1. (5 points) For \( y = f(x) \), provide the definition the elasticity of \( y \) with respect to \( x \).

\[
\text{El}_x f(x) =
\]

2. (5 points) Please evaluate the determinant of the matrix, showing your work:

\[
\begin{vmatrix}
6 & 12 & 4 & 0 \\
2 & 6 & 1 & 0 \\
4 & 6 & 3 & 0 \\
0 & 0 & 0 & 1
\end{vmatrix}
\]

3. (5 points) Invert the matrix:

\[
\begin{vmatrix}
1 & 2 & 3 \\
0 & 1 & 2 \\
0 & 0 & 1
\end{vmatrix}
\]

4. a) (10 points) Use Cramer's rule (showing your work) to find the derivative \( \frac{dy}{dz} \) for the 2 equation model, where \( g(x) \) and \( f(y) \) are increasing and continuously differentiable functions with \( f'(y) > 1 \) and \( g'(x) > 1 \):

\[
ge(x) + y = z \\
x + f(y) = 2z
\]

b) Determine the sign of the \( \frac{dy}{dz} \) derivative (explain), or show why the assumptions given are not sufficient to determine the sign.
5. (5 points) Find the derivative with respect to x of:

\[ \exp(f(x)g(x)) \]

\[ A^{f(x)} \]

\[ Ax^{10} + 1/e^x \]

6. (5 points) Please provide a formal definition of what it means for n vectors, \( x_1, x_2, \ldots, x_n \), to be linearly independent.

7. (5 points) A utility function, \( u(x) \), has a measure of risk aversion defined to be

\[ -\frac{u''(x)}{u'(x)} \]

The risk aversion measure may be constant or it may be a function of \( x \). Determine whether risk aversion is increasing in \( x \), constant, or decreasing in \( x \) for the following utility functions:

\[ u(x) = x^{1-r}, \text{ where } 0 < r < 1, \text{ and } x > 0 \]

\[ u(x) = 1 - e^{-rx} \]
8. (10 points) A monopoly firm produces output $Q$ at a constant average cost, $c$, so total cost is $cQ$. This product is sold in a market with an inverse demand curve: $p = D(Q)$, with $D'(Q) < 0$ and $D''(Q) < 0$. Show the first-order condition for profit maximization, and show that the price elasticity of demand (i.e. the elasticity of quantity with respect to price) is greater than 1 in absolute value at the optimal quantity. (That is, you are to verify the statement: "A monopolist never operates on the inelastic portion of the demand curve.")

9. (5 points) State the first-order and second-order necessary conditions for the a relative maximum of the function $f(x)$, where $x$ can be any real number.

10. (5 points) Prove that $(AB)^{-1} = B^{-1}A^{-1}$ if the matrices $A$ and $B$ have inverses.
11. (5 points) A diagonal matrix is defined to be one with zeros everywhere except on the diagonal, i.e. $a_{ij} = 0$ if $i \not= j$. What are sufficient conditions for a diagonal matrix to be idempotent.

12. (10 points) Consider the function, $f(x) = (x-2)^4$, which has a stationary point at $x = 2$. Use a third-order Taylor expansion with a remainder term involving the fourth derivative of $f(x)$ to show that this stationary point is a relative minimum. (If you have trouble, begin by writing the formula for the Taylor series expansion of a function $f(x)$ around a point $x_0$, and then try to apply it to the present problem in which the first three derivatives of the function are zero when evaluated at the stationary point.)