Prob 3.101

Draw the FBD using polar coordinates. Note that two forces are present - the normal force exerted on the block by the circular surface and the weight of the block.

Given: \( M A g = 1 \text{ lb} \Rightarrow M A = \frac{1}{32.2} \text{ slugs} = 0.031 \text{ slugs} \)

Initial Conditions: When \( t = 0 \), \( V_0 = 14 \text{ ft/sec} \), \( \Theta = 0 \)

Find: \( V \) when \( \Theta = 60^\circ \)

Summing forces gives: \( \Sigma F_r = M g \cos \Theta - N = M a_r \)

\[ \Sigma F_\theta = -m g \sin \Theta = M a_\theta \]

where \( a_r = \frac{d^2 r}{d t^2} = -r w^2 \) because \( r \) is constant = 4 ft,

\[ a_\theta = r \alpha - 2 \frac{d r}{d t} w = r \alpha \]

again because \( r \) is constant.

Therefore, \( m g \cos \Theta - N = -m r w^2 \)

and \( -m g \sin \Theta = m r \alpha \)

Now we have 3 unknowns \((N, r, \alpha)\) and 2 equations. The third equation will come from the Chain Rule.
Prob 3.101 (continued)

The definition of angular acceleration gives us:

\[ \alpha = \frac{d\omega}{dt} = \frac{d\omega}{d\theta} \cdot \frac{d\theta}{dt} = \omega \frac{d\omega}{d\theta} \]

Plugging this into the second equation gives:

\[ -mg \sin \theta = mr \frac{d\omega}{d\theta} \]

Multiply both sides by \( d\theta \) and integrate:

(Note that \( \omega_0 = \frac{v_0}{L} = \frac{14}{4} = 3.5 \text{ rad/sec} \))

\[ -mg \int_0^{60^\circ} \sin \theta \, d\theta = mr \int_{3.5}^{\omega} \, d\omega \]

Evaluate:

\[ mg \left[ \cos 60^\circ - \cos 0^\circ \right] = \frac{mv}{2} \left[ \omega^2 - 3.5^2 \right] \]

\[ (1) \left[ 0.5 - 1 \right] = \frac{0.031(4)}{2} \left[ \omega^2 - 12.25 \right] \]

\[ \therefore \omega^2 = \frac{2(-0.5) + 12.25}{0.031(4)} = 4.19 \Rightarrow \omega = 2.05 \text{ rad/sec} \]

So, the angular velocity at \( 60^\circ \) is 2.05 rad/sec. To find the velocity at \( 60^\circ \) we use the relationship:

\[ v = v \omega = 4(2.05) = 8.2 \text{ ft/sec} \]