topic of the next section.

III. The Nash Equilibrium

As was the case in the chapter on private-value auctions, the equilibrium bids will end up being linear functions of the signals, of the form:

\[ b_i = \beta v_i, \quad \text{where } 0<\beta <1 \text{ and } i = 1,2. \]

Suppose that bidder 2 is using a special case of this linear bid function by bidding exactly one half of the signal \( v_2 \). In the next several pages, we will now use this assumed behavior to find the expected value of bidder 1’s earnings for any given bid, and then we will show that this expected payoff is maximized when bidder 1 also bids one half of the signal \( v_1 \), i.e. \( \beta = 0.5 \). Similarly, when bidder 1 is bidding one half of their signal, the best response for the other bidder is to bid one half of their signal too. Thus the Nash equilibrium for two risk-neutral bidders is to bid half of one’s signal.

Since the arguments that follow are more mathematical than most parts of the book, it is useful to break them down into a series of steps. We will assume for simplicity that bidders are risk neutral, so we will need to find bidder 1’s expected payoff function before it can be maximized. (It turns out to be the case that risk aversion has no effect on the Nash equilibrium bidding strategy in this case, as noted later in the chapter.) In an auction where the payoff is 0 in the event of a loss, the expected payoff is the probability of winning times the expected payoff conditional on winning. So the first step is finding the probability of winning given that the other bidder is bidding one half of their signal value. The second step is to find the expected payoff, \textit{conditional on winning with a particular bid}. The third step is to multiply the probability of winning times the expected payoff conditional on winning, to obtain an expected payoff function, which will be maximized using simple calculus in the final step. The result will show that a bidder’s best response is to bid half of the signal when the other one is bidding in the same manner, so that this strategy is a Nash equilibrium.

Before going through these steps, it may be useful to review how we will go about maximizing a function. Think of the graph of a function as a hill, which...
is increasing on the left and decreasing on the right, as for Figure 19.2 in the chapter on private-value auctions. At the top of the hill, a tangent line will be horizontal (think of a graduation cap balanced on the top of someone’s head). So to maximize the function, we need to find the point where the slope of a tangent line is 0. The slope of a tangent line can be found by drawing the graph carefully, drawing the tangent line, and measuring its slope, but this method is not general since specific numbers are needed to draw the lines. In general, the slope of the tangent line to a function can be found by taking the derivative of the function. Both the probability of winning and the conditional expected payoff will turn out to be linear functions of the person’s bid \( b \), so the expected-payoff product to be maximized will be quadratic, with terms involving \( b \) and \( b^2 \). A linear term, like \( b/2 \), is a straight line through the origin with a slope of 1/2, so the derivative is the slope, \( 1/2 \). The quadratic term, \( b^2 \), also starts at the origin, and it increases to 1 when \( b = 1 \), to 4 when \( b = 2 \), to 9 when \( b = 3 \), and to 16 when \( b = 4 \). Notice that this type of function is increasing more rapidly as \( b \) increases, i.e. the slope is increasing in \( b \). Here all you need to know is that the derivative of a quadratic expression like \( b^2 \) is \( 2b \), which is the slope of a straight line that is tangent to the curved function at any point. This slope is, naturally, increasing in \( b \). Armed with this information, we are ready to find the expected payoff function and maximize it.

**Step 1. Finding Bidder 1’s Probability of Winning for a Given Bid**

Suppose that the other person (bidder 2) is known to be bidding half of their signal. Since the signal is equally likely to be any value from $0 to $10, the second bidder’s bid is equally likely to be any of the 1000 penny amounts between $0 to $10, as shown by the dashed line with height of 0.001 in Figure 21.1. When \( b_2 = v_2/2 \), then bidder 1 will win if \( b_1 > v_2/2 \), or equivalently, when the other’s value is sufficiently low: \( v_2 < 2b_1 \). The probability of winning with a bid of \( b_1 \) is the probability that \( v_2 < 2b_1 \). This probability can be assessed with the help of Figure 21.1. Suppose that bidder 1 makes a bid of $2, for example, as shown by the short vertical bar. We have just shown that this bid will win if the other’s value is less than \( 2b_1 \), i.e. less than $4 in this example. Notice that four-tenths of the area under the dashed line is to the left of $4. Thus the probability that the other’s value is less than $4 is 0.4, calculated as \( 4/10 = 2b_1/10 \). A bid of 0 will never win, and a bid of $5 will always win, and in general, we have the result:

\[
(21.3) \quad \text{Probability of Winning (with a bid of } b_1) = \frac{2b_1}{10}.
\]
Step 2. Finding the Expected Payoff Conditional on Winning
Suppose that bidder 1 bids $b_1$ and wins. This happens when $v_2 < 2b_1$. For example, when the bid is $2 as in Figure 21.1, winning would indicate that $v_2 < 4$, i.e. to the left of the vertical bar in the figure. Since $v_2$ is uniformly distributed, it is equally likely to be any penny amount less than 4$, so the expected value of $v_2$ would be $2 once we find out that the bid of $2 won. This generalizes easily; the expected value conditional on winning with a bid of $b_1$ is just $b_1$ (question 1). Bidder 1 knows the signal $v_1$ and expects the other signal to be equal to the bid $b_1$ if it wins, so the expected value of the prize is the average of $v_1$ and $b_1$:

(21.4) Conditional Expected Prize Value (winning with a Bid of $b_1$) = $\frac{b_1 + v_1}{2}$.

Step 3. Finding the Expected Payoff Function
The expected payoff for a bid of $b_1$ is the product of the probability of winning in (21.3) and the difference between the conditional expected prize value in (21.4) and the bid:

(21.5) Expected Payoff = $\frac{2b_1}{10} \left( \frac{b_1 + v_1}{2} - b_1 \right) = \frac{b_1^2}{10} - \frac{b_1 v_1}{10}$.

Step 4. Maximizing the Expected Payoff Function
In order to maximize this expected payoff, we will set its derivative equal to 0. The expression on the far right side of (21.5) contains two terms, one that is quadratic in $b_1$ and one that is linear. Recall that the derivative of $(b_1)^2$ is $2b_1$ and the derivative of a linear function is its slope coefficient, so:

Figure 21.1. A Uniform Distribution on the Interval [0, 10]
\[(21.6) \quad \text{Expected Payoff Derivative} = \frac{2b_1}{10} - \frac{v_1}{10}.
\]

Setting this derivative equal to 0 and multiplying by 10 yields: \(2b_1 - v_1 = 0\), or equivalently, \(b_1 = v_1/2\). To summarize, if bidder 2 is bidding half of value, then bidder 1’s best response is to bid half of value as well, so the Nash equilibrium bidding strategy is for each person to behave in this manner.

\[(21.7) \quad b_i = \frac{v_i}{2} \quad i=1,2. \quad \text{(equilibrium bid)}.
\]

Normally in a first-price auction, one should “bid below value,” and it can be seen that the bid in (21.7) is less than the conditional expected value in (21.4). Finally, recall that this analysis began with an assumption that bidder 2 was using the strategy in (21.2) with \(\beta = 1/2\). This may seem like an arbitrary assumption, but it can be shown that the only linear bidding strategy for this auction must have a slope of 1/2 (question 3).

**IV. The Winner’s Curse**

When both bidders are bidding half of the value estimate, as in (21.7), then the one who wins will be the one with the higher estimate, i.e. the one who overestimates the value of the prize. Another way to see this is to think about what a naïve calculation of the prize value would entail. One might reason that since the other’s value estimate is equally likely to be any amount between $0 and $10, the expected value of the other’s estimate is $5. Then knowing one’s own estimate, say \(v_1\), the expected prize value is \((v_1 + 5)/2\). This expected prize value, however, is not conditioned on winning the auction. Notice that this unconditional expected value is greater than the conditional expected value in (21.4) whenever the person’s bid is less than $5, which is always the case when bids are half of the value estimate. Except for this boundary case where the bid is exactly $5, the naïve value calculation will result in an overestimate of value, which can lead to an excessively high bid and negative earnings.

Holt and Sherman (2000) conducted a number of common-value auctions with a prize value that was the average of the signals. Subjects began with a cash balance of $15 to cover any losses. Figure 21.2 shows bids and signals for the final 5 rounds of a single session. Two of the bidders can be distinguished from the others by the diamond and square marks. Notice that both of these are bidding in an approximately linear manner, but that they and the others are bidding above the \(v/2\) line, which represents the Nash equilibrium. The dashed line shows the
regression line that was fit to all bids over all sessions, so we see that this session was a little high but not atypical in the nature of the bid/value relationship.

![Graph showing bids for eight subjects in the final five rounds of a common-value auction.](image)

**Figure 21.2.** Bids for Eight Subjects in the Final Five Rounds of a Common-Value Auction  
Key: Solid Line: Nash Equilibrium, Dashed Line: Regression for All Sessions  
(Holt and Sherman, 2000)

The experiment session shown in Figure 21.2 was a research experiment, with students recruited from a variety of economics classes at the University of Virginia. Figure 21.3 shows analogous data for a classroom experiment with one person paid partial earnings at random. These were students in an experimental economics class, who had already read about private-value auctions and were relatively well-versed in game theory. Notice that the bids are lower, but still generally above the Nash prediction. Some people, like the person with bids marked as diamonds, were bidding about half of their signals, however.
V. Extensions and Further Reading

The winner’s curse was first discussed in Wilson (1969), and applications to oil lease drilling were described in Capen, Clapp, and Campbell (1971). Kagel and Levine (1986) provided experimental evidence that it could be reproduced in the laboratory, even after subjects obtain some experience. The literature on common-value auctions is surveyed in Kagel (1995).

Questions
1. Sketch a version of Figure 21.1 for the case where bidder 1’s bid is $1. Show the probability distribution for the other person’s value, $v_2$, conditional on the event that the first person’s bid is the high bid. Then calculate the expected value of $v_2$, conditional on bidder 1 winning with a bid of $1$. Then explain in words why the expected value of $v_2$ conditional on the winning bid $b_1$ is equal to $b_1$.
2. (This question is somewhat mechanical, but it lets you test you knowledge of the bid derivations in the reading.) Suppose that the prize value in
equation (21.1) is altered to be the sum of the signals instead of the average. The signals are still uniformly distributed on the range from $0$ to $10$. a) What is the probability of winning with a bid of $b_1$ if the other person bids an amount that equals their signal, i.e. if $b_2 = v_2$. b) What is the expected prize value conditional on winning with a bid of $b_1$? c) What is the expected payoff function for bidder 1? d) (Calculus required.) Show that the bidder 1’s expected payoff is maximized with a bid that equal’s the bidder’s own signal.

3. The goal of this question is to show that the only linear strategy of the form in equation (21.2) is one with a slope of 1/2 when the prize value is the average of the signals. a) What is the probability of winning with a bid of $b_1$ if the other person bids: if $b_2 = \beta v_2$. (Hint: your answer should imply (21.3) when $\beta = 1/2$.) b) What is the expected prize value conditional on bidding with a bid of $b_1$? c) What is the expected payoff function for bidder 1? d) (Calculus required.) Show that the bidder 1’s expected payoff is maximized with a bid that equals one half of the bidder’s own signal.
Chapter 22. An Irrigation Reduction Auction

The game in this chapter is loosely based on an auction used to determine which tracts of land in draught-stricken Southwest Georgia would not be irrigated during the 2001 growing season. Participants are put in the role of being farmers with one or more tracts of land, each consisting of a specified number of acres. In each round of the auction, the bids are ranked from low to high, with low bids being provisionally accepted (up to a total expenditure set by the experimenter). Bidders do not know which round will be the final round that actually determines who must forego irrigation for that growing season and be compensated. The game can be used to motivate a discussion of the advantages of auctions, and of the many possible alternative ways that such an auction can be run. The actual auction was conducted with a web-based network of computers at eight different locations, and a similar structure is used in the Veconlab “WA” program. Alternatively, a discussion of whether or not to use a uniform price can be motivated by the hand-run instructions in the appendix. These instructions require that participants first be endowed with some small items, like ball point pens, which can be sold to the experimenter.

I. Introduction

In April 2000 the state legislature in Georgia passed a law mandating that an “auction-like process” be used to pay some farmers for not irrigating certain tracts of land during an officially designated severe draught year. The law set aside $10 million from Tobacco Settlement funds to fund these payments. As soon as the law passed, Susan Laury, Ron Cummings and I began running laboratory experiments to evaluate possible ways to set up such an auction. By early June, we had narrowed our focus on a multi-round procedure in which
Part VI. Bargaining and Fairness

Bilateral bargaining is pervasive, even in a developed economy, especially for large purchases like automobiles or specialty items like housing. Bargaining is also central in many legal and political disputes, and it is implicit in family relations, care of the elderly, etc. The highly fluid give and take of face-to-face negotiations makes it difficult to model specify convincing structured models that permit the calculation of Nash equilibria. Experimental research has responded in two ways. First, it is possible to look at behavior in unstructured bargaining situations to spot interesting patterns of behavior, like the well known “deadline effect,” the tendency to delay agreements until the last moment. The second approach is to limit the timing and sequence of decisions, in order to learn something about fairness and equity considerations in a simplified setting. The ultimatum bargaining game discussed in Chapter 23 is an example of this latter approach. In an ultimatum game, one person makes a proposal about how to split a fixed amount of money, and the other either accepts or rejects, in which case both earn zero. Seemingly irrational rejections in such games have fascinated economists, and more recently, anthropologists. A generalization of the ultimatum game is one where the responder can either accept the original proposal or make a counter proposal, which is then either accepted or rejected by the original proposer. Chapter 23 also presents some of what is known about behavioral patterns in such multi-stage games.

The last two chapters in this part also have an alternating decision structure, but the focus is on manipulations that highlight issues of fairness, trust, and reciprocity. The “trust game” in Chapter 24 begins when one person is endowed with some cash, say $10, of which part or all may be passed to the other person. The money passed is augmented, e.g. tripled, and any part of the resulting amount may be passed back to the original person. A high level of trust would be indicated if the first person passed the full $10, expecting the second person to reciprocate and pass back at least a third of the resulting $30. Of course, the proposer might prefer to pass nothing if no pass-back is anticipated. The “reciprocity game” in Chapter 25 has more of a market context, but the underlying behavioral factors are similar. Here, employer announces a wage, and seeing this, the worker chooses an effort level, which is costly for the worker but which benefits the employer. The issue is whether fairness considerations, which are clearly present in bilateral negotiations, will have an effect on market clearing in more impersonal market settings.
Chapter 23. Ultimatum Bargaining

This game provides one person with the ability to propose a final offer that must be accepted or rejected, just as a monopolist may post a price on a take-it-or-leave-it basis. The difference between the usual monopoly situation and ultimatum bargaining is that there is only one buyer of a single item in ultimatum bargaining, so a rejection results in zero earnings for both buyer and seller. Although rejections of positive amounts of money, however small, may surprise some, they will not surprise anyone who has participated in this type of bargaining, e.g. with the game “UG.” The webgame also has a setup option that allows the responder a range of intermediate responses that involve an equally proportional reduction (“squish”) of the proposed earnings for each person. This squish would be analogous to a partial purchase in the bilateral monopoly situation, at least under constant cost and value conditions.

I. “This is My Final Offer, Take it or Leave It”

Many economic situations involve a final offer from one person to another, where rejection means zero earnings on the transaction for both. For example, suppose that a local monopolist can produce a unit of a commodity for $5. The sole buyer needs the product and is willing to pay any amount up to $15 but not a penny more, since $15 is what it would cost to buy the product elsewhere and pay to have it shipped into the local market. Then the “surplus” to be divided is $10, since this is the difference between the value and the cost. The buyer knows the seller’s cost, and hence knows that a price of $10 would split the surplus. A higher price corresponds to offering a lower amount to the buyer. The seller has a strategic advantage if the seller can make a take-it-or-leave-it offer, which we assume to be the case. This setup is called an “ultimatum game” since the proposer (seller) makes a single offer to the responder (buyer), who must either accept the proposed split of the surplus (as determined by the price) or reject, which results in zero earnings for both.

The ultimatum game was introduced by Guth et al. (1982) because it highlights an extreme conflict between the dictates of selfish, strategic behavior and notions of fairness. If each person only cares about his or her own earnings, and if more money is preferred to less, then the proposer should be able to get away with offering a very small amount to the responder. The monopolist in the previous paragraph’s example, could offer a price of $14.99, knowing that the buyer would rather pay this price than a price of $15.00 (including shipping) from a seller in another market. The $14.99 price is unfair in the sense that $9.99 of the available surplus goes to the seller, and only a penny goes to the buyer. Even so,
the buyer who only cares about getting the lowest possible price should accept any price offer below $15.00, and hence the seller should offer $14.99.

It is easy to think of situations in which a seller might hesitate to exploit a strong strategic advantage. Getting a reservation for dinner after graduation in a college town is the kind of thing one tries to do six months in advance. Restaurants seem to shy away from allocating the scarce table space on the basis of price, which is usually not raised on graduation day. Some moderate price increases may be hidden in the form of requiring the purchase of a “special” graduation meal, but this kind of price premium is nowhere near what would be needed to remove excess demand, as evidenced by the long lead time in accepting reservations. A possible explanation is that an exorbitant price might be widely discussed and reported, with a backlash that might harm future business. The higher the price, the lower the cost of rejecting the deal. In the monopoly example discussed above, a price of $14.99 would be rejected if the buyer is willing to incur a one-cent cost in order to punish the seller for charging such a price.

Several variants of the ultimatum game have been widely used in laboratory experiments because this game maximizes the tension between fairness considerations and the other extreme where people only care about their own earnings. Moreover, in the lab it is often possible to set up a “one-shot” situation with enough anonymity to eliminate any considerations of reputation, reward, and punishment. The next section describes an ultimatum experiment where laboratory control was a particularly difficult problem.

II. Bargaining in the Bush

Jean Ensminger (2001) conducted an ultimatum experiment in a number of small villages in East Africa. All participants were members of the Orma clan. The Orma offer an interesting case where there is considerable variation in the extent of integration into a market economy, which may affect attitudes towards fairness. The more nomadic families raise cattle and live largely off of the milk and other products, with very little being bought or sold in any market. Although some nomadic families have high wealth, which is kept in the herd, they typically have low incomes in terms of wage payments. Other Orma, in contrast, have chosen a more sedentary lifestyle for a variety of reasons, including the encroachment of grazing lands. Those who live sedentary lives in villages typically purchase food with money income obtained as wages or crop revenues. Thus exposure to a market economy is quite variable and is well measured as direct money income. Such income is not highly correlated with wealth, since some of the wealthiest are self-sufficient nomadic families with large herds. Many bargaining situations in a market transaction end up with individuals
agreeing to “split the difference,” so a natural conjecture is that behavior in an ultimatum game will be related to the degree of exposure to a market economy.

A survey was completed for each household, and at least one adult was recruited from each household to play “fun games for real money.” The experiments were conducted in grass houses, which enabled Ensminger to isolate groups of people during the course of the experiment. A “grand master,” who was known to all villagers, read the instructions. This person would turn away to avoid seeing decisions as they were made. The amount of money to be divided between the proposer and responder in each pair was set to be approximately equal to a typical day’s wages (100 Kenyan shillings). The game was only played once. Each proposer would make an offer by moving some of the shillings to the other side of the table, before leaving the room while the responder was allowed to accept or reject this offer. Ensminger reports that the people enjoyed the games, despite some amusement at the “insanity” and “foolishness” of Western ways. She tried to move from one village to another before word of results arrived.

The data for 56 bargaining pairs are shown in Figure 23.1. The average offer was 44 percent of the stake, with a clear mode at 50 percent. The lowest offer was 30 percent, and even these unequal splits were rarely rejected (striped bar). If people had foreseen that the 40 percent would all be accepted, then they might have lowered their offers. It is clear that the modal offer is not optimal in
the sense of expected-payoff maximization against the actual *ex post* rejection pattern. People who made these generous offers almost always mentioned fairness as the justification in follow-up interviews. Ensminger was suspicious, however, and approached some reliable informants who revealed a different picture of the “talk of the village.” The proposers were apparently *obsessed* with the possibility that low offers would be rejected, even though rejections were thought to be unlikely. Such obsessions suggest that expected-payoff maximization may not be an appropriate assumption for large amounts of money, e.g., a day’s income. This conjecture would be consistent with the payoff-scale effects on risk aversion discussed previously in Chapter 4.

Nevertheless, it cannot be the case that individuals making relatively high offers were doing so solely out of fairness considerations, since offers were much lower in a second experiment in which the proposer’s split of the same amount of money was automatically implemented. This game, without any possibility of rejection, is called a “dictator game.” The average offer fell from 44 percent in the ultimatum game to 31 percent when there was no possibility of rejection. Even though there seems to be some strategic reaction by proposers to their advantage in the dictator game, the modal offer was still at the “fair” or “fifty-fifty” division, and less than a tenth of the proposers kept all of the money.

Ensminger used a multiple regression to evaluate proposer offers in both the ultimatum and dictator games. In both cases, the presence of wage income is significantly related to offers; those with such market interactions tend to make higher offers. Her conjecture is that people who are exposed to face-face market transactions may be more used to the notion of “splitting the difference” in negotiations. Variables such as age, gender, education, and wealth (in cattle equivalents) are not significant. This intra-cultural finding is also evident in a cross-cultural study involving 15 small-scale societies on five continents (Henrich et al., 2001). This involved one-shot ultimatum games with comparable procedures that were conducted by anthropologists and economists. There was considerable variation, with the average offer ranging from 0.26 to 0.58 of the stake. The societies were ranked in two dimensions: the extent of economic cooperation and the extent of market integration. Both variables were highly significant in a regression, explaining about 61 percent of the variation, whereas individual variables like age, sex, and relative wealth were not. The lowest offers were observed in societies where very little production occurred outside of family units (e.g., the Machiguenga of Peru). The mean offers exceeded one half for the Lamelara of Indonesia, who are whale hunters in large sea-going canoes.

A one-shot ultimatum game, played for money, is a strange new experience for these people, and behavior seemed to be influenced by parallels with social institutions in some cases. The Ache of Paraguay, for example, made generous offers, above 0.5 on average. The proposed sharing in the ultimatum
game has some parallels with a practice whereby Ache hunters with large kills will leave them at the edge of camp for others to find and divide.

III. Bargaining in the Lab

Ultimatum games have been conducted in more standard, student subject pools in many developed countries, and with stakes that are usually about $10. The mean offer, as a fraction of the stake, is typically about 0.4, and there seems to be less variation in the mean offer than was observed in the small-scale societies. Roth et al. (1991) report ultimatum game experiments that were run in four universities, each in a different country. The modal offer was 0.5 in the U.S. and Slovenia, as was the case for the Orma and for other studies in the U.S. (e.g., Forsythe et al., 1988). The modal offer was somewhat lower, 0.4, in Israel and Japan. Rejections in these countries were no higher than in the other two countries, which led the authors to conjecture that the differences in behavior across countries were due to different cultural norms. These differences in behavior reported by Roth et al. (1991) were, however, lower than the differences observed in the 15 small-scale societies, which are less homogeneous in the nature of their economic production activities.

Recall that the Orma made no offers below 0.3. In contrast, some student subjects in developed countries made offers of 0.2 or lower, and these low offers were rejected about half of the time. For example, consider the data in Figure 23.2, which is for a one-shot ultimatum game that was done in class, but with full money payments and a $10 stake. The mean offer was about 0.4 (as compared with 0.44 for the Orma), but about a quarter of the offers were 0.2 or below, and these were rejected a third of the time.

Ultimatum bargaining behavior is relatively sensitive to various procedural details, and for that reason extreme care was taken in the Henrich et al. cross-cultural study. Hoffman et al. (1994) report that the median offer of 5 in an ultimatum game was reduced to 4 by putting the game into a market context. The market terminology had the proposer play the role of a seller who chose a take-it-or-leave-it price for a single unit. Interactions with posted prices are more anonymous than face-to-face negotiations, and market price terminology in the laboratory may stimulate less generous offers for this reason. The median offer fell again, to 3, when high scores on a trivia quiz were used to decide which person in each pair would play the role of the seller in the market. Presumably, this role-assignment effect is due to the fact that people be more willing to accept an aggressive (low) offer from a person who earned the right make the offer.
It is somewhat unusual for a seller to offer only one unit for sale to a buyer, and a multi-unit setup provides the buyer with an option for partial rejection. For example, suppose that there are 10 units for sale. Each costs $0 to produce, and each unit is worth $1 to the buyer. The seller posts a price for the 10 units as a group, but the buyer can decide to purchase a smaller number, which reduces each party’s earnings proportionately. For example, if the seller posts a price of $6, which would provide earnings of $6 for the seller and $4 for the buyer. By purchasing only half of the units, the buyer reduces these earnings to $3 for the seller and $2 for the buyer. This partial rejection option is implemented in the Vconlab software as a “squish” option (Andreoni, Castillo, and Petrie, 1998). If this option is permitted, the responder can choose a fraction that indicates the extent of acceptance: with 0 being full rejection and 1 being full acceptance. This option was used in the classroom experiment shown in Figures 23.3 (first round) and 23.4 (fifth and final round). The extreme offer of 0.2 in the first round was squished by a half. Offers in the fifth round were more clustered about 0.5.
Figure 23.3. Distribution of Offers in Round 1 of a Classroom Ultimatum Game  
(6 Pairs, University of Virginia, Spring 2002)

Figure 23.4. Distribution of Offers in Round 5 of a Classroom Ultimatum Game  
(6 Pairs, University of Virginia, Spring 2002)
IV. Multi-Stage Bargaining

Face-to-face negotiations are typically characterized by a series of offers and counter-offers. The ultimatum game can be transformed into a game with many stages by letting the players take turns making proposals about how to split an amount of money. If there is an agreement at any stage, then the agreed split is implemented. A penalty for delay can be inserted by letting the size of the stake shrink from one stage to the next. For example, the amount of money that could be divided in the first stage may be $5, but a failure to reach an agreement may reduce this stake to $2 in the second stage. If there is a pre-announced final stage and the responder in that stage does not agree, then both earn zero. Thus the final stage is like an ultimatum game, and can be analyzed as such.

Now consider a game with only two stages, with a money stake, $Y, in the first stage and a lower amount, $X, in the final stage. For simplicity, suppose that the initial stake is $15, which is reduced to $10 if no agreement is reached in the first round. The analysis in this paragraph is based on the (very unreasonable) assumption that people are perfectly rational and care only about their own payoffs. Put yourself in the position of being the proposer in the first stage of this game, with the knowledge that both of you would always prefer the action with the highest payoff, even if that action only increases one’s payoff by a penny. If you offer the other player too little in the first stage, then this offer will be rejected, since the other player becomes the proposer in the final (ultimatum game) stage. So you must figure out how much the other person would expect to earn in the final stage when they make a proposal to split the $10 that remains. In theory, the other person could make a second-stage offer of a penny, which you would accept because you prefer more money (a penny) to less (zero). Hence the other person would expect to earn $9.99 in the final stage, assuming perfect rationality and no concerns for fairness. If you offer them less than $9.99 in the first stage, they will reject, and if you offer them more they will accept. The least you could get away with offering in the first stage is, therefore, just a little more than $9.99, so you offer $10, which is accepted.

In the previous example, the theoretical prediction is that the first-stage offer will equal the amount of the stake that would remain if bargaining were to proceed to the second stage. This result can be generalized. If the size of the money stake in the final stage is $X, then the person making the offer in that stage “should” offer the other a penny, which will be accepted. The person making the proposal in the final stage can obtain essentially the whole stake, i.e. $X − $0.01. Thus the person making an offer in the first stage can get away with offering a slightly higher amount, i.e. $X, which is accepted. In this two-stage game, the initial proposer earns the amount by which the pie shrinks, $Y − $X, and the other person earns the amount remaining in the second stage, $X.
An experiment with this two-stage structure is reported in Goeree and Holt (2001). The size of the stake in the initial stage was $5 in both cases. In one treatment, the pie was reduced to $2.00, so the first-stage offer should be $2.00, as shown in the middle column of Table 21.1. The average first-stage offer was $2.17, close to this prediction. In a second treatment, the pie shrunk to $0.50, so the prediction is for the first-stage offer to be very inequitable ($0.50). The average offer was somewhat higher, at $1.62, as shown in the right column of the table. The reduction in the average offer was much less than predicted by the theory, and rejections were quite common in this second treatment. Notice that the cost of rejecting a low offer is low, and knowing this, the initial proposers were reluctant to exploit their advantage fully in this second treatment.

<table>
<thead>
<tr>
<th>Size of Pie in First Stage</th>
<th>$5.00</th>
<th>$5.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>Size of Pie in Second Stage</td>
<td>$2.00</td>
<td>$0.50</td>
</tr>
<tr>
<td>Selfish Nash First-Stage Offer</td>
<td>$2.00</td>
<td>$0.50</td>
</tr>
<tr>
<td>Average First-Stage Offer</td>
<td><strong>$2.17</strong></td>
<td><strong>$1.62</strong></td>
</tr>
</tbody>
</table>

The effects of payoff inequities are even more dramatic in a second two-stage experiment reported by Goeree and Holt (2000). The initial proposers in this design made seven choices corresponding to seven different bargaining situations, with the understanding that only one of the situations would be selected afterwards, at random, before the proposal for that situation was communicated to the other player. The initial pie size was $2.40 in all seven cases, but the second-stage pie size varied from $0.00 to $2.40, with five intermediate cases.

The two extreme treatments for this experiment are shown in Table 21.2. In the middle column, the pie shrinks from $2.40 to $0.00. This gives the initial proposer a large strategic advantage, so the initial offer would be a penny in a game between two selfish, rational players. This outcome would produce a sharp asymmetry in the payoffs in favor of the proposer. Adding insult to injury, the instructions for this case indicated that the initial proposer would receive a fixed payment of $2.65 in addition to the earnings from the bargaining, whereas the initial responder would only receive $0.25. Only if the initial proposer were to offer the whole pie of $2.40 in the first stage would final earnings be equalized. This is listed as the “egalitarian first-stage offer” in the middle column. The
average of the actual offers, shown in the bottom row, is closer to the egalitarian offer.

The other extreme treatment is shown in the right-hand column of Table 21.2. Here the pie does not shrink at all in the second stage, so the theoretical prediction is that the first-stage offer will be $2.40. The initial responder now has the strategic advantage, since the pie remains high in the final (ultimatum) stage when this person has the turn to make the final offer. To make matters even more asymmetric, this strategic advantage is complemented with a high fixed payment to the initial responder. The only way for the disadvantaged initial proposer to obtain equal earnings would be to offer nothing in the first stage, despite the fact that the theoretical prediction (assuming selfish behavior) is $2.40. The average of the observed offers, $0.63, is much closer to the egalitarian offer.

Table 21.2. A Two-Stage Bargaining Game with Asymmetric Fixed Payments
(Source: Goeree and Holt, 2000)

| Size of Pie in First Stage | $2.40 | $2.40 |
| Size of Pie in Second Stage | $0.00 | $2.40 |
| Proposer Fixed Payment | $2.65 | $0.25 |
| Responder Fixed Payment | $0.25 | $2.65 |
| Egalitarian First-Stage Offer | $2.40 | $0.00 |
| Selfish Nash First-Stage Offer | $0.01 | $2.40 |
| Average First-Stage Offer | $1.59 | $0.63 |

To summarize, the first-stage offers in this experiment should be equal to the remaining pie size, but the asymmetric fixed payments were structured so that the egalitarian offers would be inversely related to the remaining pie size. This inverse relationship was generally present in the data for the seven treatments. The authors show that the data patterns are roughly consistent with an enriched model in which people care about relative earnings as well as their own earnings. For example, a person may be willing to give up some money to avoid having the other person earn more, which is an aversion to disadvantageous inequity. Roughly speaking, think of this as an “envy effect.” It is also possible that people might wish to avoid making significantly more than the other person, which would be an aversion to advantageous inequity. Think of this as a kind of “guilt effect,” which is likely to be weaker than the envy effect. These two effects are captured by a formal model of inequity aversion proposed by Fehr and Schmidt.
Goeree and Holt (2000) estimate guilt and envy parameters for their data and conclude that the envy effect is more pronounced.

V. Extensions and Further Reading

The first ultimatum experiment was reported by Guth et al. (1982), and the results were replicated by Forsythe et al. (1988), who introduced the dictator game. Economists and others have been fascinated by behavior in these games where there is a high tension between notions of fairness and strategic, narrowly self-interested behavior. Ensminger (2000) reports that one of her African subjects jovially remarked: “I will be spending years trying to figure out what this all meant.”

Theorists were initially skeptical of the high degree of seemingly non-strategic play and costly rejections. One reaction was that “irrational” rejections would diminish when the stakes of the game are increased. Hoffman et al. (1995) increased the stakes from $10 to $100, which did not have much effect on initial proposals. The effects of high stakes have also been studied by Slonim and Roth (1998) and List and Cherry (2000).

Rejections are not, of course, irrational if individuals have preferences that depend on relative earnings. For example, a responder may prefer that both earn equal zero amounts to a situation with inequitable positive earnings. Bolton and Ockenfels (1998) proposed a model with preferences based on relative earnings, and Fehr and Schmidt (1999) developed a closely related model of inequity aversion.

Finally, there have been many experiments in other, less-structured negotiations, where the order of proposal and response is not imposed by the experimenter. For example, Hoffman and Spitzer (1982, 1985) used an open, unstructured setting to evaluate the ability of bargainers to agree on efficient outcomes, irrespective of property rights (the Coase theorem). See Roth (1995) for a survey of the literature on bargaining experiments.
Chapter 24. The Trust Game

Although increases in the size of the market may increase productive specialization and trade, the accompanying increase in anonymity raises the need to trust in trading relationships. The trust game sets up a stylized situation where one person can decide how much of an initial stake to keep and how much to pass to the other person. All money passed is augmented, and the responder then decides how much of this augmented amount to keep and how much to pass back to the initial decision maker. The webgame “TG” puts participants into this situation so that they can experience the tension between private motives and the potential gains from trust and cooperation.

Chapter 25. The Reciprocity Game

The “gift-exchange” view of wage setting is that employers set wages above market-clearing levels in an effort to elicit high effort responses, even though those responses are not explicitly rewarded \textit{ex post} after wages have been set. The reciprocity game is one where each employer is matched with a worker who first sees the wage that is offered and then decides on the level of costly effort to supply. The issue is whether notions of trust and reciprocity may have noticeable effects in a market context.
Part VII. Public Choice

Many public programs and policies are designed to remedy situations where the actions taken by some people affect the well being of others. The classic example is that of the provision of a “public good” like national defense, which can be consumed freely by all without crowding and the possibility of exclusion. Chapter 26 introduces a model of voluntary provision to a public good, where the private cost to each person is less than the social benefit. The setup of the game allows the independent alteration of (internal) private cost and (external) public benefit. Nonselfish motives for giving are discussed in the context of laboratory experiments.

A similar situation with “external” benefits arises when it only takes one “volunteer” to provide the good outcome preferred by all. The “volunteer’s dilemma” discussed in Chapter 27 is whether to incur the private cost to achieve this outcome. This game is different from a standard public goods setup in that the private benefit to the volunteer exceeds the cost. For example, suppose that you would rather jump in the icy river and rescue someone, but you would prefer not to jump if one or more other bystanders decide to volunteer. Data from volunteer’s dilemma games are often used to evaluate Nash predictions of the effects of changing the number of potential volunteers.

External effects on others’ well being may arise in a negative sense as well, as is the case with pollution or overuse of a common resource. The “common pool resource” game in Chapter 28 is a setting where each person’s efforts to secure benefits form a shared resource tend to diminish the benefits that others derive from their efforts, as might happen with excessive harvests from a fishery. The resource is like a public good in the sense that the problem arises from non-exclusion, but the difference is the presence of congestion effects, i.e. it is not non-rivaled. The simplest common pool resource game is one in which people select efforts independently, and the benefits achieved are diminished as total effort increases. The focus of the discussion is on the extent of overuse, in comparison with the Nash equilibrium.

The final chapter introduces a problem of wasteful competition that may arise in non-market allocation procedures. In “beauty contest” competitions for a broadcast license, for example, the contenders may spend considerable amounts of real resources in the process of lobbying. These lobbying expenses are examples of “rent seeking,” and the total cost of such activities may even exceed the value of the prize (full dissipation of rents). Chapter 29 is based on Gordon Tullock’s model of rent seeking, where the probability of obtaining the prize is the proportion of total effort that one exerts. Factors that affect rent dissipation in theory are evaluated in the context of laboratory experiments.
Chapter 26. Voluntary Contributions

This chapter is based on the standard voluntary contributions game, in which the private net benefit from making a contribution is negative unless others reciprocate or unless the person receives satisfaction from the benefit provided to others. The setup makes it possible to evaluate independent variation of the private internal benefit and the external public benefit to others. These and other treatment manipulations in experiments are used to evaluate alternative explanations for observed patterns of contributions to a public good. The public goods webgame, PG, should be conducted prior to class discussion, or alternatively, a hand-run version using playing cards is easy to implement using the instructions provided in the appendix.

I. Introduction

The selfish caricature of the homo economicus implies that individuals will “free ride” off of the public benefits provided by others’ activities, and that such free riding may result in the under-provision of public goods. A pure public good, like national defense, does not become congested, i.e. it is “non-rivaled,” and access cannot be controlled, i.e. it is “non-excludable.” When a single individual provides a public good, like shoveling a sidewalk, the private provision cost may exceed the private benefit, even though the social benefit for all others’ combined may exceed the provision cost. The resulting misallocations have been recognized since Adam Smith’s (1776) discussion of the role of government in the provision of street lamps.

In many cases, there is not a bright line distinction between private and public goods, e.g. parks are generally considered to be public goods, although some may become crowded and require some method of exclusion. A good that is rivaled but non-excludable is sometimes called a “common pool resource,” which is the topic of one of the chapters that follows in this section. With common-pool resources like ground water resources, fisheries, or public grazing grounds, the problem is typically one of how to manage the resource to prevent overuse, since individuals may not take into account the negative effects that their own usage has on what is available for others. In contrast, most public goods are not provided by nature, and the major problems often pertain to provision of the appropriate amounts of the good.

Under-provision can remain a problem when a good imparts private benefits to the provider, as long as there are some public benefits to others that may not be fully valued by the provider. Education, for example, offers clear economic advantages to the student, but the public at large also benefits from
having a well-educated citizenry. A public goods problem remains when not all of the benefits of provision are enjoyed by the provider, and this is one of the rationales for the heavy public involvement in school systems. The mere presence of public benefits does not justify public provision of such goods, since public provision may involve inefficiencies and distortions due to the need to collect taxes. The political problems associated with public goods are complicated when the benefits are unequally distributed, e.g., public broadcasting of cultural materials.

In close-knit societies, public goods and common-pool resource problems may be mitigated by the presence of social norms that dictate other-regarding behavior. Economists are sometimes ridiculed for ignoring such norms and for assuming that individuals will typically free-ride on others’ generosity. One of the first laboratory experiments involving public goods was reported by two sociologists, Marwell and Ames (1981). Their experiment involved groups of high school students who could either invest an initial endowment in a private or a public exchange. Investment in the public exchange produced a net loss to the individual, even though the benefits to others were substantially above the private cost to the individual subject. The authors observed significant amounts of investment in the public exchange, with the major exception being the experiment was done with a group of economics doctoral students. The resulting paper title began: “Economists Free Ride, Does Anyone Else?”

The Marwell and Ames paper provoked paper initiated a large literature on the extent to which subjects in experiments incur private costs in activities that benefit others. A typical experiment involves dividing subjects into groups, and giving each one an endowment of “tokens” that can be invested in a private account, with earnings per token that exceed the earnings per token obtained from investment in the public account. For example, each token might produce 10 cents when invested in the private account and only 4 cents when invested in the public account. The difference, however, might be that all members of the group receive 5 cents from a token invested in the public account, whereas only the investor receives the 10 cents from investment in the private account. In this example, the social optimum would be to invest all tokens in the public account as long as the number of individuals in the group, $N$, is greater than 2, since the social benefit $5N$ would be greater than the private benefit of 10 for each token. In this example, the 10-cent private return can be thought of as the opportunity cost of investment in the public account. The ratio of the per capita benefit to the opportunity cost is sometimes called the “marginal per capita benefit” or MPCR, which would be 0.5 in this example. A higher MPCR reduces the net cost of making a contribution to the public account. For example, if the private account returns 10 cents and the return on the public account is raised to 9 cents, then there is only a 1 cent private loss associated with investment. A larger group size
increases the social benefit of such investments. Think about what you would do if you could give up ten dollars in order to return a dollar to every member of the student body at your university, including yourself. Here the MPCR is only 0.1, but the public benefit is extremely large. The motives for contribution to a public good are amplified if others are expected to reciprocate in some future period. These considerations suggest that contributions might be sensitive to factors such as group size, the MPCR, and whether or not the public goods experiment involves more than one round with the same group. In multi-round experiments, individuals are given new endowments of tokens at the start of each period, and groupings can either be fixed or randomly reconfigured in subsequent rounds.

The upshot is that the extent of voluntary contributions to public goods depends on a wide variety of procedural factors, although there is considerable debate still on whether contributions are primarily due to kindness, to reciprocating others’ kindness, or to confusion.

For example, there is likely to be more confusion in a single-shot investment decision, and many people may initially divide their endowment of “tokens” equally between the two types of investment, public and private. This is analogous to the typical choice of dividing one’s retirement fund contributions equally between stocks and bonds at the start of one’s career. Many of the Marwell and Ames experiments involved a single decision, e.g. administered by a questionnaire that was mailed to high school students. Subsequent experiments by economists showed that repetition typically produced a declining pattern of contributions. If some people observe that others are not contributing, then they may become frustrated and stop contributing themselves. One motive for making contributions may be to prevent others from behaving in this manner, in the hope that contributions will be reciprocated in subsequent rounds. This reciprocity motive is obviously weaker in the final rounds. Although contributions tend to decline in the final periods, some people do contribute even in the final period. We begin with a summary of an experiment with a one-shot setup, and multi-round experiments will be considered in later sections.

II. A Single-Round Public Goods Experiment

In many public goods settings, the person making a voluntary contribution may enjoy a greater personal benefit than others. For example, benefactors typically give money for projects that they value for some reason. It is quite common, for example, for gifts to medical research teams to be related to illnesses that are present in the donor’s family. Even in Adam Smith’s streetlight example, a person who erects a light over the street would pass by that spot more often, and hence may receive a greater benefit than any other randomly selected person in the town.
This difference between donor benefits and other public benefits can be introduced in an experiment by

Notation

Field experiments seed money
Chapter 27. The Volunteer’s Dilemma

A volunteer’s dilemma arises when it only takes a single volunteer to provide a public benefit. The per-capita value of this benefit exceeds the private cost of volunteering, but each person would prefer that someone else incur this cost. For example, each major country on the UN Security Council may prefer that a proposal by a small country is vetoed, but each would rather that another country incur the political cost of a veto that is unpopular with many small member nations. This dilemma raises interesting questions, such as whether volunteering is more or less likely in cases with large numbers of potential volunteers. The volunteer’s dilemma game provides data that can be compared with both intuition and theoretical predictions. This is implemented as the webgame VG, or it can be run by hand with playing cards and the instructions provided in the appendix.

Chapter 28. Common Pool Resources

Some of the most worrisome environmental problems result from excessive use or “exploitation” of a shared resource. For example, an increase in fishing activity may reduce the catch per hour for all fishermen. Each individual may tend to ignore the negative impact of their activity on other’s harvests, especially if there are a lot of others. This negative externality is typically not priced in a market, and overuse can result. The common pool resource game provides a highly stylized paradigm of overuse. In this game, individual efforts to secure more benefits from the resource have the effect of reducing the benefits received by others. In technical terms, the average and marginal products of effort are decreasing in the total effort of all participants. The game is static, with nothing that corresponds to a stock of fish, but it can nevertheless be used to structure a discussion of external effects. A hand-run version is provided in the appendix, or the Veconlab game CP may be used to compare behavior in alternate treatments.
Chapter 29. Rent Seeking

Administrators and government officials often find themselves in a position of having to distribute a limited number of prized items (locations, licenses, etc.). Contenders for these prizes may engage in lobbying or other costly activities that increase their chances for success. No single person would spend more on lobbying than the prize is worth, but with a large number of contenders, expenditures on lobbying activities may be considerable. This raises the disturbing possibility that the total cost of lobbying by all contenders may "dissipate" a substantial fraction of the prize value. Rent seeking is thought to be more prevalent in developing countries, where some estimates are that such non-market competitions consume a significant fraction of national income (Krueger, 1974). One only has to serve as a Department Chair in a U.S. university, however, to experience the frustrations of rent seeking. The game considered in this chapter is one in which the probability of obtaining the prize is equal to one’s share of the total lobbying expenditures of all contenders. The game illustrates the potential costs of administrative (non-market) allocation procedures. It can be implemented with playing cards, as indicated in the appendix, or it can be run on the Veconlab software (game RS).

I. Government with “a Smokestack on Its Back”

One of the most spectacular successes of recent government policy has been the auctioning off of bandwidth used to feed the exploding growth in the use of cell phones and pagers. The US Federal Communications Commission (FCC) has allocated major licenses with a series of auctions that raised billions of dollars without adverse consequences. Some European countries followed with similarly successful auctions, which also raised many more billions than expected in some cases. The use of auctions collects large numbers of potential competitors, and the commodities can be allocated quickly to those with the highest valuations. In the U.S., this bandwidth was originally reserved for the armed forces, but it was underused at the end of the Cold War, and the transfer created a large increase in economic wealth, without significant administrative costs. It could have easily been otherwise. Radio and television broadcasting licenses were traditionally allocated via administrative proceedings, which are sometimes called “beauty contests.” The successful contender would have to convince the regulatory authority that service would be of high quality and that social values would be protected. This often required establishing a technical expertise and an effective lobbying presence. The lure of extremely high potential profits was strong enough to induce large expenditures by the aspiring providers. Those who had acquired licenses in this manner were strongly opposed to market-based
allocations that forced the recipients to pay for the licenses. Similar economics pressures may explain why beauty contest allocations continue to be prevalent in many other countries.

The first crack in the door appeared in the late 1980’s, when the FCC decided to skip the administrative proceedings in the allocation of hundreds of regional cell phone licenses. The forces opposed to the pricing of licenses did manage to block an auction, and the licenses were allocated by lottery instead. There were about 320,000 applications for 643 licenses. Each application involved paper work, legal and accounting services to the extent that firms specializing in providing completed applications began offering this service for about $600 per application. The services used to provide this service have opportunity costs, and Hazlett and Michaels (1993) estimated that the total cost of all submitted applications to be about $400,000. The lottery winners often sold their licenses to more efficient providers, and resales were used to estimate that the total market value of the licenses at that time was about a billion dollars. Each individual lottery winner earned very large profits on the difference between the license value and the application cost, but the costs incurred by others were lost, and the total cost of the transfer of this property was about forty percent of the market value. There are, of course, the indirect costs of subsequent transfers of licenses to more efficient providers, a process of consolidation that may take many years. In the meantime, inefficient provision of the cellular services may have created ripples of inefficiency in the economy. It is episodes like this that once caused Milton and Rose Friedman (1989), in a discussion of the unintended side effects of government policies, to remark: “Every government measure bears, as it were, a smokestack on its back.”

In a classic paper, Gordon Tullock (1967) pointed out that the real costs associated with competitions for government granted “rents” may destroy or “dissipate” much of the value of those rents in the aggregate, even though the winners in such contests may earn large profits. This destruction of value is often invisible to those responsible, since the contestants participate willingly, and the administrators may enjoy the process of being lobbied. Moreover, some of the costs of activities like waiting in line and personal lobbying are not directly priced in the market. These costs can be quite apparent in a laboratory experiment of the type to be described next.

II. Rent Seeking in the Classroom Laboratory

In many administrative (non-market) allocation processes, the probability of obtaining a prize or monopoly rent is an increasing function of the amount spent in the competition. In the FCC auctions, the chances of winning a license were approximately equal to the applicant’s efforts as a proportion to the total efforts of the other contestants. This provides a rationale for a standard
mathematical model of “rentseeking” with \( N \) contestants. The effort for person \( i \) is denoted by \( x_i \) for \( i = 1, \ldots, N \). The total cost of each effort is a cost \( c \) times the effort, i.e. \( cx_i \). In the simplest symmetric model, the value of the rent, \( V \), is the same for all, and each person’s probability of winning the prize is equal to their effort as a fraction of the total effort of all contestants:

\[
\text{expected payoff} = -\frac{x_i}{\sum_{j=1}^{N} x_j} V - cx_i.
\]

Here the expected payoff is the probability of success, which is the fraction of total effort, times the prize value \( V \), minus the cost of effort. This latter cost is not multiplied by any probability since it must be paid whether or not the prize is obtained.

Goeree and Holt (1999) used the payoff function in (1) in a classroom experiment with \( V = 16,000 \), \( c = 3,000 \), and \( N = 4 \). Each of the four competitors consisted of a team of 2-3 students. Efforts were required to be integer amounts. This requirement was enforced by giving 13 playing cards of the same suit to each team. Rent-seeking effort was determined by the number of cards that the person placed in an envelope, as described in the instructions to this chapter. The cards were collected, shuffled, and one was drawn to determine which contender would win the $16,000 prize. Each team incurred a cost of $3,000 for each card played, regardless of whether or not they obtained the prize. The number of cards varied from team to team, but on average there were slightly more than three cards played by each team. Thus a typical team incurred 3x$3,000 in expenses, and the total lobbying cost for all four teams was over $36,000, all for a prize worth only $16,000.

Similar results were obtained in a separate classroom experiment using the game RS from the Veconlab setup. The parameters were the same (four competitors, a $16,000 value, and a $3,000 cost per unit of effort), and the 12 teams were randomly put into markets of four competitors in a series of rounds. The average number of lobbying effort units was 3 in the first round, which was reduced to a little over 2 in rounds 4 and 5. Even with two units expended by each team, the total cost would be 2 (number of effort units) times 4 (teams) times the $3,000 cost of effort, for a total cost of $24,000. This again resulted in over-dissipation of the rent, which was only $16,000.

III. The Nash Equilibrium

A Nash equilibrium for this experiment is a lobbying effort for each competitor such that nobody would want to alter their expenditure given that of the other competitors. First, consider the case of four contenders, a $16,000
value, and a $3,000 effort cost. Note that efforts of 0 cannot constitute an equilibrium, since each person could deviate to an effort of 1 and obtain the $16,000 prize for sure at a cost of only $3,000. Next suppose that each person is planning to choose an effort of 2 at a cost of $6,000. Since the total effort is 8, the chances of winning are 2/8 = 1/4 so the expected payoff is 16,000/4 – 6,000, which is minus $2,000. This cannot be a Nash equilibrium, since each person would have an incentive to deviate to 0 and earn nothing instead of losing $2,000. Next, consider the case where each person’s strategy is to exert an effort of 1, which produces an expected payoff of 16,000/4 – 3,000 = 1,000. To verify that this is an equilibrium, we have to check to be sure that deviations are not profitable. A reduction to an effort of 0 with a payoff of 0 is obviously bad. A unilateral increase to an effort of 2, when the others maintain efforts of 1, will result in a 2/5 chance of winning, for an expected payoff of 16,000(2/5) – 6,000 = 400, which is also worse than the payoff of 1,000 obtained with an effort of 1.

In the symmetric equilibrium, the effort of 1 for each of the four contestants results in a total cost of 4(3,000) = 12,000, which dissipates three-fourths of the 16,000 rent. This raises the issue of whether a reduction in the cost of rent seeking efforts might reduce the extent of wasted resources. In the context of the FCC lotteries, for example, this could correspond to a requirement of less paper work for each separate application. Suppose that the resource cost of each application is reduced from $3,000 to $1,000. This cost reduction causes the Nash equilibrium level to rise from 1 to 3 (see question 1).

III. A Mathematical Derivation of the Equilibrium (Optional)

The Nash equilibrium calculations done thusfar were based on considering a particular level of rent-seeking activity, common to each person, and showing that a deviation by one person alone would not increase that person’s expected payoff. This is a straightforward, but tedious, approach, and it has the additional disadvantage of not explaining how the candidate for a Nash equilibrium was found in the first place. A calculus derivation is relatively simple, and is offered here as an option for those who are familiar with basic rules for taking derivatives. First, consider the expected payoff function in (29.1), modified to let the decisions of the N−1 others be equal to a common level, \( x^* \).

\[
(29.2) \quad \text{expected payoff} = \frac{x_i}{x_i + (N-1)x} - cx_i.
\]
This will be a “hill-shaped” (concave) function of the person’s own rent-seeking activity, \( x_i \), and the function is maximized “at the top of the hill” where the slope of the function is zero. To find this point, the first step is to take the derivative and set it equal to zero. The resulting equation will determine player \( i \)’s best response when the others are choosing \( x^* \). In a symmetric equilibrium with equal rent-seeking activities, it must be the case that \( x_i = x^* \), which will yield an equation that determines the equilibrium level of \( x^* \). This analysis will consist of two steps: finding the derivative of player \( i \)’s expected payoff and then using the symmetry condition.

**Step 1.** In order to use the rule for taking the derivative of a power function, it is convenient to express (29.2) as a power function:

\[
(29.3) \quad \text{expected payoff} = x_i (x_i + (N-1)x^*)^{-1}V - cx_i.
\]

The final term on the right side of (29.3) is linear in \( x_i \), so its derivative is \(-c\). The first term on the right side of (29.3) is the product of \( x_i \) and a power function that contains \( x_i \), so the derivative is found with the product rule (derivative of the first function times the second, plus the first function times the derivative of the second), which yields the first two terms on the left side of (29.4):

\[
(29.4) \quad (x_i + (N-1)x^*)^{-1}V - x_i (x_i + (N-1)x^*)^{-2}V - c = 0.
\]

**Step 2.** Next we use the fact that \( x_i = x^* \) in a symmetric equilibrium. Making this substitution into (29.4) yields a single equation in the common equilibrium level, \( x^* \):

\[
(29.5) \quad (x^* + (N-1)x^*)^{-1}V - x^* (x^* + (N-1)x^*)^{-2}V - c = 0,
\]

which can be simplified to:

\[
(29.6) \quad \frac{1}{N}V - \frac{1}{N^2}x^*V - c = 0.
\]

Finally, we can multiply both sides of (29.6) by \( N^2 \) \( x^* \), which yields an equation that is linear in \( x^* \) that reduces to:
It is straightforward to verify that \( x^* = 1 \) when \( V = $16,000, \ c = $3,000, \) and \( N = 4. \) When the cost is reduced to $1,000, the equilibrium level of rent-seeking activity is raised to 3. Notice that the predicted effect of a cost reduction is that the total amount spent on rent-seeking activity is unchanged.

Finally, consider that total cost of all rent-seeking activity, which will be measured by the product of the number of contenders, the effort per contender, and the cost per unit of effort: \( N x^* c. \) It follows from (29.7) that this total cost is: \( (N-1)/N \) times the prize value, \( V. \) Thus the fraction of the rent that is dissipated is a fraction, \( (N-1)/N, \) which is an increasing function of the number of contenders. With two contenders, half of the value is dissipated in a Nash equilibrium, and this fraction increases towards 1 as the number of contenders gets large. In general, rent dissipation is measured as the ratio of total expenditures on rent-seeking activity by all contenders to the value of the prize.

Notice that the fraction of rent dissipation is predicted to be independent of the cost of lobbying effort, \( c. \) For example, recall the previous example with four contenders, where the reduction in \( c \) from $3,000 to $1,000 raised the Nash equilibrium lobbying effort from 1 to 3, thereby maintaining a constant total level of expenditures.

These predictions were tested with a Veconlab experiment run on a single class with a “2x2” design with high and low lobbying costs, and with high and low numbers of competitors. There were 20 rounds (5 for each treatment). The prize value was $16,000 in all rounds, and each person began with an initial cash balance of $100,000. One person was selected at random ex post to receive a small percentage of earnings. The treatments involved a per-unit cost of either $500 or $1,000, and either 2 or 4 contenders. The four treatment combinations are shown in the two columns on the left side of Table 29.1.

Table 29.1. A Classroom Rent-seeking Experiment with a Prize Value of $16,000

<table>
<thead>
<tr>
<th>Number of Contenders: ( N )</th>
<th>Per-unit Cost: ( C )</th>
<th>Nash Predictions: ( x^* )</th>
<th>Observed Averages: ( x )</th>
<th>total cost ( (Ncx^*) )</th>
<th>total cost ( (Ncx) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>$1,000</td>
<td>3</td>
<td>$12,000</td>
<td>6</td>
<td>$24,000</td>
</tr>
<tr>
<td>4</td>
<td>$500</td>
<td>6</td>
<td>$12,000</td>
<td>8</td>
<td>$16,000</td>
</tr>
<tr>
<td>2</td>
<td>$1,000</td>
<td>4</td>
<td>$8,000</td>
<td>6</td>
<td>$12,000</td>
</tr>
<tr>
<td>2</td>
<td>$500</td>
<td>8</td>
<td>$8,000</td>
<td>10</td>
<td>$10,000</td>
</tr>
</tbody>
</table>

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The Nash prediction for a contender’s lobbying effort, as calculated from (29.7), is shown in the third column. Multiplying this by the number of contenders and then by the cost per unit of effort yields the predicted total cost of rent seeking activities, which is shown in the fourth column. These total costs are $12,000 with \( N = 4 \) (top two rows) and $8,000 for \( N = 2 \) (bottom two rows). Thus the Nash prediction is that an increase in the number of contenders makes the situation more competitive and increases rent-seeking activity and total costs. For each fixed level of \( N \), however, a reduction in the per-unit lobbying cost will double the effort, leaving the predicted total cost unchanged.

The Nash predictions can be compared with data from the classroom experiment, shown in the final two columns. The average effort decisions have been rounded off to the nearest integer, to make the table easier to read. Several conclusions are apparent:

1) The total costs of rent seeking activity are significant, and are greater than 50% of the prize value in all treatments.
2) The total costs of rent-seeking activity are greater than the Nash predictions.
3) An increase in the number of contenders tends to increase the total costs of rent seeking.
4) A decrease in per-unit effort costs does raise efforts, but not enough to offset the cost reduction.

V. Extensions and Further Reading

Rent seeking models have been used in the studying of political lobbying (Hillman and Samet, 1987), which is a natural application because lobbying expenditures often involve real resources. In fact, any type of contest for a single prize may have similar strategic elements, e.g. political campaigns or research and development contests. There is a large and growing literature that uses the rent-seeking paradigm to study non-market allocation activities.

There are been a number of rent seeking experiments using the payoff structure in equation (21.1). Millner and Pratt (1989) report that rent dissipation was significantly higher than the Nash prediction. In a companion paper, Millner and Pratt (1991) used a lottery choice decision to separate people into groups according to their risk aversion. Groups with more risk-averse individuals tended to expend more on rent-seeking activity.

Anderson, Goeree, and Holt (1998) provide a theoretical analysis of a model of rent seeking where the prize goes to the person with the highest effort. This is like a race or auction, where it only takes a small advantage to obtain a sure win (unlike the model contained in this chapter where a higher effort increases the probability of obtaining the prize but does not ensure a win). The
Anderson, Goeree and Holt model is based on logit stochastic response functions that were introduced in chapter 11. This incorporation of randomness in the game is used to derive a theoretical that rent-seeking efforts will be above the levels predicted in a Nash equilibrium.

Questions
1. Suppose that the cost per unit of effort is reduced from $3,000 to $1,000, with four competitors and a prize of $16,000. Show that a common effort of 3 is a Nash equilibrium for the rent seeking game with payoffs in equation (21.1). Hint: Check to be sure that a unilateral deviation to an effort of 2 or 4 would not be profitable, given that the other three people keep choosing 3. An alternative is to use calculus.
2. Now suppose that the effort cost is reduced to $500. Show that a common effort of 6 is a Nash equilibrium.
3. Calculate the total amount of money spent in rent-seeking activities for the setup in question 1. Did the reduction in the per-unit (marginal) cost of rent seeking from $1,000 to $500 reduce the total cost of rent seeking activity.
4. Now suppose that the number of competitors for the setup in question 1 is reduced from 4 to 2. The per-unit effort cost is $1000 and the prize value is $16,000. Show that a common effort of 4 is a Nash equilibrium.
5. Did the reduction in competitors from 4 to 2 in question 4 reduce the extent of rent dissipation? (Rent dissipation is the total cost of all rent-seeking activity as a proportion of the prize value.)
Part VIII. Information and Finance

Information specific to individuals is often not easily observed by others. Such information may be conveyed at a cost, but misrepresentation and strategic non-revelation is sometimes a problem. Informational asymmetries yield rich economic models that may have multiple equilibria and unusual patterns of behavior. One example considered in an earlier part is the case where individuals may decide to rely on information inferred from others’ decisions and “follow the crowd,” which creates information cascades. A diversity of investor information is also important in finance, where an issue may be the extent to which asset prices aggregate diverse bits of information. Many other interesting issues may arise, since market processes may fail when information cannot be priced. And Chapter 30 introduces the standard signaling game, where the proposer has a privately known attribute or “type.” This type is not observed by the responder, who can only see a “signal” made by the proposer. For example, an employer may attempt to use some educational credential or detailed portfolio to infer something about a worker’s productivity and energy. There are several different types of behavior that may emerge, depending on whether the signals effectively distinguish the different types of people. The webgame SG can be configured to generate some of these behavioral patterns, and the results are compared with Nash predictions. Signaling models have been widely applied to other situations in economics, politics, law, and finance.

In a signaling model, the individual productivity trait is exogenous and the signal is endogenously chosen. The setup in the next chapter is different in that the worker’s productivity is endogenously determined by an investment decision, but the observed signal is fixed exogenously, like gender or skin color. Statistical discrimination can arise if the employer uses past correlations between the fixed signal and worker productivity to make job assignment decisions. “Biased” expectations may be confirmed in equilibrium as workers of one type become discouraged and stop investing. The statistical discrimination model presented in Chapter 31 is implemented in the webgame SD by assigning a color (purple or green) to each person in the worker role and letting them make an unobserved investment decision. Those in the employer role must then make job assignments on the basis of an imperfect productivity “test.” Experience-based discrimination is most likely to show up when the test is inconclusive. This exercise provides an excellent opportunity for informed and non-emotional class discussions of issues like race and gender discrimination.
Chapter 30. Signaling

When individuals have traits that cannot be directly observed by others, they may engage in costly activities in order to “signal” the presence of these traits. Many educational credentials, for example, are thought to provide signals of a prospective employee’s productivity and enthusiasm. The simplest signaling game is one where the signals are not directly productive, but differential costs of signaling may allow one type of person to distinguish themselves from others. There may also be “pooling” equilibria in which both types of people engage in signaling, so that the signal loses meaning. The Veconlab signaling game SG facilitates the discussion of how these behavioral patterns may arise.

I. Introduction

The total cost of attending a single class for most students is several hundred dollars (if you consider the costs of tuition, room, and board, and the opportunity cost of what you might be earning if you were not attending college). On hearing this, many people wonder whether what they have learned from a single class is worth the cost. The “signaling” literature suggests that your educational investment may pay off even if the direct value added of a single class is not large. The intuition is that it is more enjoyable for talented students to acquire education, so the more educated people tend to be more talented. In this case, employers may be rational in offering a wage premium for education, regardless of whether the education is directly productive. The signaling game discussed in this chapter gives students the opportunity to send a costly signal (analogous to education) that the employer may use to infer something about the worker’s productivity.
Chapter 31. Statistical Discrimination

Economists and other social scientists have long speculated that discrimination based on observable traits (race, gender, etc.) could become self-perpetuating in a cycle of low expectations and low achievement by workers of one type. In such a case, the employers may be reacting rationally, without bias, to bad employment experiences with one type of worker. Workers of that “disadvantaged” type, in turn, may correctly perceive that job opportunities are diminished, and may reduce their investments in human capital. The result may be a type of “statistical discrimination” that is based on experience, not on any underlying bias. The game implemented by the Veconlab program SD sets up this kind of situation, with each worker being assigned a color, Purple or Green. The game can be used to stimulate a class discussion of topics that might be too sensitive for many students.

I. “Brown-eyed People Are More Civilized”

As Jane Elliott approached her Riceville, Iowa classroom one Friday in April 1968, she was going over the provocative experiment that she had stayed up late planning. Martin Luther King had been murdered in Memphis the day before. She anticipated a lot of questions and confusion, and she was desperate to do something that might make a difference. Her ideas had begun to take shape when she recalled an argument about racial prejudice with her father that had caused his hazel eyes to flare. She remembered commenting to her roommate that her father would be in trouble if hazel eyes ever went out of favor.

As soon as her third grade class was seated, Ms. Elliott divided them into two groups based on eye color, as described in A Class Divided (Peters, 1971). At first, brown-eyed people were designated as being superior, with the understanding that the roles would be reversed the next day. She began: “What I mean is that brown-eyed people are better than blue-eyed people. They are cleaner … more civilized …. And they are smarter than blue-eyed people.” As the brown-eyed people were seated in the front of the room and allowed to drink from the water fountain without using paper cups, the blue-eyed students slumped in their chairs and exhibited other submissive types of behavior. Objections and complaints were transformed into rhetorical questions about whether blue-eyed children were impolite, etc., which resulted in a chorus of enthusiastic replies from the brown-eyed people. When one student questioned Ms. Elliott about her own eye color (blue) and she tried to defend her intelligence, the reply was that she was not as smart as the brown-eyed teachers. What began as a role-playing exercise began to take on an eerie reality of its own. These patterns were reversed when blue-eyed people were designated as superior on the following day.
Psychologists soon began conducting experiments under more controlled conditions (e.g., Tajfel, 1970; Vaughan, Tajfel, and Williams, 1981). A typical setup involved a task of skill like a trivia quiz or estimation of the lengths of some lines. The subjects would be told that they were being divided into groups on the basis of their skill, but in fact the assignments were random. Then the subjects would be asked to perform a task like division of money or candy that might indicate differential status (see the review in Anderson, Goeree, and Holt, 2002). Such effects were often manifested as lower allocations to those with lower status. Moreover, people sometimes offered preferential treatment to people of their own group, which is known as an “in-group bias.”

Economists are typically interested in whether group and status effects will carry over into market settings. For example, Ball et al. (2001) examined the effects of status assignments in double auctions where the demand and supply functions were “box” shapes with a large vertical overlap, which produced a range of market-clearing prices. In the earned-status treatment, traders on one side of the market (e.g. buyers) were told that they obtained a gold star as a result of their performance on a trivia quiz. The random-assignment treatment began with some people being selected to receive stars that were awarded in a special ceremony. The subjects were not told that the star recipients were selected at random. In each treatment, the people with stars were all on one side of the market (all buyers or all sellers). This seemingly trivial status permitted them to earn more of the total surplus, whether or not they were buyers or sellers.

Group differences were largely exogenous in the status experiments, but economists have long worried about the possibility that differences in endogenously acquired skills may develop and persist in a vicious circle of self-confirming differential expectations. Formal models of “experienced-based” economic discrimination were developed independently by Arrow (1972) and Phelps (1973), and have been refined by others. The intuition is that if some historical differences in opportunity cause employment opportunities for one group (race, gender, etc.) to be less attractive, then members of that group may rationally choose not to invest as much in human capital. In response, employers may form lower expectations for members of that group. Expectations are based on statistics from past experience, so these have been called models of “statistical discrimination.”

The obvious question is why a member of the disadvantaged group could not invest and break out of the cycle. This strategy may not work if the employer attributes a good impression made in an interview to random factors, and then bases a decision not to hire on past experience and the fear that the interview did not reveal potential problems. (Think of the human capital being discussed as any aspect of productivity that is learned and that cannot be observed without error in the job application process. For example, the employer may be able to ascertain
that a person attended a particular software class, but not the extent to which the person mastered the details of working with spreadsheets.) Even if investment by members of the disadvantaged group results in good job placement some of the time, a lower success rate for people in this group will nevertheless result in a lower investment rate. After all, investment is costly in terms of lost income and lost opportunities to have children, etc. These observations raise the disturbing possibility that two groups with \textit{ex ante} identical abilities will be discriminated against because of differential rates of investment in acquired skills. In such a situation, the employers need not be prejudiced in any way other than reacting rationally to their own experiences. It is, of course, easy to imagine how real prejudice might arise (or continue) and accentuate the economic forces that perpetuate discrimination.

II. Being Purple or Green

The focus in this chapter is on experiments in which some group aspects are exogenous (color) and some are endogenous (investments). The participants are assigned a role, employer or worker. Workers are also assigned a color, Purple or Green. Workers are given a chance to make a costly investment in skills that would be valuable to an employer, but only if they were hired. The employers can observe workers’ colors and the results of an imperfect pre-employment test before making the hiring decision. Investment is costly for the worker, but it increases the chances of avoiding a bad result on the pre-employment test, and hence, of getting hired. In this setup, the employers prefer to hire workers who invested, and not to hire those who did not.

<table>
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<tr>
<th>Table 31.1. Experiment Parameters</th>
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<td>Worker’s payoffs:</td>
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<td>$1.50 if hired</td>
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<tr>
<td>$0.00 if not hired</td>
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<tr>
<td>Employer’s payoffs:</td>
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<td>$1.50 if hired worker invested</td>
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<tr>
<td>$-1.50 if hired worker did not invest</td>
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<td>$0.50 if worker not hired</td>
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This experimental setup used by Fryer, Goeree, and Holt (2002) matches a theoretical model (Coate and Loury, 1993) in which statistical discrimination a possible outcome. The experiment consisted of a number of rounds with random pairings of employers and workers. Each round began with workers finding out their own (randomly determined) costs of making an investment in some skill. Investment costs are uniform on the range form $0.00 to $1.00, except as noted below. The employer could observe the worker’s color, but not the investment decision. A test given to each worker provided noisy information about the
worker’s investment, and on the basis of this information the employer decided whether or not to hire the worker. The payoff numbers are summarized in Table 31.1.

Workers prefer to be hired, which yields a payoff of $1.50 per period, as opposed to the zero payoff for not being hired. The investment cost (if any) is deducted from this payoff, whether or not the worker is hired. Investment is good in that it increases the probability of getting hired at $1.50. Thus, a risk-neutral worker would want to invest if the investment cost is less than the wage times the increase in the hire probability that results from investing. If the investment cost is C, then the decision rule is:

**Worker decision:** invest if \( C < 1.50 \) (increase in hire probability).

Employers prefer to hire a worker who invested, which yields a payoff of $1.50. Hiring someone who did not invest yields a payoff of \(-1.50\), which is worse than the $0.50 payoff the employer receives if no worker is hired. (Think of the fifty cents as what the manager can earn without any competent help.) The manager wishes to hire a worker as long as the probability of investment, \( p \), is such that the expected payoff from hiring the worker, \( p(1.50) + (1-p)(-1.50) \), is greater than the $0.50 earnings from not hiring. It is straightforward to show that \( p(1.50) + (1-p)(-1.50) > 0.50 \) when \( p > 2/3 \):

**Employer decision:** hire when the probability of investment > 2/3.

The decision rules just derived, are of course, incomplete, since the probabilities of investment and being hired are determined by the interaction of worker and employer decisions. 

The employer’s beliefs about the probability that a worker invested are affected by a pre-employment test, which provides an informative but imperfect indication of the worker’s decision. If the worker invests, then the employer sees two independent draws, with replacement, from the “invest cup” which has 3 Blue marbles and 3 Red marbles. If the worker does not invest, the employer sees two draws with replacement from a cup with only 1 Blue and 5 Reds. Thus the cups are:

<table>
<thead>
<tr>
<th>Invest Cup</th>
<th>No-Invest Cup</th>
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</table>

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Obviously, the Invest Cup provides three times as great a chance that each draw will be Blue (B), so Red (R) is considered a bad signal. If the decision is to invest (Inv), the probabilities of the draw combinations are just products: \( \Pr(BB \mid \text{Inv}) = (1/2)(1/2) = 1/4 \), \( \Pr(RR \mid \text{Inv}) = (1/2)(1/2) = 1/4 \), and therefore, the residual probability of a mixed signal (RB or BR) is 1/2. Similarly, the probabilities of the signal combinations for no investment (No) are: \( \Pr(BB \mid \text{No}) = (1/6)(1/6) = 1/36 \), \( \Pr(RR \mid \text{No}) = (5/6)(5/6) = 25/36 \), and \( \Pr(RB \text{ or } BR) = 10/36 \), which is the residual.

### IV. An Unfair Equilibrium

The signal combination probabilities just calculated can be used to determine whether or not it is worthwhile for a worker to invest, but we must know how employers react to the signals. The discussion will pertain to an asymmetric equilibrium outcome in which one color gets preferential treatment in the hiring process. (Symmetric equilibria without discrimination will be discussed afterwards.) The theoretical question is whether asymmetric equilibria (with discriminatory hiring decisions) can exist, even if workers of each color have the same investment cost opportunities and payoffs. Consider the discriminatory hiring strategy:

<table>
<thead>
<tr>
<th>Signal</th>
<th>Action</th>
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<tbody>
<tr>
<td>BB</td>
<td>Hire both colors.</td>
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<tr>
<td>BR or RB</td>
<td>Hire Green, not hire Purple.</td>
</tr>
<tr>
<td>RR</td>
<td>Do not hire either color.</td>
</tr>
</tbody>
</table>

(31.1) There are two steps to complete the analysis of this equilibrium: 1) to figure out the ranges of investment costs for which workers of each color will invest, and 2) to verify that the conjectured hiring strategy given above is optimal for employers.

#### Step 1: Worker Investment Decisions

Recall that the worker will invest if the cost is less than the wage $1.50 times the increase in the probability of being hired due to a decision to invest. The Purple worker is only hired if both draws are blue, which happens with probability \( (1/2)(1/2) = 1/4 \) following investment and \( (1/6)(1/6) = 1/36 \) following no investment, so the increase in the probability of being hired is \( 1/4 - 1/36 = 8/36 = 2/9 \). Hence \( (2/9)\$1.50 = $0.33 \) is the expected net gain from investment, which is the optimal decision if the investment cost is less than $0.33. In contrast, the favored Green workers are hired whenever the draw combination is not RR. So investment results in a hire with probability \( 1 - (1/2)(1/2) = 3/4 \). Similarly, the probability of a Green worker being hired without investing is the probability of not getting two R draws from the No Invest Cup (BRR), which is \( 1 - (5/6)(5/6) = 11/36 \). The increase in the chances of getting a job due to
investment is $3/4 - 11/36 = 27/36 - 11/36 = 16/36 = 4/9$. Hence the expected net gain from investment by a Green worker is $(4/9) \times 1.50 = 0.67$. To summarize, given the hiring strategy that discriminates in favor of Green workers, the Greens will invest whenever their investment costs are less than $0.67$, and the Purple workers will invest half as often, i.e. when their costs are less than $0.33$.

**Step 2: Employer Hiring Decisions**

Recall that the employer’s payoffs in Table 31.1 are such that it is better to hire whenever the probability that the worker invested is greater than $2/3$. Since Greens invest twice as often as Purples (a probability of $2/3$ versus $1/3$), the employer’s posterior probability that a worker invested will be higher for Green, regardless of the combination of draws. These posterior probabilities are calculated with Bayes’ rule and are shown in Table 31.2.

<table>
<thead>
<tr>
<th>Decision is to Hire if the Probability &gt; 2/3</th>
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<tbody>
<tr>
<td>Green Workers</td>
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<tr>
<td>Pr(worker invested</td>
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For example, consider the top row of the Green column, which shows the probability that a Green worker with a BB signal invested. This probability, 18/19, is calculated by taking the ratio, with the Greens who invested and received the BB signal in the numerator and all Greens (who invested or not) who received the BB signal in the denominator. Since two-thirds of Greens invest and investment produces the BB signal with probability $(1/2)(1/2) = 1/4$, the numerator is $(2/3)(1/4)$. Since one-third of the Greens do not invest and not investment produces BB signal with probability $(1/6)(1/6) = 1/36$, the denominator also includes a term that is the product: $(1/3)(1/36)$. It follows that the probability of investment for a Green with the BB signal is: $(2/3)(1/4)$ divided by $(2/3)(1/4) + (1/3)(1/36)$, which reduces to 18/19, as shown in the Table. The employer’s optimal decision is to hire when the investment probability is greater than $2/3$, so the Green worker with a BB test result should be hired. The other probabilities and hire decision in the table are calculated in a similar manner (question 1). A comparison of the middle and right columns of the table show that these decisions discriminate against Purple in a manner that matches the original specification in (31.1).

It can be shown that there is also a symmetric equilibrium where both colors invest when the cost is less than $0.67$ and both colors are treated like the
employer treated Greens in Table 31.1, i.e. always hire a worker of either color unless the test outcome is RR (question 2).

IV. Data

The advantage of running experiments is obvious in this case, since the model has both asymmetric and symmetric equilibria, and since exogenous sources of bias can be controlled. Moreover, the setup is sufficiently complex that behavior may not be drawn to any of the equilibria. Fryer, Goeree, and Holt (2002) have run a number of sessions using the statistical discrimination game. In some of the sessions there is little distinction between the way workers with different color designations are treated. This is not too surprising, given the symmetry of the model and the presence of a symmetric equilibrium.

Many cases where unequal treatment is likely to persist have evolved from social situations where discrimination was either directly sanctioned or indirectly subsidized by legal rules and social norms. Therefore, we began some of the sessions with 10 rounds of unequal investment cost distributions. In particular, one color (e.g. Green) might be favored initially by drawing investment costs from a distribution that is uniform on [0, 0.50], and Purples may draw from a distribution that is uniform on [0.50, 1.00]. The vertical dashed line at round 10 in Figure 31.1 shows the round were these unequal-cost-opportunity ended. Subjects were not told of this cost difference; they were just told that all cost draws would be between 0.00 and 1.00, which was the case. All draws after round 10 were from a common uniform distribution on [0.00, 1.00].

Figure 31.1 shows five-period average investment and hiring rates. Notice that the initial cost asymmetry works in the right direction; Greens start out investing at about twice the rate as that for Purples, as can be seen from the left panel. There is a slight crossover in rounds 10-15, which seems to have been caused by a “relative cost effect,” i.e. the tendency for Purples to invest a lot when encountering costs that are lower than initially high levels that they were used to seeing. Similarly, investment rates for Greens fell briefly after period 10 when they began drawing from a higher cost distribution. As can be seen from the right panel in the figure, the employers seemed to notice the lower initial investment rates for Purples, and they continued hiring Greens at a higher rate. The inertia of this preference for Green eventually caused the investment rates for Green to rise above those for Purple again, and separation continued.

The color-based hiring preference was clear for a majority of the employers, although the separation was not as uniform as that predicted in the previous section’s equilibrium analysis. After an employer saw mixed signals (BR or RB), Greens were hired more frequently (96% versus 58% in the final 20 rounds). In fact, Greens were even hired more frequently after a negative RR signal (53% versus 0% in the final 20 rounds).
The individual employer decisions for the final 15 rounds of this session are shown in Table 31.3. Most of the employers discriminate to some extent, either in the case of a mixed (RB or BR) signal, or in the case of a negative (RR) signal. Employers 1, 2, and 3 hired all Green workers encountered, even when the test result was RR. Employers 4, and 5 tend to discriminate after mixed signals, and employers 2 and 3 tend to discriminate after the negative signal. Employer 6 is essentially color-blind in the way workers are treated.

The experience of employer 2 indicated the problems facing Purple workers. All Purples encountered by this employer had invested, but the employer was unable to spot this trend since no Purples were hired after round 40. (The investment decisions of the hired workers are shown in dark bold letters (Inv or No) when the worker was hired, and the unobserved investment decisions are shown in light gray when the worker was not hired.) Such differential treatment, when it occurs, is particularly interesting when the experiment is conducted in class for teaching purposes. In classroom experiments, one person once remarked “Purple workers just can’t be trusted…they won’t invest.” Another student remarked: “I invested every time, even when costs were high, because I felt confident that I would get the …job – because I am Green.”
Table 31.3. Employer Information and Decisions for the Final 15 Rounds
Key: \text{Inv} = \text{investment observed ex post by employer.}\n\text{Inv} = \text{investment not observed ex post by employer.}\n
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<tr>
<th>Round</th>
<th>Employer 1</th>
<th>Employer 2</th>
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The initial cost differences produced the same initial patterns in a second experiment, shown in Figure 31.2. The vertical dashed line again indicates the point after which the cost distributions became symmetric. Notice that the surge caused by the relative cost effect after 10 rounds carries over, causing a fascinating reverse separation where the initially disadvantaged group (Purple) ends up investing more for the remainder of the session. This differential investment rate seems to cause the hiring rates to diverge in the final ten periods. A similar crossover effect was observed in a classroom experiment with 5 rounds of asymmetric costs followed by 15 rounds of symmetric costs.

![Figure 31.2. A Cross-Over Effect: Five-Period Average Investment and Hire Rates by Color (Green is Solid, Purple is Dashed) with Reverse Statistical Discrimination Induced by an Initial Investment Cost Asymmetry (Source: Fryer, Goeree, and Holt, 2002)](image)

**Extensions and Further Reading**

There are a number of related theoretical models of statistical discrimination. These models are presented with a uniform notation in Fryer (2001). The classroom experiments mentioned in the previous section are discussed in more detail in Fryer, Goeree, and Holt (2001). Anderson and Harpert (1999) describe a classroom experiment with exogenously determined worker skill levels. Davis (1987) provides an experimental test of the idea that perceptions about a group may be influenced by the highest skill level encountered in the past. In this case, a majority group produces more workers, so statistically speaking, the highest skill level from that group should be higher than from the other group.
Questions

1. Verify the Bayes’ rule probability calculations for Purple workers in the right column of Table 31.2.
2. Consider a symmetric equilibrium where employers hire a worker regardless of color if the test result is BB, BR, or RB, and not otherwise. To do this, first show that workers of either color will invest as long as the cost is less than $0.67. Then calculate the probabilities of investment conditional on the test results, as in Table 3.2. Finally, show that the employer’s best response is to hire unless the test result is RR.
Appendices: Instructions for Class Experiments

Pit Market Instructions (Chapter 2)

We are going to set up a market in which the people on my right will be buyers, and the people on my left will be sellers. There will be equal numbers of buyers and sellers, which is the reason that some of you had to switch to the other side of the room. Several assistants have been selected to help record prices. I will now give each buyer and seller a numbered playing card. Some cards have been removed from the deck(s), and all remaining cards have a number. Please hold your card so that others do not see the number. The buyers' cards are red (Hearts or Diamonds), and the sellers' cards are black (Clubs or Spades). Each card represents a "unit" of an unspecified commodity that can be bought by buyers or sold by sellers.

Trading: Buyers and sellers will meet in the center of the room (or other designated area) and negotiate during a 5 minute trading period. When a buyer and a seller agree on a price, they will come together to the front of the room to report the price, which will be announced to all. Then the buyer and seller will turn in their cards, return to their original seats, and wait for the trading period to end. There will be several market periods.

Sellers: You can each sell a single unit of the commodity during a trading period. The number on your card is the dollar cost that you incur if you make a sale. You will be required to sell at a price that is no lower than the cost number on the card. Your earnings on the sale are calculated as the difference between the price that you negotiate and the cost number on the card. If you do not make a sale, you do not earn anything or incur any cost in that period. Think of it this way: its as if you knew someone who would sell you the commodity for a price that equals your cost number, so you can keep the difference if you are able to resell the commodity for a price that is above the acquisition cost. Suppose that your card is a 2 of Clubs and you negotiate a sale price of $3. Then you would earn: $3 — 2 = $1. You would not be allowed to sell at a price below $2 with this card (2 of Clubs). If you mistakenly agree to a price that is below your cost, then the trade will be invalidated when you come to the front desk; your card will be returned and you can resume negotiations.
**Buyers:** You can each buy a single unit of the commodity during a trading period. The number on your card is the dollar value that you receive if you make a purchase. You will be required to buy at a price that is no higher than the value number on the card. Your earnings on the purchase are calculated as the difference between the value number on the card and the price that you negotiate. If you do not make a purchase, you do not earn anything in the period. Think of it this way: its as if you knew someone who would later buy the unit from you at a price that equals your value number, so you can keep the difference if you are able to buy the unit at a price that is below the resale value. Suppose that your card is a 9 of Diamonds and you negotiate a purchase price of $4. Then you would earn: $9 - 4 = $5. You would not be allowed to buy at a price above $9 with this card (9 of Diamonds). If you mistakenly agree to a price that is above your value, then the trade will be invalidated when you come to the front desk; your card will be returned and you can resume negotiations.

**Recording Earnings:** Some buyers and sellers may not be able to negotiate a trade, but do not be discouraged since new cards will be passed out at the beginning of the next period. Remember that earnings are zero for any unit not bought or sold (sellers incur no cost and buyers receive no value). When the period ends, I will collect cards for the units not traded, and you can calculate your earnings while I shuffle and redistribute the cards. Your total earnings equal the sum of earnings for units traded in all periods, and you can use the Earnings Record Form on the back of this sheet to keep track of your earnings. Sellers use the left side of the Earnings Record Form, and buyers use the right side. At this time, please draw a diagonal line through the side that you will not use. All earnings are hypothetical. Please do not talk with each other until the trading period begins. Are there any questions?

**Final Observations:** When a buyer and a seller agree on a price, both should immediately come to the front table to turn in their cards together, so that we can verify that the price is neither lower than the seller's cost nor higher than the buyer's value. If there is a line, please wait together. After the price is verified, the assistant at the board will write the price and announce it loudly. Then those two traders can return to their seats to calculate their earnings. The assistants should come to their positions in the front of the room. Buyers and sellers, please come to the central trading area NOW, and begin calling out prices at which you are willing to buy or sell. The market is open, and there are 5 minutes remaining.
### Seller Earnings

(sellers use this side)

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<td>Sixth Period</td>
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### Buyer Earnings

(buyers use this side)

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#### total earnings, for all periods: $  
#### total earnings, for all periods: $ 

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Game Instructions (Chapter 3)

We are going to play a card game in which everybody will be matched with someone on the opposite side of the room. I will now give each of you a pair of playing cards, one red card (Hearts or Diamonds) and one black card (Clubs or Spades). The numbers or faces on the cards will not matter, just the color. You will be asked to play one of these cards by holding it to your chest (so we can see that you have made your decision, but not what that decision is). Your earnings are determined by the card that you play and by the card played by the person who is matched with you. If you play your red card, then your earnings in dollars will increase by $2, and the earnings of the person matched with you will not change. If you play your black card, your earnings do not change and the dollar earnings of the person matched with you go up by $3. In other words, you can either "pull" $2 to yourself by playing the red card, or you can "push" $3 to the other person by playing the black card. If you each pull your red card, you will each earn $2. If you each push the black card, you will each earn $3. If you push your black card and the other person pulls his or her red card, then you earn zero and the other person earns the $5. If you pull red and the other person pushes black, then you earn the $5, and the other person earns zero. All earnings are hypothetical, except as noted below.

After you choose which card to play, hold it to your chest. We then tell you who you are matched with, and you can each reveal the card that you played. Record your earnings in the space below. (Option: After we finish all periods, I will pick one person with a random throw of dice and pay that person 10% of his or her total earnings, in cash. All earnings for everyone else are hypothetical. To make this easier, please write your name: ______________________ and the identification number that I will now give each of you: _____________. Afterwards, I will throw a 10-sided die twice, with the first throw determining the "tens" digit, until I obtain the ID number of one of you, who will then be paid 10% of his or her total earnings in cash.) Any questions?

To begin: Would the people in the row that I designate please choose which card to play and write the color (R or B) in the first column. Show that you have made your decision by picking up the card you want to play and holding it to your chest. Everyone finished? Now, I will pair you with another person, ask you to reveal your choice, and calculate your earnings. Remember to keep track of earnings in the space provided below. Finally, please note that in period 2 you will be matched with a different person, and payoffs change. In period 3 you will be matched with a different person and payoffs change again, but you get to play with him/her in all three subsequent periods.
<table>
<thead>
<tr>
<th>Round</th>
<th>Payoffs</th>
<th>Your card (R or B)</th>
<th>Other’s card (R or B)</th>
<th>Your Earnings</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Red: pull $2, Black: push $3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Red: pull $2, Black: push $8</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Red: pull $2, Black: push $3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Red: pull $2, Black: push $3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Red: pull $2, Black: push $3</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Lottery Choice Instructions (Chapter 4)

Your decision sheet shows ten decisions listed on the left. Each decision is a paired choice between "Option A" and "Option B." You will make ten choices and record these in the final column, but only one of them will be used in the end to determine your earnings. Before you start making your ten choices, please let me explain how these choices will affect your earnings, which will be hypothetical unless otherwise indicated.

Here is a ten-sided die that will be used to determine payoffs; the faces are numbered from 1 to 10 (the "0" face of the die will serve as 10.) After you have made all of your choices, we will throw this die twice, once to select one of the ten decisions to be used, and a second time to determine what your payoff is for the option you chose, A or B, for the particular decision selected. Even though you will make ten decisions, only one of these will end up affecting your earnings, but you will not know in advance which decision will be used. Obviously, each decision has an equal chance of being used in the end.

Now, please look at Decision 1 at the top. Option A pays 200 pennies if the throw of the ten sided die is 1, and it pays 160 pennies if the throw is 2-10. Option B yields 385 pennies if the throw of the die is 1, and it pays 10 pennies if the throw is 2-10. The other Decisions are similar, except that as you move down the table, the chances of the higher payoff for each option increase. In fact, for Decision 10 in the bottom row, the die will not be needed since each option pays the highest payoff for sure, so your choice here is between 200 pennies or 385 pennies.

To summarize, you will make ten choices: for each decision row you will have to choose between Option A and Option B. You may choose A for some decision rows and B for other rows, and you may change your decisions and make them in any order. When you are finished, we will come to your desk and throw the ten-sided die to select which of the ten Decisions will be used. Then we will throw the die again to determine your money earnings for the Option you chose for that Decision. Earnings (in pennies) for this choice will be added to your previous earnings (if any).

So now please look at the empty boxes on the right side of the record sheet. You will have to write a decision, A or B in each of these boxes, and then the die throw will determine which one is going to count. We will look at the decision that you made for the choice that counts, and circle it, before throwing the die again to determine your earnings for this part. Then you will write your earnings in the blank at the bottom of the page. Are there any questions?
<table>
<thead>
<tr>
<th>Decision</th>
<th>Option A</th>
<th>Option B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Decision 1</td>
<td>$2.00 if throw of die is 1 &lt;br&gt;$1.60 if throw of die is 2-10</td>
<td>$3.85 if throw of die is 1 &lt;br&gt;$0.10 if throw of die is 2-10</td>
</tr>
<tr>
<td>Decision 2</td>
<td>$2.00 if throw of die is 1-2 &lt;br&gt;$1.60 if throw of die is 3-10</td>
<td>$3.85 if throw of die is 1-2 &lt;br&gt;$0.10 if throw of die is 3-10</td>
</tr>
<tr>
<td>Decision 3</td>
<td>$2.00 if throw of die is 1-3 &lt;br&gt;$1.60 if throw of die is 4-10</td>
<td>$3.85 if throw of die is 1-3 &lt;br&gt;$0.10 if throw of die is 4-10</td>
</tr>
<tr>
<td>Decision 4</td>
<td>$2.00 if throw of die is 1-4 &lt;br&gt;$1.60 if throw of die is 5-10</td>
<td>$3.85 if throw of die is 1-4 &lt;br&gt;$0.10 if throw of die is 5-10</td>
</tr>
<tr>
<td>Decision 5</td>
<td>$2.00 if throw of die is 1-5 &lt;br&gt;$1.60 if throw of die is 6-10</td>
<td>$3.85 if throw of die is 1-5 &lt;br&gt;$0.10 if throw of die is 6-10</td>
</tr>
<tr>
<td>Decision 6</td>
<td>$2.00 if throw of die is 1-6 &lt;br&gt;$1.60 if throw of die is 7-10</td>
<td>$3.85 if throw of die is 1-6 &lt;br&gt;$0.10 if throw of die is 7-10</td>
</tr>
<tr>
<td>Decision 7</td>
<td>$2.00 if throw of die is 1-7 &lt;br&gt;$1.60 if throw of die is 8-10</td>
<td>$3.85 if throw of die is 1-7 &lt;br&gt;$0.10 if throw of die is 8-10</td>
</tr>
<tr>
<td>Decision 8</td>
<td>$2.00 if throw of die is 1-8 &lt;br&gt;$1.60 if throw of die is 9-10</td>
<td>$3.85 if throw of die is 1-8 &lt;br&gt;$0.10 if throw of die is 9-10</td>
</tr>
<tr>
<td>Decision 9</td>
<td>$2.00 if throw of die is 1-9 &lt;br&gt;$1.60 if throw of die is 10</td>
<td>$3.85 if throw of die is 1-9 &lt;br&gt;$0.10 if throw of die is 10</td>
</tr>
<tr>
<td>Decision 10</td>
<td>$2.00 if throw of die is 1-10</td>
<td>$0.10 if throw of die is 1-10</td>
</tr>
</tbody>
</table>
BS Game Instructions (Chapter 5)

We are going to play a card game in which everybody will be matched with someone on the opposite side of the room. I will now give each of you a pair of playing cards, one Red card (Hearts or Diamonds) and one Black card (Clubs or Spades). (Instead, these may be index cards with numbers written in red or black). The people on the left side of the room have a red 8 and a black 2, whereas the people on the right side of the room have a red 2 and a black 8.

<table>
<thead>
<tr>
<th>Left Side</th>
<th>Right Side</th>
</tr>
</thead>
<tbody>
<tr>
<td>Red 8, Black 2</td>
<td>Red 2, Black 8</td>
</tr>
</tbody>
</table>

You will be asked to play one of these cards by holding it to your chest (so we can see that you have made your decision, but not what that decision is).

Your earnings are determined by the card that you play and by the card played by the person who is matched with you:

If the colors of the cards do not match (Red and Black), you each earn nothing.
If the colors match, earnings are equal to the number on your card:

After you choose which card to play, hold it to your chest. We then tell you who you are matched with, and you can each reveal the card that you played. Record your earnings in the space below. All earnings are hypothetical, except as noted below. (Option: After we finish all periods, I will pick one person with a random throw of dice and pay that person ___% of his or her total earnings, in cash. All earnings for everyone else are hypothetical. To make this easier, please write your name: __________ and the identification number that I will now give each of you: ________. Afterwards, I will throw a 10-sided die twice, with the first throw determining the "tens" digit, until I obtain the ID number of one of you, who will then be paid 10% of his or her total earnings in cash.) Any questions?

To begin: Would the people in the row that I designate please choose which card to play and write the color (R or B) in the first column. Show that you have made your decision by picking up the card you want to play and holding it to your chest. Everyone finished? Now, I will pair you with another person, ask you to reveal your choice, and calculate your earnings. Remember to keep track of earnings in the space provided below. Finally, please note that in period 2 you will be matched with a different person, and payoffs change. In period 3 you will be matched with a different person and payoffs change again, but you get to play with him/her in all three subsequent periods.
BS Game Payoffs:

If the cards match in color (RR or BB), you earn a dollar amount (to be announced) for the card you played.
If the cards to not match in color (RB or BR), you do not earn anything.

Initial payoffs for round 1:
You play R, other plays R, you earn $____.
You play B, other plays B, you earn $____.
You play R, other plays B, you earn $0.00.
You play B, other plays R, you earn $0.00.

<table>
<thead>
<tr>
<th>Round</th>
<th>Payoffs</th>
<th>Your card (R or B)</th>
<th>Other’s card (R or B)</th>
<th>Your Earnings</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>R card: $____</td>
<td>B card: $____</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>R card: $____</td>
<td>B card: $____</td>
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</tr>
<tr>
<td>3</td>
<td>R card: $____</td>
<td>B card: $____</td>
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<tr>
<td>4</td>
<td>R card: $____</td>
<td>B card: $____</td>
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<td>5</td>
<td>R card: $____</td>
<td>B card: $____</td>
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<td>6</td>
<td>R card: $____</td>
<td>B card: $____</td>
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<td>7</td>
<td>R card: $____</td>
<td>B card: $____</td>
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<td>8</td>
<td>R card: $____</td>
<td>B card: $____</td>
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Binary Prediction Game (Chapter 6)

This is an exercise in which you will be asked which of two random events will occur. The events will be called L (left) and R (right). We will use the throw of dice to determine which event will occur in each period. Please look at the decision sheet below. The period number is on the left, and you will use the second column to record your prediction, L or R. After everybody has written their prediction for round 1 into the top row, we will throw the dice in the front of the room to determine the event, L or R. This throwing of the dice will be done behind a screen so that you cannot see what method is being used. The main thing to remember here is that the person throwing the dice will not know your predictions, so these decisions cannot affect the likelihood of future events. When we announce the event (L or R) for the current period, please write in your earnings: +25 cents if you were correct, and -25 cents if you were incorrect. You can keep track of your cumulative earnings in the final column. Your instructor will make an announcement about whether or not the earnings will actually be paid in cash. To summarize, each period consists of 1) the prediction stage, 2) the throwing of the dice to determine the event, 3) the announcement of the event and the calculation of results. Are there any questions?

<table>
<thead>
<tr>
<th>Round</th>
<th>Your Prediction (L or R)</th>
<th>Observed Event (L or R)</th>
<th>Your Earnings</th>
<th>Total Earnings</th>
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</thead>
<tbody>
<tr>
<td>1</td>
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Bayes’ Rule Instructions (Chapter 7)

Note: The setup requires a 6-sided die, a 10-sided die, three light marbles, 3 dark marbles, a plastic cup for making draws, two paper envelopes marked A and B, and a way to hide the cup selection process from view. The reading of the instructions and the experiment itself can be done in about 25 minutes.

In this experiment, you will observe "balls" (colored marbles) drawn from one of two possible "cups." You will see the balls drawn, but you will not know for sure which cup is being used. Then you will be asked to indicate your beliefs about which cup is being used. I will begin by choosing one of you to serve as a “monitor” who assists in setting things up and drawing the marbles.

The two cups will be called cup A and cup B. Cup A contains 2 light balls and 1 dark ball, and Cup B contain 1 light ball and 2 dark balls. The cup will be chosen by the throw of a 6 sided die: cup A is used if the roll of the die yields a 1, 2, or 3, and cup B is used if the roll of the die yields a 4, 5, or 6. Thus it is equally likely that either cup will be selected.

<table>
<thead>
<tr>
<th>Cup A</th>
<th>Cup B</th>
</tr>
</thead>
<tbody>
<tr>
<td>(used if the die is 1, 2, or 3)</td>
<td>(used if the die is 4, 5, or 6)</td>
</tr>
<tr>
<td>2 Light Balls</td>
<td>1 Light Ball</td>
</tr>
<tr>
<td>1 Dark Ball</td>
<td>2 Dark Balls</td>
</tr>
</tbody>
</table>

Once a cup is determined by the roll of the die, we will empty the contents of that cup into a container. The container is always the same, regardless of which cup is being used, so you cannot guess the cup by looking at the container. Then we will draw one or more balls from the container. (If more than one draw is to be made, then the first ball drawn will be put back into the container, which is then shaken before a second draw is made, etc.)

Recording Your Beliefs

After the draw has been made, we will ask you to tell us your beliefs about the chances that cup A is being used. You will indicate a number between 0 and 100, which we will call \( P \), such that the chances that cup A is being used are \( P \) out of 100." If you could be sure that cup A is being used, you should choose \( P = 100 \) to indicate that the chances are 100 out of 100 that cup A is being used. If you could be sure that cup A is not being used, you should choose \( P = 0 \) to indicate that the chances are 0 out of 100 that cup A is being used. Thus the magnitude of \( P \) corresponds to the chances that cup A is being used. For
example, if the manner in which the cup is selected and the balls that you see drawn cause you think that cup A is just as likely as cup B, then you should choose $P = 50$, indicating that the chances of A are 50 out of 100.

You will use the attached Decision Sheets to record your information and decisions. At the start of each "period," the monitor will throw a 6-sided die to select the cup (A if the throw is 1, 2, or 3, and B if the throw is 4, 5, or 6). The monitor will then place the 3 balls for that cup in the plastic container from which we make the draws. The period number is shown in column (1) of the decision sheet. The results of the draws are to be recorded in column (2). Please look at the decision sheet for periods 1-7. In the first period, there will be no draws, as indicated in column (2), so the only information that you have is the information about how the cup is selected.

In subsequent periods, you will see one or more draws, and you can use column (2) to record the draw(s). Write L (for Light) or D (for Dark) in this column at the time the draw is made. In periods 2 and 3, there will be only a single draw. In periods 4 and 5, there will be two draws from the same cup (with replacement after the first draw). In periods 6 and 7, there will be 3 draws (with each ball drawn being put back into the cup before the next draw is made).

After seeing the draw(s), if any, for the period, you will be asked to indicate the chances that cup A is being used. Do this by writing a number, P, between 1 and 100 in column (3).

Earnings

Next, let me describe a procedure that will help you make your decision. The procedure is complicated, but the underlying idea is simple. Its as if you send your friend to a fruit stand for tomatoes, and they ask you what to do if there are both red and yellow tomatoes. You should tell them the truth about your preference, since only then can they make the best choice for you. We'll set up the incentives so that your report about the chances of cup A will enable us to choose a lottery that gives you the highest chance of winning a $1 prize.

Suppose that, after seeing the draw or draws from the unknown cup, you think the chances are $P$ out of 100 that cup A is being used. If you were to receive $1 in the event that the cup actually used was A, then this determines a “$P$ lottery” which pays $1 with chances of $P$ out of 100, nothing otherwise.

After you write down the number $P$ that represents your beliefs about likelihood of cup A, we will use a ten-sided die to determine a second lottery, the “dice lottery.” To do this, we throw the die twice, with the first throw giving the “tens” digit (0, 1, …9) and the second throw giving the “ones” digit (0, 1, …9). Thus the random number will be between 0 and 99. If the number is $N$, then the “dice lottery” will give you an $N$ out of 100 chance of earning $1. (To play the dice lottery for a given value of $N$, we would throw the ten-sided die twice more
to get a number that is equally likely to be 0, 1, … 99, and you would earn $1 if this second throw is less than or equal to N.)

Up to this point, you will have written down a number P for what you think are the chances out of 100 that cup A is being used. The resulting P lottery will pay $1 if cup A is used. Then we will have used a throw of dice to determine a dice lottery that will pay $1 with chance N out of 100. Which would you rather have? It is better to have the dice lottery if N is greater than P, and it is better to have the P lottery otherwise. We’ll give you the one that is best for you.

**Case 1: P < N**

So if you write down a number P and the number N determined by the dice throws is greater, you will get to play the dice lottery (throwing the dice again to see if you get the $1, i.e. when the number determined by the second throw is less than or equal to N.)

**Case 2: P ≥ N**

So if you write down a number P that is greater than or equal to the number N determined by the dice throws, then your earnings are determined by the P lottery, i.e. you will get $1 if the cup used turns out to be cup A.
<table>
<thead>
<tr>
<th>Period</th>
<th>Your draws</th>
<th>Chances of urn A</th>
<th>Dice throw N</th>
<th>if $P &lt; N$, use Dice Lottery</th>
<th>if $P \geq N$, use the $P$ Lottery</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>L or D</td>
<td>$P$</td>
<td>$N$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>---</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>,</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>,</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>,</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>, ,</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>, ,</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>, ,</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Lottery Choice Instructions (Chapter 8)

Your decision sheet shows ten decisions listed on the left, which are labeled 0, 1, 2, .. 9. Each decision is a paired choice between a randomly determined payoff described on the left and another one described on the right. You will make ten choices and record these in the final column, but only one of them will be used in the end to determine your earnings. Before you start making your ten choices, please let me explain how these choices will affect your earnings. Unless otherwise indicated, all earnings are hypothetical.

Here is a ten-sided die that will be used to determine payoffs; the faces are numbered from 0 to 9. After you have made all of your choices, we will throw this die three times, once to select one of the ten decisions to be used, and then two more times to determine what your payoff is for the option you chose (on the left side or on the right side) for the particular decision selected. Even though you will make ten decisions, only one of these will end up affecting your earnings, but you will not know in advance which decision will be used. Obviously, each decision has an equal chance of being used in the end.

Now, please look at Decision 0 at the top. If this were the decision that we ended up using, we would throw the ten-sided die two more times. The two throws will determine a number from 0 to 99, with the first throw determining the “tens” digit and the second one determining the “ones” digit. The option on the left side pays $6.00 if the throw of the ten-sided die is 0-99, and it pays $0.00 otherwise. Since all throws are between 0 and 99, this option provides a sure $6.00. The option on the right $8.00 if the throw of the die is 0-79, and it pays $0.00 otherwise. Thus the option on the right side of Decision 0 provides 80 chances out of 100 (a four-fifths probability) of getting $8.00. The left and right options for the other decision rows are similar, but with differing payoffs and chances of getting each payoff. In addition, you will receive $10.00 for participating, so any earnings will be added to this amount. Some of the decisions involve losses, indicated by minus signs. If the decision selected ends up with a loss, this loss will be subtracted from the initial $10.00 payment to determine your final earnings.

To summarize, you will make ten choices: for each decision row you will have to choose between the option on the left and the option on the right. Make your choice by putting a check by the option you prefer. If you change your mind, cross out the check and put it on the other side. Thus there should be one check mark in each row. You may change your decisions and make them in any order. When you are finished, mark your final choices, L for left or R for right, in the far-right column, and we will come to your desk and throw the ten-sided die to select which of the ten Decisions will be used. We will circle that decision before
throwing the die again to determine your money earnings for the Option you chose for that Decision.

Before you begin, let me mention that different people may make different choices for the same decision, in the same manner that one person may purchase a sweater that differs from that purchased by someone else. We are interested in your preferences, i.e. in which option you prefer to have, so please think carefully about each decision, and please do not talk with others in the room. Are there any questions?

<table>
<thead>
<tr>
<th>Decision</th>
<th>Left Side</th>
<th>Right Side</th>
<th>Your Choice L or R</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$6.00 if throw of die is 0-99</td>
<td>$8.00 if throw of die is 0-79 $0.00 if throw of die is 80-99</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>$2.00 if throw of die is 0-99</td>
<td>$4.00 if throw of die is 0-79 $-4.00 if throw of die is 80-99</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>$6.00 if throw of die is 0-24 $0.00 if throw of die is 25-99</td>
<td>$8.00 if throw of die is 0-19 $0.00 if throw of die is 20-99</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>$30.00 if throw of die is 0-99</td>
<td>$40.00 if throw of die is 0-79 $0.00 if throw of die is 80-99</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>$4.00 if throw of die is 0-49 $3.20 if throw of die is 50-99</td>
<td>$7.70 if throw of die is 1-49 $0.20 if throw of die is 50-99</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>$-6.00 if throw of die is 0-99</td>
<td>$-8.00 if throw of die is 0-79 $-0.00 if throw of die is 80-99</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>$2.00 if throw of die is 0-49 $1.20 if throw of die is 50-99</td>
<td>$5.70 if throw of die is 1-49 $-1.80 if throw of die is 50-99</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>$-6.00 if throw of die is 0-24 $-0.00 if throw of die is 25-99</td>
<td>$-8.00 if throw of die is 0-19 $-0.00 if throw of die is 20-99</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>$-4.00 if throw of die is 0-49 $-3.20 if throw of die is 50-99</td>
<td>$-7.70 if throw of die is 1-49 $-0.20 if throw of die is 50-99</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>$30.00 if throw of die is 0-24 $0.00 if throw of die is 25-99</td>
<td>$40.00 if throw of die is 0-19 $0.00 if throw of die is 20-99</td>
<td></td>
</tr>
</tbody>
</table>
Search Instructions (Chapter 9)

In this game, we will use throws of dice to determine a series of money amounts, and you have to choose which amount to accept, with the understanding that each additional throw entails a cost that will be deducted from the money amount that you finally accept.

Please look at this 10-sided die. The sides are marked from 0 to 9, so if I throw it twice I will determine a number from 00 to 89 cents (with the first throw providing the "tens" digit, ignoring the 9, and the second throw providing the "ones" digit). If you pay a cost of 5 cents, I will throw the dice for you, and you can either keep the amount determined, or you can decide to pay another 5 cents and I'll throw again to get a second number in the range from 0 to 89 cents. (If the throw for the "tens" digit is a 9, I will throw the die again.) You can do this as many times as you wish (with no limit), but you have to pay 5 cents for each new number. When you decide to stop, you can use the highest 2-digit number that you have received up to that point, and your earnings will be that number minus the total cost of the search process, which is 5 cents times the number of times that you asked me to throw the dice. There is no limit on the number of times you can pay the 5 cent cost in search of a higher number.

Use the table below to keep records, and you can use another sheet if you need additional space. Each new 2-digit number will be called an "offer". For the 1st offer that you pay to obtain, look at the "1st" column. After I throw the dice, write the number in the top row for "value of current offer". The highest offer received up to now is written in the second row. (At first, the current offer is also the highest offer.) Moving down to the next row, the total search cost is the number of draws times 5 cents. The "earnings if you stop" are calculated by subtracting the total search cost from the highest draw thusfar. After calculating these earnings, you can decide whether to stop or to pay and search again. Write "S" for stop or "C" for continue. When you stop, please circle your earnings at that point and stop recording the new numbers. I will keep throwing the dice and announcing numbers until everyone had decided to stop. Then the whole process will be repeated, for another "search sequence", etc….

All earnings are hypothetical unless otherwise indicated. Are there any questions before we begin?
Your Name: ___________________

Record of the First Search Sequence

<table>
<thead>
<tr>
<th>order of offer</th>
<th>1st</th>
<th>2nd</th>
<th>3rd</th>
<th>4th</th>
<th>5th</th>
<th>6th</th>
<th>7th</th>
<th>8th</th>
<th>9th</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>value of current offer (in cents)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>highest offer thusfar</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>total search cost (5 for each search)</td>
<td>5 10 15 20 25 30 35 40 45 50</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>earnings if you stop (use ! for losses)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>will you stop now? (Stop/Continue)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Record of the Second Search Sequence

<table>
<thead>
<tr>
<th>order of offer</th>
<th>1st</th>
<th>2nd</th>
<th>3rd</th>
<th>4th</th>
<th>5th</th>
<th>6th</th>
<th>7th</th>
<th>8th</th>
<th>9th</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>value of current offer (in cents)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>highest offer thusfar</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>total search cost (5 for each search)</td>
<td>5 10 15 20 25 30 35 40 45 50</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>earnings if you stop (use ! for losses)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>will you stop now? (Stop/Continue)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Please note: the cost of search for the next sequence has increased to 20 cents.

Record of the Third Search Sequence

<table>
<thead>
<tr>
<th>order of offer</th>
<th>1st</th>
<th>2nd</th>
<th>3rd</th>
<th>4th</th>
<th>5th</th>
<th>6th</th>
<th>7th</th>
<th>8th</th>
<th>9th</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>value of current offer (in cents)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>highest offer thusfar</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>total search cost (20 for each search)</td>
<td>20</td>
<td>40</td>
<td>60</td>
<td>80</td>
<td>100</td>
<td>120</td>
<td>140</td>
<td>160</td>
<td>180</td>
<td>200</td>
</tr>
<tr>
<td>earnings if you stop (use ! for losses)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>will you stop now? (Stop/Continue)</td>
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<td></td>
<td></td>
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</tr>
</tbody>
</table>

Finally, I will do this again for a search cost of 20 cents, but there is a maximum of three offers, which are equally likely to be any amount between 0 and 89 cents. You can stop before the first draw and earn 0, or you can pay 20 cents for the first offer, and stop or pay 20 more for a second, and stop with the highest of the two or pay 20 more for a third, and take the highest of the three. Before doing this, please decide how high the first offer must be for you to stop after only one, and how high the best of the two initial offers must be for you to stop after only two. You may want to discuss this with your neighbor. Please record your decisions below.

On the first offer, I will stop if it is at least _______.

On the 2nd draw, I will stop if the highest of the first two offers is at least _______.

Now I will throw the dice for the 1st offer, etc., and you can record your earnings below:
Multi-Unit Auction (Chapter 22)

(These instructions are loosely adapted from a practice procedure in the instructions used for some of the experiments reported in Cummings, Holt, and Laury (2001). The number of units to be purchased in the auction is announced in advance. An alternative to the fixed purchase quantity is for the instructor to announce a budget amount that will be spent, which corresponds more closely to the irrigation reduction auction. We use a fixed quantity here to clarify the incentives in the uniform price auction. The instructor may wish to give the discriminatory auction instructions to half of the class, and to give the uniform price instructions that follow to the other half.)

A Discriminatory Auction
Each person has been given a single colored pen. This is yours to keep when you leave today, unless you decide to sell it back to us. Here I have an amount of money that is sufficient to purchase _______ of these pens. What I will do is let each of you write down an offer to sell your pen, using the form:

__________________________
Your Name:
__________________________
Sale offer price:
__________________________

Please write your name and offer in the two boxes. After you have turned in your offer, all offers will be ranked from low to high, regardless of pen color, and the _____ lowest offers will be accepted. Each person with an accepted offer will receive an amount that equals that person’s own offer amount, and they must, in turn, give up their pens. People with rejected offers will keep their pens.

For example, if the quantity goal were two pens and the bids were $10, $13, $15, and $20, then two bids ($10 and $13) would be accepted. This is because we start with the lowest bid and stop when the quantity goal is reached. The people with accepted offers get reimbursed an amount that equals their own offer, which means that one person receives $10 and the other receives $13. Each of these two people must then give up the pen. The others keep their pens and receive no payment. In the actual auction, the quantity goal will be _____ pens instead of the two pens used in this example.
A Uniform Price Auction
Each person has been given a single colored pen. This is yours to keep when you leave today, unless you decide to sell it back to us. Here I have an amount of money that is sufficient to purchase _______ of these pens. What I will do is let each of you write down an offer to sell your pen, using the form:

____________________________________________________________________

Your Name:  

____________________________________________________________________

Sale offer price:

____________________________________________________________________

Please write your name and offer in the two boxes. After you have turned in your offer, all offers will be ranked from low to high, regardless of pen color, and the ____ lowest offers will be accepted. Each person with an accepted offer will receive an amount that equals the lowest rejected offer, and they must, in turn, give up their pens. People with rejected offers will keep their pens.

For example, if the quantity goal were two pens and the bids were $10, $13, $15, and $20, then two bids ($10 and $13) would be accepted. This is because we start with the lowest bid and stop when the quantity goal is reached. The people with accepted offers get reimbursed an amount that equals the lowest rejected bid, which is $15 in this case. These people must then turn each give up the pen. The others keep their pens and receive no payment. In the actual auction, the quantity goal will be ______ pens instead of the two pens used in this example.
Ultimatum Bargaining (Chapter 23)

Proposer Number: _____

I have $10 to split. Those in the front of the room are “responders,” and those in the back are “proposers.” There are 11 ways that the $10 can be divided between two people. The proposer must suggest one of these by circling one (and only one) of the listed options. The proposer sheets will then be collected and shuffled. One of the sheets will be given to each responder, who must either accept or reject. If the responder accepts, then the proposal is enacted. If the responder rejects, then the money is not divided and each person, proposer and responder, will earn nothing. Then I will collect the sheets and use the throw of a ten-sided die to select one, and the earnings (if any) determined by the decisions on that sheet will actually be paid to the proposer and responder who made those decisions. This payment will be in cash.

$0 for the proposer, $10 for the responder  
$1 for the proposer, $9 for the responder  
$2 for the proposer, $8 for the responder  
$3 for the proposer, $7 for the responder  
$4 for the proposer, $6 for the responder  
$5 for the proposer, $5 for the responder  
$6 for the proposer, $4 for the responder  
$7 for the proposer, $3 for the responder  
$8 for the proposer, $2 for the responder  
$9 for the proposer, $1 for the responder  
$10 for the proposer, $0 for the responder

Responder Number: _____

_______ I accept, and earnings will be determined by the proposal.

_______ I reject, and both of us will earn nothing.
Lobbying Game Instructions (Chapter 29)
Source: These instructions are adapted from Goeree and Holt (1999).

This is a simple card game. Each of you has been assigned to a team of investors bidding for a local government communications license that is worth $16,000. The government will allocate the license by choosing randomly from the applications received. The paperwork and legal fees associated with each application will cost your team $3,000, regardless of whether you obtain the license or not. (Think of this $3,000 as the opportunity cost of the time and materials used in completing the required paperwork.) Each team is permitted to submit any number of applications, up to a limit of thirteen per team. Each team begins with a working capital of $100,000.

There will be four teams competing for each license, each of which is provided with thirteen playing cards of the same suit. Your team will play any number of these cards by placing them in an envelope provided. Each card you play is like a lottery ticket in a drawing for a prize of $16,000. All cards that are played by your team and the other three teams will be placed on a stack and shuffled. Then one card will be drawn from the deck. If that card is one of your suit, your team will win $16,000. Otherwise you receive nothing from the lottery. Whether or not you win, your earnings will decrease by $3,000 for each card that you play. To summarize, your earnings are calculated:

\[
\text{earnings} = \begin{cases} 
$16,000 & \text{if you win the lottery} \\
-3,000 \times \text{the number of cards you play.} & \text{if you do not win the lottery}
\end{cases}
\]

Earnings are negative for the teams that do not win the lottery, and negative earnings are indicated with a minus sign in the record table below. The cumulative earnings column on the right begins with $100,000, reflecting your initial financial capital. Earnings should be added to or subtracted from this amount. Are there any questions?

<table>
<thead>
<tr>
<th>Round</th>
<th>Number Cards Played</th>
<th>Cost per Card Played</th>
<th>Total Cost</th>
<th>License Value</th>
<th>Your Earnings</th>
<th>Cumulative Earnings $100,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>$3,000</td>
<td>$16,000</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The next lottery is for a second license. Your team begins again with thirteen cards, but the cost of each card played is reduced to $1,000, due to a government
efficiency move that requires less paperwork for each application. This license is worth $16,000 as before, whether or not your team already acquired a license.

<table>
<thead>
<tr>
<th>Round</th>
<th>Number Cards Played</th>
<th>Cost per Card Played</th>
<th>Total Cost</th>
<th>License Value</th>
<th>Your Earnings</th>
<th>Cumulative Earnings</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td></td>
<td>$1,000</td>
<td></td>
<td>$16,000</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In the lottery, the value of the license may differ from team to team. Your team begins again with thirteen cards, and the cost of each card played remains at $1,000. Your instructor will inform you of the license value, which you should write in the appropriate place in the table below. Each license is worth $16,000 as before, whether or not your team already acquired a license.

<table>
<thead>
<tr>
<th>Round</th>
<th>Number Cards Played</th>
<th>Cost per Card Played</th>
<th>Total Cost</th>
<th>License Value</th>
<th>Your Earnings</th>
<th>Cumulative Earnings</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td></td>
<td>$1,000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In the final round, the license will be worth the same to you as it was in round 3, but there is no lottery and no application fee. Instead, I will conduct an auction by starting with a low price of $8,000 and calling out successively higher prices until there is only one team actively bidding. The winning team will have to pay the amount of its final bid. The losing teams do not have to pay anything for the license that they did not purchase; the winning team earns an amount that equals its license value minus the price paid. The revenue from the auction will be divided equally among the teams:

<table>
<thead>
<tr>
<th>Round</th>
<th>Your Earnings (your license value minus your bid if you win, $0 otherwise)</th>
<th>Cumulative Earnings $100,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
References


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Scraps:

Cox and Oaxaca (1989)  
Plott, The Application of Laboratory Experimental Methods to the Public Choice  
Rabin and Thaler (2001) JEP  
Adam Smith  