Marriage, Divorce, and Asymmetric Information

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Abstract. In answers to unique questions from the National Survey of Families and Households, people reveal their valuations of their options outside of marriage as well as their beliefs about their spouses’ options. We use this data to demonstrate several features of household bargaining. First, we document marriages in which one spouse would be happier outside the marriage and the other spouse would be unhappier. This provides a new type of evidence that bargaining takes place. Second, we show that spouses have private information about their outside options, and we estimate a bargaining model that quantifies the extent of resulting inefficiencies. However, estimation predicts unreasonably low variation in divorce rates across the sample arising because spouses drive too hard a bargain in the presence of asymmetric information and linear utility. Third, therefore, we allow for interdependent and diminishing marginal utility from marital surplus, both of which are identified by incorporating divorce data, and we obtain reasonable divorce predictions. Our results show that agents forgo their own utility in order to raise the utility of their spouses, and, in doing so, offset much of the inefficiency generated by their imperfect knowledge. In contrast, a social planner with only public information about spouses’ outside options would reduce welfare considerably by keeping far too many couples together. In sum, we find evidence about two key features of marriage – asymmetric information and interdependent utility – which are difficult to identify in most studies of interpersonal relationships.

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1 Introduction

A burgeoning empirical literature provides evidence that spouses bargain over household decisions. The existence of intrahousehold bargaining has two important implications for our understanding of individual welfare and behavior. First, the welfare of household members depends on the distribution of bargaining power and not just on total household resources. Second, decisions like consumption and saving that are observed at the household level are not the outcome of a single individual maximizing utility.

A limitation of most bargaining studies is that, as Lundberg and Pollak (1996, p.140) pointed out, “empirical studies have concentrated on debunking old models rather than on discriminating among new ones.” In this paper, we use unique questions from the National Survey of Families and Households to shed light on the nature of bargaining. Both spouses in NSFH households are asked about their happiness in case of divorce as well as their perception of their spouse’s happiness in case of divorce. We interpret these answers as shedding light on the valuation of outside options, with the former revealing the spouse’s private information and the latter the public information within the marriage. This interpretation relies on the premise that such questions elicit informative and unbiased answers. Given our reasonable estimation results when we fit these answers to our model of household bargaining, we conclude that questions like these offer a promising approach to seeing inside the “black box” of household decision-making.

We use the NSFH data to demonstrate several features of asymmetric information and bargaining. We begin by noting that, in some marriages, one spouse reports that they would be happier outside the marriage, and the other reports that they would be unhappier. Since such couples are in fact married (and a large fraction remain married five years later), this provides a new kind of evidence that bargaining takes place.

We also use the data to investigate some important characteristics of marital bargaining that have not been identifiable in most earlier studies. One of the key unresolved questions is whether bargaining is efficient. Despite important work that assumes efficient bargaining (for example, Browning et al., 1994; Chiappori, Fortin, and Lacroix, 2002; Mazzocco, 2007; and Del Boca and Flinn, 2009), indirect evidence of inefficiency is suggested by the rise in divorce rates following the transition from mutual to unilateral divorce laws in the U.S. and Europe (Friedberg, 1998; Wolfers, 2006; González and Viitanen, 2006). However, those papers do not indicate sources of inefficiency. The NSFH data reveal that spouses have private information about their outside options. The theoretical implication is that some transfers of marital surplus between spouses will be inefficiently small, generating too many divorces. We use the data on outside options to estimate a model of bargaining and quantify the extent to which asymmetric information generates bargaining inefficiencies.

When we evaluate this basic specification, we find that divorce probabilities appear too high and too homogeneous within the sample. This suggests that the model makes spouses drive too hard a bargain with each other in the pres-
ence of asymmetric information and linear utility from marital surplus. For that reason, we generalize the model to include interdependent utility, which is identified by using divorce data from the Current Population Survey. Estimates from the full specification show that agents forgo utility in order to raise the utility of their spouses with only very mild limits on transferable utility resulting from slightly diminishing marginal utilities in marital surplus. The resulting divorce predictions are reasonable, so caring preferences offset the bargaining inefficiencies arising from asymmetric information. The results further show that limited government involvement may be justified, as many couples in our sample appear to benefit from the level of divorce costs implicit in their answers about marital happiness, though our model does not quantify the optimal divorce cost. In contrast, a social planner with only public information about spouses’ outside options reduces welfare considerably by keeping far too many couples together.

While it is obvious that dynamics are important in marital bargaining, we mostly ignore dynamics for a number of reasons. First, and perhaps most important, our data are not rich enough to identify interesting dynamics. The NSFH data has two waves, separated by five years. The important dynamics about bargaining and learning would have to be observed at greater frequency to identify parameters of interest. Second, and still important, adding dynamics and asymmetric information in a bargaining model, much less an empirical one, is a major step beyond the literature. Some other papers have models (though no structural estimates) of repeated bargaining, but most lack a substantive role for private information. A few papers have multi-step bargaining and private information (Sieg, 2000; Watanabe, 2006, 2008) but with very limited time horizons in one-shot litigation games. Perhaps the paper most closely aligned to our problem is Hart and Tirole (1988). It has a model with repeated bargaining and private information; yet, in its set-up, a failure to agree in any period does not sever the relationship, which is unrealistic with marriage.

To sum up, we have found evidence about two key features of marriage – asymmetric information and interdependent utility – which are important in studying many kinds of interpersonal relationships. Moreover, our results suggest very mild limits on the transferability of utility, another concern raised in the household literature as an impediment to efficiency (Fella, Manzini, and Mariotti, 2004; Zelder, 1993). There has been little direct evidence in any area of economic research about the existence of information asymmetries. Some papers have tested for the presence of asymmetric information by analyzing mar-

\footnote{Some papers in the literature use the word “dynamics” to focus on the dynamics of a particular bargaining outcome (e.g. Rubinstein, 1985; Cramton, 1992). Our interest is in the dynamics associated with repeated bargaining.}


\footnote{The proposer in Hart and Tirole uses information on rejected offers to update beliefs about the other side, a feature of marital bargaining that would be relevant in a model where the couple can disagree without divorce (Lundberg and Pollak, 1993; Zhelyansky, 2012).}
ket outcomes,\textsuperscript{4} while some show that agents have private information, though without demonstrating an effect on market outcomes.\textsuperscript{5}

Although our evidence about interdependent utilities is indirect, it arises in the context of real world outcomes rather than experimental settings, which have generated abundant results about altruism.\textsuperscript{6} Thus, the evidence here justifies incorporating “love” into economic theory.\textsuperscript{7} Yet, our results show that, even when a couple is in love, they neither know everything about each other nor behave completely selflessly (perhaps retaining a measure of victory for cynical economists?), and this can justify limited government involvement, at least in the form of divorce costs.

The rest of this paper is organized as follows. We discuss the raw data from the NSFH in Section 2. We present a simple model of marital bargaining in Section 3 and estimates of the simple model in Section 4. These results lead us to develop the model further by adding caring preferences to the model in Section 5 and to the estimation results in Section 6. We conclude in Section 7.

2 Data on Happiness in Marriages

We use data from the National Survey of Families and Households (NSFH).\textsuperscript{8} The sample consists of 13008 households surveyed in 1987-88 and again in 1992-94. We use data from the first wave of the NSFH and, for descriptive purposes, information about subsequent divorces between the first and second waves. The first wave asked about individuals’ and their partners’ well-being in marriage relative to separation.\textsuperscript{9} This information is obtained from responses by both spouses to the following questions:

1. Even though it may be very unlikely, think for a moment about how various areas of your life might be different if you separated. How do you think your overall happiness would change? [1-Much worse; 2-Worse; 3-Same; 4-Better; 5-Much better]

\textsuperscript{4}For example, characteristics of markets for insurance (Finkelstein and Poterba, 2004) and used durables (Engers, Hartmann, and Stern, 2004) exhibit features that are consistent with the presence of asymmetric information.

\textsuperscript{5}For example, subjective expectations reported by individuals about life spans (Hurd and McGarry, 1995) and long-term care needs (Finkelstein and McGarry, 2006) are informative about future outcomes, even when controlling for population average outcomes. Scott-Morton, Zettelmeyer, and Risso (2004) finds that car shoppers with superior information obtain a better price than uninformed shoppers, but we do not know of other papers that directly measure information asymmetries when two agents act strategically.

\textsuperscript{6}Selﬁss behavior is a leading explanation for results obtained in a range of experiments, including ultimatum and public goods games.

\textsuperscript{7}Hong and Ríos-Rull (2004) is similar in spirit, though very different in the details. It uses life insurance purchases to identify interdependent preferences and a restricted form of bargaining in a general-equilibrium overlapping-generations model.

\textsuperscript{8}Sweet, Bumpass, and Call (1988) offers a thorough description of the data.

\textsuperscript{9}While some NSFH data were collected through verbal responses, the questions that we focus on were collected through a written, self-administered survey component.
2. How about your partner? How do you think his/her overall happiness might be different if you separated? [same measurement scale]

In the rest of this section, we will discuss what the answers may reveal about bargaining and information asymmetries. We will report statistics for our estimation sample of 4242, postponing until later a description of our sample selection criteria.

2.1 Evidence of Bargaining

Figure 1 illustrates the joint density of each spouse’s reported happiness or unhappiness associated with separation, based on question #1. Spouses appear happy in their marriages on average, relative to their outside options, with husbands being a little happier. Almost identical percentages – 77.0 percent of husbands and 77.4 percent of wives – say they would be worse or much worse off if they separated, while only 5.9 percent of husbands and 7.5 percent of wives say they would be better or much better off. 40.9 percent of couples report the same level of happiness (denoted by bars that are outlined with heavy black). While husbands would be worse off than wives in 27.0 percent of couples and wives would be worse off in the other 32.0%, only about a quarter of all the discrepancies in overall happiness are “serious” (differing by more than one category).

We interpret this data as reflecting the overall value of marriage relative to separation – including concerns such as one’s children’s well-being, religious values, or losses associated with divorce – before any side payments that redistribute marital surplus. Otherwise, if the answers reflected happiness after side payments, it would be difficult to understand why a spouse is married if he or she would be better off divorcing, since most U.S. states have unilateral divorce laws. Moreover, we find support for the assumption that answers do not include side payments in our finding later that one spouse’s happiness does

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10More husbands report “worse” while more wives report “much worse.”
not move with the other spouse’s reported happiness (as it would if answers were reported net of side payments). Under the assumption that answers precede side payments, the data provide evidence that spouses bargain with each other. Consider the 7.0 percent of couples in which one spouse would be better or much better off if the couple separates, while the other spouse would be worse or much worse off. The fact that we observe them as intact couples shows that the spouse who prefers marriage must be compensating the spouse who prefers separation. This is reinforced by the fact that only 15.4 percent of those couples divorce by the time of wave 2 of the NSFH, roughly six years later, so a large fraction remains together, presumably with the relatively happy spouse compensating the relatively unhappy one.

2.2 Evidence of Asymmetric Information

Perceptions about one’s spouse’s happiness or unhappiness outside of marriage are also interesting. The joint density of perceptions about husbands’ happiness or unhappiness, as reported by both spouses, appears in Table 1-A, and the joint density of perceptions about wives’ happiness or unhappiness appears in Table 1-B.

While 77 percent of individuals in Figure 1 say they would be worse or much worse off if they separated, wives slightly overestimate and husbands slightly underestimate how much worse off their spouses would be if they separated – 79.4 percent of wives and 73.5 percent of husbands think that their spouses would be worse or much worse off. Overall, as shown in the tables’ bottom rows, somewhat less than half of spouses have the same perceptions about their partners’ happiness as their partner reports. About one-quarter of those misperceptions are “serious” (again, differing by more than one category), with wives overestimating their husbands’ unhappiness and husbands underestimating their wives’ unhappiness, on average. Lastly, we note that the accuracy of a spouse’s perceptions is highest when the other spouse would be unhappiest in case of separation, suggesting that asymmetric information in cases of spouses who would be relatively happy in divorce is indeed relevant.

The NSFH provides other information that helps us understand the nature of asymmetric information and of disputes more generally. Stern (2003) shows that (a) spouses have very accurate perceptions of the time spent by the other

\[\text{If instead we assumed that the answers reflect happiness inclusive of marital surplus transfers, then we must incorporate some other friction that prevents divorce, but that is not identifiable from the available data without imposing additional structure. The assumption that the answers incorporate any costs of divorce gain support from Zhylyevskyy (2012), who finds that the NSFH answers are significantly affected by state divorce and child support laws. The final alternative is to view the answers as incomplete or biased reports of marital happiness, in which case they are unusable without stronger assumptions as well. Nevertheless, our approach leaves us at a loss to explain why both spouses in 1.6 percent of couples report that they would be “better” or “much better” off if they separated; and why one spouse answers “same” and the other answers “better” or “much better” in 3.7 percent of couples.}\]

\[\text{Spouses are accurate about 50 percent of the time when partners report that they would be much worse or worse off. The accuracy rate declines monotonically as partners report being the same or better off.}\]
Table 1A: Joint Density, Perceptions of Husband’s Overall Happiness if Spouses Separated

<table>
<thead>
<tr>
<th>Husband’s Answer about Self:</th>
<th>W’s Answer About Husband</th>
<th>Much Worse</th>
<th>Worse</th>
<th>Same</th>
<th>Better</th>
<th>Much Better</th>
<th>W’s Answer (Total)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Much worse</td>
<td>0.179</td>
<td>0.124</td>
<td>0.032</td>
<td>0.008</td>
<td>0.002</td>
<td>0.345</td>
<td></td>
</tr>
<tr>
<td>Worse</td>
<td>0.139</td>
<td>0.205</td>
<td>0.082</td>
<td>0.018</td>
<td>0.004</td>
<td>0.449</td>
<td></td>
</tr>
<tr>
<td>Same</td>
<td>0.030</td>
<td>0.065</td>
<td>0.043</td>
<td>0.011</td>
<td>0.004</td>
<td>0.155</td>
<td></td>
</tr>
<tr>
<td>Better</td>
<td>0.007</td>
<td>0.016</td>
<td>0.009</td>
<td>0.003</td>
<td>0.002</td>
<td>0.041</td>
<td></td>
</tr>
<tr>
<td>Much better</td>
<td>0.002</td>
<td>0.003</td>
<td>0.002</td>
<td>0.003</td>
<td>0.000</td>
<td>0.009</td>
<td></td>
</tr>
<tr>
<td>H’s answer (Total)</td>
<td>0.357</td>
<td>0.413</td>
<td>0.171</td>
<td>0.047</td>
<td>0.012</td>
<td>1.000</td>
<td></td>
</tr>
<tr>
<td>H Better than W Thinks</td>
<td>0.000</td>
<td>0.124</td>
<td>0.114</td>
<td>0.037</td>
<td>0.012</td>
<td>0.287</td>
<td></td>
</tr>
<tr>
<td>H, W agree</td>
<td>0.179</td>
<td>0.205</td>
<td>0.045</td>
<td>0.007</td>
<td>0.000</td>
<td>0.436</td>
<td></td>
</tr>
<tr>
<td>H Worse than W Thinks</td>
<td>0.178</td>
<td>0.084</td>
<td>0.012</td>
<td>0.003</td>
<td>0.000</td>
<td>0.276</td>
<td></td>
</tr>
</tbody>
</table>

Table 1B: Joint Density, Perceptions of Wife's Overall Happiness if Spouses Separated

<table>
<thead>
<tr>
<th>Husband’s Answer about Wife</th>
<th>Wife’s Answer About Self:</th>
<th>Much Worse</th>
<th>Worse</th>
<th>Same</th>
<th>Better</th>
<th>Much Better</th>
<th>Wife’s Answer (Total)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Much Worse</td>
<td>0.139</td>
<td>0.071</td>
<td>0.020</td>
<td>0.006</td>
<td>0.002</td>
<td>0.258</td>
<td></td>
</tr>
<tr>
<td>Worse</td>
<td>0.209</td>
<td>0.109</td>
<td>0.065</td>
<td>0.019</td>
<td>0.006</td>
<td>0.477</td>
<td></td>
</tr>
<tr>
<td>Same</td>
<td>0.048</td>
<td>0.009</td>
<td>0.049</td>
<td>0.017</td>
<td>0.006</td>
<td>0.188</td>
<td></td>
</tr>
<tr>
<td>Better</td>
<td>0.011</td>
<td>0.023</td>
<td>0.014</td>
<td>0.003</td>
<td>0.001</td>
<td>0.065</td>
<td></td>
</tr>
<tr>
<td>Much Better</td>
<td>0.002</td>
<td>0.003</td>
<td>0.003</td>
<td>0.002</td>
<td>0.001</td>
<td>0.012</td>
<td></td>
</tr>
<tr>
<td>W’s Answer (Total)</td>
<td>0.424</td>
<td>0.350</td>
<td>0.151</td>
<td>0.057</td>
<td>0.019</td>
<td>1.000</td>
<td></td>
</tr>
<tr>
<td>W Better than H Thinks</td>
<td>0.000</td>
<td>0.071</td>
<td>0.085</td>
<td>0.041</td>
<td>0.017</td>
<td>0.215</td>
<td></td>
</tr>
<tr>
<td>W, H agree</td>
<td>0.159</td>
<td>0.184</td>
<td>0.049</td>
<td>0.013</td>
<td>0.001</td>
<td>0.406</td>
<td></td>
</tr>
<tr>
<td>W Worse than H Thinks</td>
<td>0.265</td>
<td>0.095</td>
<td>0.017</td>
<td>0.002</td>
<td>0.000</td>
<td>0.379</td>
<td></td>
</tr>
</tbody>
</table>

Notes:
1. Sample size is 4242.
2. H denotes husband, W denotes wife.
3. Cells that are outlined indicate agreement between husbands’ and wives’ perceptions.

spouse on various household activities, (b) the vast majority thinks that decisions are made fairly, and (c) they fight infrequently. The first two findings suggest that there are not asymmetric views about how much each spouse contributes to household public goods or how well spouses feel they are treated; so, the asymmetries may instead involve information about options outside the marriage. The third finding downplays the importance of conflict as a reason for divorce, which leaves a role for asymmetric information.13

13Zhylevskyy (2012) shows, in a theoretical model in which conflict, cooperation, and divorce are all equilibrium states, that neither conflict nor divorce will occur without asymmetric information.
2.3 Asymmetric Information and Divorce

Inefficient divorces will arise when one spouse would be unobservably happier outside of marriage than the other believes. If so, then the side payment will be inefficiently small for the unhappy spouse, leading to some divorces. According to Tables 1-A and 1-B, 6.9 percent of husbands and 5.9 percent of wives “seriously” misperceive (by more than one category) their spouses’ happiness. We can follow marriage outcomes in Wave 2, roughly six years later, among the 3597 couples from our sample that the NSFH was able to track.

Table 2 reports divorce rates for this group, classified according to spouses’ answers about their own happiness and their perceptions of their partners’ happiness in Wave 1. The overall divorce rate was 7.3%, and it generally fell with each spouse’s reported happiness. When both spouses said they would be worse or much worse off if they separated, for example, the divorce rate was only 4.8 percent.

To demonstrate the potential relevance of asymmetric information, we compare the divorce rates of couples with accurate perceptions and those with misperceptions about their spouses. In couples where a spouse had the correct perception about their partner and thus bargaining should yield an efficient outcome, 5.4 – 5.7 percent divorced (depending on whether we consider correct perceptions of the husband or wife). In couples where a spouse has incorrect perceptions, and one spouse underestimates how unhappy the other would be if they separated, the divorce rate is 6.9 – 8.1 percent. Next, consider the strong prediction arising in a model of inefficient bargaining. In couples in which one spouse overestimates how unhappy the other spouse would be if they separated, then the mistaken spouse would try to extract too much surplus, leading some marriages with positive surplus to break up. The data support this prediction: the divorce rate was higher for couples where one spouse overestimated how unhappy the other spouse would be if they separated, at 9.0 – 11.7 percent, and especially if the misperception was serious (with answers differing by more than one category), at 13.1 – 14.5 percent.

Next, we formalize a model of bargaining with imperfect information. Later, we estimate the model using the data we have described here.

3 A Simple Bargaining Model without Caring Preferences

In this section, we describe the model which we apply to the data on happiness in marriage. We first discuss how concerns about identification motivate the choices we made in developing the model. Then, we present the detailed model with caring preferences and analyze special cases.

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14 Focusing on all misperceptions, they arise for 28.7 percent of husbands and 21.5 percent of wives.
Table 2: Divorce Rates (% of Couples Who had Divorced by Wave 2)

| Divorce Rate | N |  
|-------------|---|---
| Full Wave 1 Sample | 3597 | 7.30% 
| How would your overall happiness change if you separated? | | 
| Both spouses “worse” or “much worse” | 2297 | 4.80% 
<table>
<thead>
<tr>
<th>Divorce Rate</th>
<th>H about W</th>
<th>W about H</th>
</tr>
</thead>
<tbody>
<tr>
<td>Does ... have correct perceptions about spouse’s happiness?</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Correct perceptions</td>
<td>5.4%</td>
<td>5.7%</td>
</tr>
<tr>
<td>Incorrect perceptions</td>
<td>8.6%</td>
<td>8.6%</td>
</tr>
<tr>
<td>Understates spouse’s unhappiness</td>
<td>6.9%</td>
<td>8.1%</td>
</tr>
<tr>
<td>Overstates spouse’s unhappiness</td>
<td>11.7%</td>
<td>9.0%</td>
</tr>
<tr>
<td>Does ... have roughly correct perceptions about spouse’s happiness?</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Roughly correct perceptions</td>
<td>6.5%</td>
<td>6.5%</td>
</tr>
<tr>
<td>Seriously incorrect perceptions</td>
<td>12.0%</td>
<td>13.0%</td>
</tr>
<tr>
<td>Seriously understates spouse’s unhappiness</td>
<td>11.3%</td>
<td>11.3%</td>
</tr>
<tr>
<td>Seriously overstates spouse’s unhappiness</td>
<td>13.1%</td>
<td>14.5%</td>
</tr>
</tbody>
</table>

Notes:
1. Sample consists of those among our Wave 1 estimation sample of 4242 who also appear in Wave 2 and report information about their marital status. Wave 1 took place in 1987-88 and wave 2 in 1992-94.
2. H denotes husband, W denotes wife.
3. "Roughly correct" perceptions are defined as answers that differ by one category or less. "Seriously incorrect" perceptions are answers that differ by two categories or more.

3.1 Motivation

We assume a model that follows much of the literature on household bargaining. Spouses cooperate to maximize total surplus (before our model begins) and then bargain over the surplus, with the relative strength of each spouse’s threat point outside of marriage determining how the surplus is split.\(^{15}\) We interpret the NSFH data as revealing these threat points.\(^{16}\) Numerous papers use the Nash bargaining model (which assumes no private information and implies Pareto efficiency) to analyze how the split in the unknown marital surplus may shift as a function of factors observed by the econometrician that move otherwise unknown threat points. Given our data, we focus on how the split in the unknown surplus may lead to inefficient divorce as a function of threat points observed by the econometrician.

While most papers do not actually model a specific bargaining rule, it is important for us to do so. We choose a transparent bargaining rule that is robust in ways we discuss next in order to make predictions about inefficient divorce. We simply assume that one spouse makes an offer which the other accepts or rejects, in which case the marriage ends. This take-it-or-leave-it rule is a limiting

\(^{15}\)We ignore sorting into marriage. While this would be a problem if couples know something about their prospective happiness before they marry, the marriage decision is beyond the scope of our analysis, especially because we lack data on individuals before they marry. One way to interpret our results is that all couples start out equally happy at the beginning of marriage, while their observed happiness in the NSFH reflects new information.

\(^{16}\)In contrast, most empirical papers use, as a proxy for threat points, data indicating which spouse controls a particular source of income. In common with most such papers, though, our data would not allow us to identify a model like that in Lundberg and Pollak (1993) in which threat points depend on noncooperative bargaining within marriage.
case of the bilateral bargaining game in Chatterjee and Samuelson (1983), in
which parties make simultaneous offers and split the difference, if positive, with
exogenous share \( k \) going to one agent and \( 1 - k \) to the other. The solution to the
general game is tractable and unique only under restrictive assumptions – if, for
example, agents’ private information is uniformly distributed – but an analytical
solution is not possible under our assumption of a normal distribution. However,
we are able to implement a test of this take-it-or-leave-it bargaining assumption,
jointly with an assumption about the informational content of responses about
happiness, as we explain later, and we do not reject this joint test.

Under our take-it-or-leave-it rule, whichever agent makes the offer seeks to
extract as much surplus as is possible. To explore the implications of this,
we estimated two versions of the model – one with each spouse making the
offer – resulting in upper and lower bounds on the estimated side payments,
conditional on observables. As the distribution of private and public information
about happiness, shown earlier, is quite similar for husbands and wives, this
did not alter the parameter estimates substantively. What changed is that
different couples divorce under either alternative, depending on which spouse in
a particular couple is unobservably unhappy and which makes the offer; yet the
average predicted divorce rate remains very similar.

3.2 Model

Let the direct utility that a husband \( h \) and wife \( w \) get from marriage be, respectively,

\[
U_h = \theta_h - p + \varepsilon_h,
\]
\[
U_w = \theta_w + p + \varepsilon_w
\]

where \((\theta_h, \theta_w)\) are observable components and \((\varepsilon_h, \varepsilon_w)\) are unobservable compo-
nents of utility for the husband and wife and utility from not being married to
each other is normalized to zero. Ignoring discreteness for the moment, we will
assume that answers to Question #1 above, about one’s happiness in marriage,
reveal \((\theta_h + \varepsilon_h, \theta_w + \varepsilon_w)\) and that answers to Question #2, about one’s spouse’s
happiness, reveal \((\theta_h, \theta_w)\). As we noted earlier, \((\theta_h, \theta_w)\) and \((\theta_h + \varepsilon_h, \theta_w + \varepsilon_w)\)
include the value of household public goods and the (negative) value of any flows
associated with divorce (Weiss and Willis 1993).\(^{17}\) Without loss of generality,
we can assume that \(\varepsilon_h\) and \(\varepsilon_w\) are independent because any component that
is correlated with something observed by the other spouse could be relabeled
as part of \((\theta_h, \theta_w)\). Define \(f_h(\cdot)\) and \(f_w(\cdot)\) as the density functions and \(F_h(\cdot)\)
and \(F_w(\cdot)\) as the distribution functions of \(\varepsilon_h\) and \(\varepsilon_w\). Lastly, the variable \(p\) is
a (possibly negative) side payment from the husband to the wife that allocates
marital surplus in the sense of McElroy and Horney (1981), Chiappori (1988),
and Browning et al. (1994). Later, in Section 6.3.2, we describe an empirical

\(^{17}\) We also assume that reported happiness includes information about expected future hap-
piness in marriage. As noted above, however, we lack sufficient information and a tractable
approach to estimate a model of dynamic bargaining.
test of the assumption that answers to the question reflect happiness before the side payment \( p \), jointly with the assumption of take-it-or-leave-it bargaining; the estimates fail to reject this joint test.

### 3.3 Analytics

In this subsection, we derive the comparative statics of this simple version of the model to demonstrate some intuitive features.\(^ {18} \) We also show the impact of incorporating an explicit divorce cost, since we are interested in the welfare effects of policies that alter the cost of divorce.

In this take-it-or-leave-it model of bargaining, suppose the husband chooses \( p^* \) to maximize his expected value from marriage,

\[
    p^* = \arg \max_p \left( \theta_h - p + \varepsilon_h \right) \left[ 1 - F_w (-\theta_w - p) \right]
\]

(1)

where \( \theta_h - p + \varepsilon_h \) is the marital surplus for the husband and \( 1 - F_w (-\theta_w - p) \) is the probability of the wife accepting the offer of \( p^* \). The first order condition is

\[
    \left[ \theta_h - p + \varepsilon_h \right] f_w (-\theta_w - p) - \left[ 1 - F_w (-\theta_w - p) \right] = 0.
\]

(2)

It is straightforward to show that \( \frac{dp}{d\varepsilon_h} > 0 \), \( \frac{dp}{d\theta_h} > 0 \), \( \frac{dp}{d\theta_w} < 0 \), and

\[
    \frac{dp}{d\theta_h} = \frac{f_w (-\theta_w - p)}{2f_w (-\theta_w - p) - [\theta_h - p + \varepsilon_h] \frac{df_w (-\theta_w - p)}{dp}} > 0,
\]

(3)

so the side payment rises with the husband’s observed happiness. The probability of a divorce is

\[
    \Pr \left[ \theta_w + p \left( \varepsilon_h \mid \theta_h \right) + \varepsilon_w < 0 \right].
\]

Equation (2) implies that the husband picks \( p \) so that

\[
    U_h = \theta_h - p + \varepsilon_h = \frac{1 - F_w (-\theta_w - p)}{f_w (-\theta_w - p)} > 0.
\]

(4)

Thus, if \( (\varepsilon_h, \varepsilon_w) \) satisfy \( 0 \geq U_h + U_w = \theta_h + \theta_w + \varepsilon_h + \varepsilon_w \) (so marital surplus is negative), then

\[
    0 \geq (\theta_w + p + \varepsilon_w) + (\theta_h - p + \varepsilon_h) \Rightarrow 0 > \theta_w + p + \varepsilon_w.
\]

So, no divorces that occur with perfect information (when \( 0 \geq \theta_h + \theta_w + \varepsilon_h + \varepsilon_w \)) are avoided with asymmetric information. Plus, there are \( (\varepsilon_h, \varepsilon_w) \) that satisfy \( 0 \leq \theta_h + \theta_w + \varepsilon_h + \varepsilon_w \) and \( 0 \geq \theta_w + p \left( \varepsilon_h \mid \theta_h \right) + \varepsilon_w \). This is because \( \theta_h + \theta_w + \varepsilon_h + \varepsilon_w \) is related to Peters’ (1986) model of asymmetric information in marriage. She proposed a fixed-wage contract negotiated upon entering marriage as a second-best solution to this problem; we assume that such a contract was not negotiated or is not renegotiation-proof.
$\varepsilon_h + \varepsilon_w$ and $p (\varepsilon_h \mid \theta_h)$ are continuous in $\varepsilon_h$ and $\theta_h$, and $\theta_w + p + \varepsilon_w < 0$ when $\theta_h + \theta_w + \varepsilon_h + \varepsilon_w = 0$. Thus, some divorces could be avoided if there were no asymmetric information, as Peters (1986) shows when unilateral divorce is legal.

We can also compute expected utility for each partner as

$$EU_h = \int_{-\infty}^{\infty} \left[ \theta_h - p (\varepsilon_h \mid \theta_h) + \varepsilon_h \right] [1 - F_w (-\theta_w - p (\varepsilon_h \mid \theta_h))] dF_h (\varepsilon_h);$$

$$EU_w = \int_{-\infty}^{\infty} \left[ \theta_w + p (\varepsilon_h \mid \theta_h) + \varepsilon_w \right] dF_w (\varepsilon_w) dF_h (\varepsilon_h).$$

This implies that total expected utility from marriage is

$$EU_h + EU_w = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left[ \theta_h + \theta_w + \varepsilon_h + \varepsilon_w \right] dF_w (\varepsilon_w) dF_h (\varepsilon_h)$$

$$< \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left[ \theta_h + \theta_w + \varepsilon_h + \varepsilon_w \right] dF_w (\varepsilon_w) dF_h (\varepsilon_h),$$

because of equation (4), so it is smaller than total utility with no asymmetric information. We can show that $\partial EU_h / \partial \theta_h > 0$ and $\partial U_w / \partial \theta_h > 0$, implying that total expected utility from the marriage increases with $\theta_h$, and similarly with $\theta_w$.

### 3.4 Numerical example

Now, we present a numerical example of the model. Assume that $\varepsilon_i \sim iidN (0, 1)$, $i = h, w$. Then, $p (\varepsilon_h \mid \theta_h)$ solves

$$\left[ \theta_h - p + \varepsilon_h \right] \phi (-\theta_w - p) - [1 - \Phi (-\theta_w - p)] = 0.$$

We can solve the couple’s problem numerically. From the husband’s point of view, the offered side payment $p$ increases with his happiness $\theta_h + \varepsilon_h$ and decreases with his wife’s observed happiness $\theta_w$. The divorce probability is represented in Figure 2 and decreases in $\theta_h + \varepsilon_h$ and $\theta_w$.

The total expected value of the match, conditional on $\theta_h$ and $\theta_w$, is represented in Figure 3. It increases with both arguments. Recall, though, that the total expected match value is always diminished by the imperfect information.

The consequent loss in expected value due to information asymmetries is shown in Figure 4. The loss is quite small when $\theta_h + \theta_w$ is small because it is highly unlikely that $\varepsilon_h + \varepsilon_w$ is large enough so that a marriage should stay intact. The loss is high for large values of $\theta_h + \theta_w$, as the husband tries to take as much of the match value as he can, risking divorce.
Figure 2: Divorce Probabilities

Figure 3: Total Expected Value of Marriage

Figure 4: Loss Due to Asymmetric Information
3.5 Incorporating a Divorce Cost

Many U.S. states have altered their divorce laws since 1970 in ways that reduce the cost of divorce. We model a divorce cost $C$ as an element that respondents net out when reporting the value of their outside options. \(^\text{19}\) A divorce cost reduces welfare in the case of perfect information but has theoretically ambiguous effects when information is imperfect. Equation (1) becomes

$$p^* = \arg \max_p \left[ \theta_h - p + \varepsilon_h \right] \left[ 1 - F_w (-\theta_w - (1 - \gamma) C - p) \right]$$

$$-\gamma CF_w (-\theta_w - (1 - \gamma) C - p)$$

where the husband now maximizes his expected value from marriage minus his expected divorce cost, with $\gamma$ representing the proportion of $C$ that the husband must pay. The problem in equation (6) has the same solution as

$$p^* = \arg \max_p \left[ \theta_h + \gamma C - p + \varepsilon_h \right] \left[ 1 - F_w (x) \right] - \gamma C.$$

where $x = -\theta_w - (1 - \gamma) C - p$. The $\gamma C$ term at the end of the expression is a fixed cost and has no effect on the husband’s behavior. Thus, the effect of the divorce cost on his behavior is equivalent to the effect of increasing $\theta_h$ by $\gamma C$ and $\theta_w$ by $(1 - \gamma) C$.

One can show that

$$\frac{dp}{dC} = \gamma - \frac{f_w (x) - (\theta_h - p + \varepsilon_h) \frac{\partial f_w (x)}{\partial \theta_w}}{2f_w (x) - (\theta_h - p + \varepsilon_h) \frac{\partial f_w (x)}{\partial p}}.$$

More importantly,

$$\frac{d\left[ 1 - F_w (-\theta_w - (1 - \gamma) C - p) \right]}{dC} = f_w \cdot \left[ (1 - \gamma) + \frac{dp}{dC} \right] > 0,$$

so, as $C$ increases, divorces occur less frequently.\(^\text{20}\) Expected utility of each partner can be rewritten as

$$EU_h = \int_{-\infty}^{\infty} \left\{ [\theta_h - p (\varepsilon_h | \theta_h) + \varepsilon_h] \left[ 1 - F_w (x) \right] - \gamma CF_w (x) \right\} dF_h (\varepsilon_h);$$

$$EU_w = \int_{-\infty}^{\infty} \left\{ [\theta_w + p (\varepsilon_h | \theta_h) + \varepsilon_w] \left[ 1 - F_w (x) \right] - (1 - \gamma) CF_w (x) \right\} dF_h (\varepsilon_h).$$

\(^\text{19}\) Earlier we noted our assumption that reported happiness in marriage captures losses associated with divorce, and we cited evidence from Zhilyevskyy (2012) showing that answers about relative happiness in the NSFH are systematically related to state divorce and child support laws.

\(^\text{20}\) The denominator of the second term in brackets in equation (7) is negative because it is the second order condition. Thus, the entire term in brackets is positive.
The effect on expected utility of the divorce cost $C$ is, after applying the Envelope Theorem and equation (1),

$$\frac{\partial EU_h}{\partial C} = (1 - \gamma) \int_{-\infty}^{\infty} [\theta_h - p(\varepsilon_h | \theta_h) + \varepsilon_h] f(x) dF(\varepsilon_h)$$

$$- \gamma \int_{-\infty}^{\infty} F(x) dF(\varepsilon_h) + \gamma (1 - \gamma) \int_{-\infty}^{\infty} C f(x) dF(\varepsilon_h),$$

with a similar expression for $\frac{\partial EU_w}{\partial C}$. The first term represents the utility gain from a reduced probability of divorce (i.e., of the wife rejecting the offer $p$) which results from facing $C$. The second term represents the loss in utility from possibly having to pay $C$, while the third is the gain from the reduced probability of having to pay $C$. The total gain in expected utility is

$$\frac{\partial EU_h}{\partial C} + \frac{\partial EU_w}{\partial C} = (1 - \gamma) \int_{-\infty}^{\infty} [\theta_h + \theta_w + \varepsilon_h + \varepsilon_w] f(x) dF(\varepsilon_h)$$

$$- \int_{-\infty}^{\infty} F(x) dF(\varepsilon_h) + (1 - \gamma) \int_{-\infty}^{\infty} C f(x) dF(\varepsilon_h).$$

While this cannot be signed, we know from above that the first and third terms are positive while the second term is negative. The welfare gain arising from the first term (the gain in utility from the reduced divorce probability) rises with $\theta$ and $\varepsilon$. Also, the welfare gain from $C$ declines with $\gamma$, since a decrease in the share of the divorce cost borne by the wife raises the probability of divorce for any value of $C$.

We continue the numerical example to analyze the expected welfare gain associated with a divorce cost $C$. Figure 5 shows the expected welfare gain when the husband’s share $\gamma$ of $C$ takes the values $\{0.1, 0.9\}$. In both cases, there are some values of $\theta_h + \theta_w$ large enough that a) the probability of divorce (i.e., of large negative realizations of $(\varepsilon_h, \varepsilon_w)$) is relatively small and b) the loss associated with asymmetric information is relatively large. In such cases, the imposition of a divorce cost raises expected welfare. On the other hand, for those cases where $\theta_h + \theta_w$ is relatively small, $C$ just adds an extra cost to the likely divorce and reduces welfare. As we mentioned above, welfare gains are less likely as $\gamma$ rises, which leads the wife to avoid rejecting the husband’s offer and choose divorce.\(^{21}\)

4 Estimation of the Simple Bargaining Model

Earlier, we presented our data on how happy or unhappy each person would be if they separated along with their beliefs about how happy or unhappy their

\(^{21}\)It should be noted that the model with caring preferences, which we estimate below, yields more complicated comparative statics in $C$ and $\gamma$. 

15
partner would be. We treat this as information about the unobservable components $\varepsilon_h$ and $\varepsilon_w$ and the observable components $\theta_h$ and $\theta_w$ of utility from marriage. We use this information, along with information about divorce probabilities from other data, to estimate our model of marriage without caring preferences.

4.1 Estimation Methodology

Our estimation approach uses a generalized simulated maximum likelihood (SML) method. The objective function has two terms: a) likelihood contributions associated with our happiness data and b) moment conditions associated with divorce probabilities. The likelihood contributions associated with our happiness data resemble bivariate ordered probit terms that seek to explain the husbands’ and wives’ self-reported happiness data, conditional on the reports of their happiness by their spouses and on other family characteristics, which incorporates the structure of the simple bargaining model laid out above.

Define the set of happiness variables for each family $i$ as $\delta_i = (\varepsilon_{hi}, \varepsilon_{wi}, \varepsilon_{hi}, \varepsilon_{wi})$. We assume that they have the following properties. For the joint distribution $F_\theta (\cdot | X_i)$ of $\theta_i = (\theta_{hi}, \theta_{wi})$ given observable characteristics $X_i$, assume that $X_i$ affects $F_\theta (\cdot | X_i)$ only through the mean such that

$$E (\theta_{hi} | X_i) = X_i \beta_h; E (\theta_{wi} | X_i) = X_i \beta_w.$$ (8)

For the joint distribution $F_\varepsilon (\cdot)$ of $\varepsilon_i = (\varepsilon_{hi}, \varepsilon_{wi})$, assume that $E \varepsilon_i = 0$ and that $(\varepsilon_{hi}, \varepsilon_{wi})$ are independent of each other.\footnote{The assumption that $E \varepsilon_i = 0$ provides no loss in generality because any nonzero mean can be part of $\theta_i$. The independence assumption follows from the definition of $\varepsilon_i$ being unobserved by the partner.} Prior to any bargaining about
transfers,\textsuperscript{23} marital utilities are $u_{hi}^* = \theta_{hi} + \varepsilon_{hi}$ for the husband and $u_{wi}^* = \theta_{wi} + \varepsilon_{wi}$ for the wife, and the utilities perceived by the other spouse are

$$z_{hi}^+ = E u_{hi}^* = \theta_{hi}^*; z_{wi}^+ = E u_{wi}^* = \theta_{wi}^*.$$  

We observe a bracketed version of $(u_{hi}^*, u_{wi}^*, z_{hi}^*, z_{wi}^*)$, called $(u_{hi}, u_{wi}, z_{hi}, z_{wi})$, where, for example,

$$u_{hi} = k \quad \text{if} \quad t_{k}^u < u_{hi}^* < t_{k+1}^u, t_{k}^z \leq z_{hi}^* < t_{k+1}^z.$$  

The available data is thus $\{X_i, u_{hi}, u_{wi}, z_{hi}, z_{wi}\}_{i=1}^n$. The parameters to estimate are $\alpha = (\beta, \Omega, \theta)$ where we assume that

$$\theta_i \sim iidN(X_i, \beta, \Omega)$$  

and $\varepsilon_i \sim iidN(0, I)$.

The log likelihood contribution for observation $i$ consists of the probability of observing $(z_{hi}, z_{wi})$ and the probability of observing $(u_{hi}, u_{wi})$ conditional on $\theta_i$. The probability of observing $(z_{hi}, z_{wi})$ conditional on $X_i$ is

$$L_i (\Theta) = \log \int_{z_{hi}^*}^{z_{hi}^*+1} \int_{z_{wi}^*}^{z_{wi}^*+1} \prod_{m=h,w} \int_{t_{m,i}^u - \theta_{mi}}^{t_{m,i}^u + 1 - \theta_{mi}} d\Phi (\varepsilon_{mi}) \left[ \prod_{m=h,w} \int_{t_{m,i}^u - \theta_{mi}}^{t_{m,i}^u + 1} d\Phi (\varepsilon_{mi}) \right] dF_{\theta} (\theta_i \mid X_i)$$

\textsuperscript{23}While one might object to assuming that all data is observed prior to bargaining, as noted earlier, it is not clear otherwise how to interpret a spouse saying that his or her partner would be better off if separated after bargaining. The fact that a separation did not occur should tell the partner that his or her spouse is better off not separated; otherwise the spouse would have separated. As can be seen in Table 1A, 13.2 percent of wives’ reports are inconsistent with the interpretation that they reflect happiness after the husband’s offer of a side payment (because she would be saying that she still is happier outside of marriage and/or her husband perceives this), while a full 57.1 percent of husbands’ responses are inconsistent with the interpretation that they reflect his side payment. Furthermore, Proposition 7 implies that the wife has full information about her husband’s preferences once she observes the side payment, although the data reflect imperfect information.

\textsuperscript{24}Note that once we condition on $\theta_i$, it is not necessary to condition also on $X_i$.\textsuperscript{17}
\[ Z_t^{z+1} + Z_t^{z+1} \]

\[ X_i^{m+1} = X_i^{m+1} - X_i^{m+1} \]

\[ B(\cdot) \]

\[ \Omega_\theta \]

\[ \Theta = \{ \beta, \Omega_\theta, t \} \]

\[ L(\Theta) = \sum_{i=1}^n L_i(\Theta) \]

\[ e_k(\Theta) = \bar{e}_k - \text{Pr} [D_k^*(\varepsilon) = 1] \]

\[ \Omega_e^{-1} \]

\[ \lambda e(\Theta) \]

\[ (13) \]

\[ (12) \]

\[ (11) \]

\[ \text{Pr} \{ D_i(\varepsilon | X_i) = 1 \} \]

\[ L(\Theta) = \sum_{i=1}^n L_i(\Theta) - \lambda e(\Theta)' \Omega_e^{-1} e(\Theta) \]
of $L$ over $\Theta$ provides consistent estimates of $\Theta$ with asymptotic covariance matrix

$$A^{-1}BA^{-1};$$

$$A = \left[ \sum_i L_{i\Theta} - 2\lambda e_{\Theta}' \Omega^{-1} e_{\Theta} \right];$$

$$B = \left[ \sum_i \{L_{i\Theta} - L_{\Theta}\} \{L_{i\Theta} - L_{\Theta}\}' \right]$$

where $L_{\Theta} = \frac{1}{n} \sum_i L_{i\Theta}$. See Appendix 8.3 for more details.

### 4.2 NSFH Data

Of the 13008 households surveyed by the NSFH in 1987, we excluded 6131 households without a married couple, 4 without race information, 796 because the husband was younger than 20 or older than 65, and 1835 because at least one of the dependent variables was missing. This left a sample of 4242 married couples.

In the estimation, we use as explanatory variables $X$: age, race, and education level of the husband and differences in those characteristics between the husband and wife. Table 3 shows summary statistics for these variables. We present results later suggesting that additional covariates related to children are unnecessary. However, we do find evidence that some other variables such as religion, marital duration, and nonlinear age terms may belong in the model.

---

Table 3: Explanatory Variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Std Dev</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age</td>
<td>38.5</td>
<td>11.7</td>
<td>Age of husband (20-65)</td>
</tr>
<tr>
<td>White</td>
<td>0.82</td>
<td>0.38</td>
<td>Husband is white</td>
</tr>
<tr>
<td>Black</td>
<td>0.1</td>
<td>0.3</td>
<td>Husband is black</td>
</tr>
<tr>
<td>ΔRace</td>
<td>0.03</td>
<td>0.17</td>
<td>Spouses have different race</td>
</tr>
<tr>
<td>HS diploma</td>
<td>0.91</td>
<td>0.29</td>
<td>Husband has HS diploma</td>
</tr>
<tr>
<td>College degree</td>
<td>0.32</td>
<td>0.46</td>
<td>Husband has college degree</td>
</tr>
<tr>
<td>ΔEducation</td>
<td>0.75</td>
<td>0.43</td>
<td>Spouses have different education levels</td>
</tr>
</tbody>
</table>

Notes:
1. Sample size is 4242.
2. ΔRace is defined based on racial categories white, black, or other.
3. ΔEducation is defined based on educational categories no diploma, high school diploma, or college degree.

---

diagonal element. We began by somewhat arbitrarily setting $\lambda = 1000$. We then found that, for a wide range of values of $\lambda$, the NSFH data on outside options largely determines the values of all parameters.
4.3 Divorce Data

We incorporate divorce data from the marital history supplements of the Current Population Surveys. We use the June 1990 and June 1995 supplements to compute divorce probabilities for subsets of the population. The supplements surveyed all women aged 15 – 65 about the nature and timing of their marital transitions. From this data, we select a sample of women who were married as of the time period corresponding to Wave 1 of the NSFH (which ended in May 1988) and whose marriage did not end in widowhood. We then determine which women had divorced or separated within one year after that.

We use this sample from the CPS to compute divorce rates within demographic groups. Groups are defined by age in 1988 (18 – 27, 28 – 37, 38 – 47, 48 – 57), race (white, black), and educational attainment (did not complete high school, completed high school, attended college). The overall one-year divorce and separation rate for this group (married and aged 18 – 57 in 1987, either white or black, marriage did not end in widowhood) within one year was 2.4 percent. The divorce rate declines strongly with age and is somewhat higher for less educated and nonwhite women.

4.4 Estimation Results for the “No-Caring” Model

We estimated three versions of the model without caring preferences, each assuming that the characteristics listed in Table 4 have linear effects on observable utility $\theta$ from marriage. In the first version, explanatory variables are allowed to have distinct effects on $\theta_h$ and $\theta_w$; and, in the second and third, all the variables except the constant are restricted to have the same coefficient. Also, in the first and second, we exclude divorce information from the estimation objective function in equation (13); and, in the third we include divorce information.

For the most part, coefficient estimates from the unrestricted version excluding divorce information are either similar or insignificantly different across spouses, though the joint restrictions on the former are rejected with a $\chi^2$ likelihood ratio statistic of 17.2. The estimates show that people who are white get higher utility from marriage than people who are black or in the “other” racial group. Education increases the utility from marriage as well. The two variables measuring differences between husbands and wives, $\Delta$Race and $\Delta$Education, have insignificant effects on happiness.

---

27The first threshold is specified as $t_1 = -\exp\{\tau_1\}$, the second is set to zero, the two after are specified as $t_k = t_{k-1} + \exp\{\tau_k\}$, and the log likelihood is maximized over $\tau_k$. This ensures that the thresholds are increasing in k.

28The diagonal elements of $\Omega_\theta$ are specified as $\Omega_{ii} = \exp\{\omega_{ii}\}$, $i = 1, 2$, and the off-diagonal elements are specified as $\Omega_{12} = \Omega_{21} = \rho_{12}\sqrt{\Omega_{11}\Omega_{22}}$, where

$$\rho_{12} = \frac{2\exp\{\omega_{12}\}}{1 + \exp\{\omega_{12}\} - 1},$$

and the log likelihood is maximized over $(\omega_{11}, \omega_{22}, \omega_{12})$. This ensures that $\Omega_\theta$ is positive definite.
### Table 4: Estimation Results for Model Without Caring Preferences

<table>
<thead>
<tr>
<th>Variable</th>
<th>Excluding Divorce Information</th>
<th>Including Divorce Information (λ=10)</th>
<th>Including Divorce Information (λ=0.1)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Unrestricted</td>
<td>Restricted</td>
<td>Own, Spouse</td>
</tr>
<tr>
<td></td>
<td>Husband</td>
<td>Wife</td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>0.993 **</td>
<td>1.083 ***</td>
<td>1.068 **</td>
</tr>
<tr>
<td></td>
<td>(0.091)</td>
<td>(0.086)</td>
<td>(0.066)</td>
</tr>
<tr>
<td>Age/100</td>
<td>0.134</td>
<td>0.046</td>
<td>0.066</td>
</tr>
<tr>
<td></td>
<td>(0.059)</td>
<td>(0.036)</td>
<td>(0.015)</td>
</tr>
<tr>
<td>White</td>
<td>-0.139 **</td>
<td>-0.305 **</td>
<td>-0.273 **</td>
</tr>
<tr>
<td></td>
<td>(0.071)</td>
<td>(0.066)</td>
<td>(0.031)</td>
</tr>
<tr>
<td>Black</td>
<td>0.020</td>
<td>-0.163 **</td>
<td>0.009</td>
</tr>
<tr>
<td></td>
<td>(0.070)</td>
<td>(0.076)</td>
<td>(0.061)</td>
</tr>
<tr>
<td>HS Diploma</td>
<td>0.234 **</td>
<td>0.123 **</td>
<td>0.171 **</td>
</tr>
<tr>
<td></td>
<td>(0.050)</td>
<td>(0.038)</td>
<td>(0.053)</td>
</tr>
<tr>
<td>College Degree</td>
<td>0.033</td>
<td>0.038</td>
<td>0.003</td>
</tr>
<tr>
<td></td>
<td>(0.007)</td>
<td>(0.003)</td>
<td>(0.003)</td>
</tr>
<tr>
<td>ΔEducation</td>
<td>0.157 **</td>
<td>0.157 **</td>
<td>0.207 **</td>
</tr>
<tr>
<td></td>
<td>(0.007)</td>
<td>(0.007)</td>
<td>(0.007)</td>
</tr>
<tr>
<td>Threshold1</td>
<td>-1.577 **</td>
<td>-0.578 **</td>
<td>-0.873 **</td>
</tr>
<tr>
<td></td>
<td>(0.013)</td>
<td>(0.012)</td>
<td>(0.011)</td>
</tr>
<tr>
<td>Threshold2</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
</tr>
<tr>
<td>Threshold3</td>
<td>0.666 **</td>
<td>0.666 **</td>
<td>0.959 **</td>
</tr>
<tr>
<td></td>
<td>(0.011)</td>
<td>(0.011)</td>
<td>(0.017)</td>
</tr>
<tr>
<td>Threshold4</td>
<td>1.591 **</td>
<td>1.591 **</td>
<td>1.759 **</td>
</tr>
<tr>
<td></td>
<td>(0.011)</td>
<td>(0.011)</td>
<td>(0.017)</td>
</tr>
<tr>
<td>Var(β)</td>
<td>1.702 **</td>
<td>1.704 **</td>
<td>2.701 **</td>
</tr>
<tr>
<td></td>
<td>(0.007)</td>
<td>(0.007)</td>
<td>(0.007)</td>
</tr>
<tr>
<td>Corr(θ, R²)</td>
<td>0.242 **</td>
<td>0.240 **</td>
<td>0.871 **</td>
</tr>
<tr>
<td></td>
<td>(0.013)</td>
<td>(0.013)</td>
<td>(0.001)</td>
</tr>
<tr>
<td>Log Likelihood/ Objective Function</td>
<td>227.000</td>
<td>227.081</td>
<td>293.185</td>
</tr>
</tbody>
</table>

Notes:
1. Numbers in parentheses are asymptotic standard errors.
2. One asterisk indicates significance at the 10% level, and two asterisks indicate significance at the 5% level.
3. Variance terms are husband variance, followed by wife variance.
Also of interest is the correlation between $\theta_h$ and $\theta_w$. The estimated correlation in the first two columns is positive and substantial, at around 0.24. There are two features that may be reflected in ($\theta_h, \theta_w$). First, additional unobserved characteristics – for example, common religious beliefs – may affect the value of a marriage. If so, then omitting a measure of commonness would increase the variances of ($\theta_h, \theta_w$) and generate a positive correlation between them. Second, there may be unobserved variation in how families divide resources (à la Browning et al., 1994). Such variation also would increase the variances of ($\theta_h, \theta_w$) but would generate a negative correlation, since one spouse gains utility at the expense of the other. The estimated positive correlation suggests that the first type of variation is more important.

The last two columns of the table report the coefficient estimates when divorce rates by demographic group, computed using the CPS, are included in the estimation. The column with $\lambda = 10$ gives more weight to the divorce data, and the next column gives more weight to the happiness data. We do not have a way to test which is preferable, but we note that the coefficient values are not very stable across these specifications. In contrast, when we include the divorce data along with caring preferences in our final, and preferred set of estimates, we find that they are quite similar to the coefficient estimates that we analyzed from the first two columns, for reasons we will discuss later; therefore, we postpone further discussion of the estimates.

4.5 Interpretation of the No Caring Estimates

Using additional data from the NSFH to test the model’s out-of-sample predictive power confirms the model in some dimensions but also reveals some problems with the specification. First, we measure the correlation between predicted divorce probabilities and answers to the question, “What do you think are the chances that you and your partner will eventually separate?” Using the estimates from the restricted model excluding divorce data, Figure 6 shows that the correlations are low but follow expected patterns. The correlation between the predicted divorce probability and spouses’ pessimism is roughly monotonic. The correlation is negative when the husband or wife answers “very low,” and then it switches sign when the answer is instead “about even” or high. If we regress the predicted probability of divorce on dummy variables corresponding to each answer, almost all of the estimates are statistically significant. The coefficients increase from “very low” to “low” but then level off and show small declines from “low” to “very high.”

We also look at correlations between the total predicted side payment and one of its possible components, time spent on housework. Figure 7 shows the correlations between the predicted payment from husband to wife and the difference in hours per week spent by the husband and wife on various chores. Again, the correlations have the expected sign but are small. Almost all

---

29This supports results in Friedberg and Webb (2007) showing that shifts in bargaining power (as measured by relative wages) have little effect on the allocation of chores versus leisure time.
the correlations are positive (as the husband provides a larger side payment, he spends extra time on housework), and regressing the predicted side payment on the extra time spent by the husband yields results that are generally statistically significant.

Next, Table 5 reports predicted side payments and divorce probabilities based on the sets of estimates presented here as well as later on. We show means of these predictions as well as two measures of the variance. In the case of divorce probabilities, the first is the standard deviation across households of mean probabilities – i.e., it integrates over the distribution of unobservables that is implied by our estimates – so it captures the variation caused by observables. The second measure is based on draws of \((\theta_h, \theta_w, \varepsilon_h)\), which captures the variation within households caused by the unobservables and shows the variation of true divorce probabilities in the population.

The results for the model with no caring preferences and no divorce probabilities used in estimation are problematic. First, the mean divorce probabilities
of 0.321 are quite high. It might be explained by thinking about a long period of reference over which divorces occur, but only to the extent that current reports about happiness and hence the current bargaining situation persist just as long – and in fact, as we saw earlier, the divorce rate between Waves 1 and 2 (a period of roughly six years) is relatively low even for observably unhappy couples. The high divorce probability arises in the model because husbands adjust their offered side payments to wives to capture most of the marriage rents. Second and even more problematic are the predictions of a very small standard deviation across households in mean divorce probabilities and side payments. Lastly, there is very little variation in side payments and divorce probabilities generated by exogenous explanatory variables in our model, although those variables are in fact useful in predicting divorce. The lack of variation in divorce rates due to observed and unobserved factor occurs in the model without caring because husbands in good marriages (high s) drive harder bargains than those with weak marriages (mediocre s), thus reducing the variation in divorce rates relative to the variation in s.

Next, in the model without caring but with one-year divorce rates from the CPS, the divorce probabilities are pinned down to a much more realistic (and lower) level. But, the standard deviation across households (some of whom are observably quite happy and some of whom are not) remains extremely low. Thus, the bargaining implications of the model remain too strong for the model to fit the variation in divorce rates in the CPS.

One possible explanation for these difficulties is that we have omitted an important factor from consideration – for example, children. We use a Lagrange multiplier test to determine whether children (of any age or under age five) help explain the estimation residuals, but the results are not statistically signifi-
cant. When we compare the actual incidence of divorce minus the probability of divorce predicted from our model, a family with kids has only a slightly and insignificantly less negative difference than a family without kids. Consequently, we proceed without controlling for the presence of children.

5 A Bargaining Model with Caring Preferences

In the simple version of the model, we found that husbands drive too hard a bargain (and wives would as well, if they were making the take-it-or-leave-it offer in the model), resulting in high and relatively invariant predicted divorce probabilities. Therefore, we develop a model of caring preferences to assuage the hard bargaining. The divorce data that we incorporated earlier helps identify the extent to which caring preferences keep spouses from being too tough. Also, caring preferences allow divorce probabilities to differ reasonably across the sample, addressing the problem in the earlier estimates that divorce probabilities were similar for everyone, even if their reported happiness was quite different, because spouses were driving a hard bargain. Much of the literature on interdependent preferences assumes that individuals care about either the consumption of others (Becker’s 1974 rotten kid theorem), a gift to others (Hurd’s 1989 model of bequests) or a contribution to a public good (Andreoni 2005). We assume, instead, that individuals care directly about the utility of others, termed “caring preferences” (Browning et al., 1994). This choice is motivated by our data, which measures overall happiness rather than, say, expected consumption or income outside of marriage. To keep the model tractable, we further assume that reported happiness in marriage does not include how much one cares about the spouse’s happiness.

We define a “super-utility” function that depends on one’s own and partner’s marital utility. We allow for diminishing marginal utilities in one’s own and one’s spouse’s marital utility (so utility is not completely transferrable), and these features will be empirically identified using data on divorce rates. Suppose that individuals care not only about their direct utility from marriage \( U_k \) but also about their spouses’ utility \( U_{-k} \). The super-utility that the husband and wife get from marriage is \( V_h(U_h, U_w) \) and \( V_w(U_w, U_h) \) respectively, with partial derivatives on each function \( V_k \), with \( k = h, w \) and \( -k = w, h \), that obey

\[
V_{k1}(U_k, U_{-k}) \geq c > 0, \quad V_{k2}(U_k, U_{-k}) \geq 0; \quad (14)
\]
\[
V_{k11}(U_k, U_{-k}) \leq 0, \quad V_{k22}(U_k, U_{-k}) \leq 0. \quad (15)
\]

Conditions (14) and (15) allow for concavity in each argument. They also imply that, while spouses definitely care about themselves, they at least want no harm to come to the other. Also, we assume that \( \exists \bar{U} > 0 \):

\[
V_{k1}(U_k, U_{-k}) - V_{k2}(U_k, U_{-k}) \geq c > 0 \quad \forall (U_k, U_{-k}) : U_k < -\bar{U}, U_{-k} > \bar{U}. \quad (16)
\]

Condition (16) places an upper bound on the degree to which each spouse cares for the other, so a spouse prefers a greater share of marital resources if the
allocation favors the other spouse too much. Lastly, in the estimation we will require that
\[
V_{k11} (U_k, U_{-k}) \leq V_{k12} (U_k, U_{-k}) , V_{k22} (U_k, U_{-k}) \leq V_{k12} (U_k, U_{-k}).
\]

This last condition allows the cross-partial term to be positive or negative but bounds it from below with the own second partial derivatives. Spouses’ marginal value from their own utility either increases when the other spouse’s utility rises, or it decreases by less than it does when their own utility rises. Analogously, when the other spouse’s utility rises, the marginal value from their own utility increases or else decreases by less than does their marginal value from their spouse’s utility. These conditions together guarantee continuity in the optimal value of \(p\).

We assume further that \(V_h, V_w, f_h,\) and \(f_w\) satisfy a bounding condition:
\[
\max_{k=h,w} |V_k (U_k, U_{-k}) f_h (\varepsilon_h) f_w (\varepsilon_w)| < \infty.
\]

Equation (18) can be satisfied if, for example, second derivatives of \(V_h\) and \(V_w\) are non-positive and \(f_h\) and \(f_w\) have finite moments.

With partial information, the husband knows \(f_w (\varepsilon_w)\) rather than \(\varepsilon_w\). The husband makes an offer \(p\) to maximize his super-utility function, given the likelihood of remaining married:
\[
V^*_h (\varepsilon_h, p) = \frac{\int_{\varepsilon_w} V_w^* (\varepsilon_w, p) \geq 0 V_h (\theta_h - p + \varepsilon_h, \theta_w + p + \varepsilon_w) f_w (\varepsilon_w) d\varepsilon_w}{\int_{\varepsilon_w} V^*_w (\varepsilon_w, p) \geq 0 f_w (\varepsilon_w) d\varepsilon_w},
\]
\[(19)\]

The wife’s super-utility function, conditional on remaining married with an offer of \(p\), is
\[
V^*_w (\varepsilon_w, p) = \frac{\int_{\varepsilon_h} V_h^* (\varepsilon_h, p) \geq 0 V_w (\theta_w + p + \varepsilon_w, \theta_h - p + \varepsilon_h) f_h (\varepsilon_h | p) d\varepsilon_h}{\int_{\varepsilon_h} V^*_h (\varepsilon_h, p) \geq 0 f_h (\varepsilon_h | p) d\varepsilon_h}.
\]
\[(20)\]

If \(V^*_h (\varepsilon_h) < 0\) or \(V^*_w (\varepsilon_w) < 0\), then there is no agreement, and divorce occurs. Otherwise, the marriage continues with side payment \(p\). Note that the wife conditions her belief about \(\varepsilon_h\) on the husband’s offer \(p\).

The husband chooses \(p\) to maximize his expected utility, so \(p^*\) satisfies
\[
p^* (\varepsilon_h) = \arg \max_p V^*_h (\varepsilon_h, p) \Pr [V^*_w (\varepsilon_w, p) \geq 0].
\]

We now discuss the equilibrium of this bargaining game.

**Proposition 1** ∃ an equilibrium with the following properties:

1) \((\text{monotonicity}) \quad \frac{\partial V^*_h (\varepsilon_h, p)}{\partial \varepsilon_w} > c > 0 \quad \text{and} \quad \frac{\partial V^*_w (\varepsilon_w, p)}{\partial \varepsilon_h} > c > 0;\)
2) \((\text{reservation values}) \quad \exists \varepsilon^*_h (p) : V^*_h (\varepsilon^*_h, p) > 0 \forall \varepsilon_h > \varepsilon^*_h (p) \quad \text{and} \quad V^*_h (\varepsilon_h, p) < 0 \forall \varepsilon_h < \varepsilon^*_h (p), \quad \text{and} \quad \exists \varepsilon^*_w (p) : V^*_w (\varepsilon_w, p) > 0 \forall \varepsilon_w > \varepsilon^*_w (p) \quad \text{and} \quad V^*_w (\varepsilon_w, p) < 0 \forall \varepsilon_w < \varepsilon^*_w (p);\)
Proposition 2 (husband’s monotonicity) If condition (2) is satisfied, then \( \frac{\partial V^*_w(\varepsilon_h, p)}{\partial \varepsilon_h} \geq c > 0 \).

Proposition 3 (husband’s reservation values) If condition (2) is satisfied, then \( \exists \varepsilon^*_h(p) : V^*_h(\varepsilon_h, p) > 0 \ \forall \varepsilon_h > \varepsilon^*_h(p) \) and \( V^*_w(\varepsilon_w, p) < 0 \ \forall \varepsilon_w < \varepsilon^*_w(p) \).

Proposition 4 (effect of \( p \) on husband’s reservation values) If condition (2) is satisfied, then \( \frac{\partial \varepsilon^*_h(p)}{\partial p} > 0 \).

To elaborate on what we established in Propositions 2 through 4, the husband chooses an offer

\[
p^*(\varepsilon_h) = \arg\max_p V^*_h(\varepsilon_h)[1 - F_w(\varepsilon^*_w(p))] \tag{21}
\]

\[
= \frac{\partial V^*_h(\varepsilon_h)}{\partial p} [1 - F_w(\varepsilon^*_w(p))] - V^*_h(\varepsilon_h) f_w(\varepsilon^*_w(p)) \frac{\partial \varepsilon^*_w(p)}{\partial p} = 0.
\]
The second order condition (SOC) for the husband’s optimization problem can be written as

\[
\frac{\partial^2 V_h^*}{\partial p^2} - \left( \frac{\partial V_h^*}{\partial p} \right)^2 + \frac{\partial}{\partial w} f_w (\varepsilon_w^*(p)) \frac{\partial \varepsilon_w^*(p)}{\partial p}.
\]  

(22)

Sufficient conditions for the SOC to be negative everywhere are that (a) \(\frac{\partial^2 V_h^* (\varepsilon_h)}{\partial p^2} < 0\) (the first term is negative), (b) \(- \left( \frac{\partial V_h^*}{\partial p} \right)^2\) is negative, which is obvious, (c) \(f_w (\cdot) / [1 - F_w (\cdot)]\) is increasing in its argument (the first part of the third term is positive), and (d) \(\frac{\partial \varepsilon_w^*(p)}{\partial p} < 0\) (the second part of the third term is negative). Condition (c) is a common assumption made in the literature and is equivalent to assuming that \(F_w\) satisfies the monotone likelihood ratio property (Milgrom, 1981a). It is satisfied by many distributions including the normal, exponential, chi-square, uniform, and Poisson (Milgrom, 1981b). Condition (d) can be assumed and later shown to be consistent with equilibrium. However, condition (a) is problematic. In particular, while it is reasonable to assume that \(\frac{\partial^2 V_h}{\partial p^2} < 0\), this is not equivalent to condition (a). While we have not been able to produce a minimal sufficient set of conditions to imply that equation (22) is satisfied everywhere, we still can prove it is satisfied at the place where the husband chooses the optimal \(p\) and, therefore, at one place at least where equation (21) is solved. We can now demonstrate the properties of the second order condition and several of the properties from Proposition 1 for the wife.

**Proposition 5** (second order condition) Conditional on \((\theta_h, \theta_w, \varepsilon_h)\), if \(\exists p : V_h^* > 0\), then \(\exists p^* : \text{equation (21)}\) and equation (22) both are satisfied.

**Proposition 6** (comparative statics for optimal offer) If condition (2) is satisfied, then \(\frac{\partial p^* (\varepsilon_h)}{\partial \theta_h} > 0\), \(\frac{\partial p^* (\varepsilon_h)}{\partial \theta_w} < 0\), and \(\frac{\partial p^* (\varepsilon_h)}{\partial \varepsilon_h} > 0\).

**Proposition 7** (information in \(p\)) \(p^* (\varepsilon_h) \Rightarrow \varepsilon_h\).

**Proposition 8** (wife’s monotonicity) If \(p^* (\varepsilon_h) \Rightarrow \varepsilon_h\), then \(\frac{\partial V_w^* (\varepsilon_w, p)}{\partial \varepsilon_w} > 0\).

**Proposition 9** (reservation values) \(\exists \varepsilon_w^* (p) : V_w^* (\varepsilon_w, p) > 0\) \(\forall \varepsilon_w > \varepsilon_w^* (p)\) and \(V_w^* (\varepsilon_w, p) < 0\) \(\forall \varepsilon_w < \varepsilon_w^* (p)\).

**Proposition 10** (effect of \(p\) on reservation values) \(\frac{d \varepsilon_w^*(p)}{dp} < 0\).

We are now ready to apply a Schauder fixed point theorem to establish the existence of an equilibrium.

**Proposition 11** Given (exogenous) \(V_h, V_w, \text{and } F_{\varepsilon} (= F_h F_w)\), \(\exists\) an equilibrium characterized by an optimal side payment rule for the husband \(p^* (\varepsilon_h)\) and an optimal reservation value for the wife \(\varepsilon_w^* (p)\). These two together define expected value functions for the husband and wife, \(V_h^* (\varepsilon_w^*, p)\) and \(V_w^* (\varepsilon_w^*, p)\).
To wrap up, we will mention some comparative statics of the equilibrium. We can prove that the probability of divorce falls with each spouse’s observable and unobservable happiness.

**Proposition 12** *(comparative statics for divorce probabilities)* \(\exists\) an equilibrium with

\[
\begin{align*}
\frac{\partial}{\partial \theta_h} \Pr [V^*_w (\varepsilon_w, p) \geq 0] &> 0, \\
\frac{\partial}{\partial \theta_w} \Pr [V^*_w (\varepsilon_w, p) \geq 0] &> 0, \\
\frac{\partial}{\partial \varepsilon_h} \Pr [V^*_w (\varepsilon_w, p) \geq 0 \mid \varepsilon_h] &> 0.
\end{align*}
\]

6 Estimation of the Caring Preferences Model

6.1 Estimation Methodology

In order to estimate parameters related to caring preferences, we need to specify the functions \(V_h(U_h, U_w)\) and \(V_w(U_w, U_h)\) that indicate the total value of marriage. Each should be an increasing concave function with cross-partial derivatives that limit the extent to which individual \(i\) is either selfish (getting much more utility from \(U_i\) than \(U_j\)) or selfless (vice versa).

We simplify notation below by referring to \(V\) instead of \(V_h\) or \(V_w\) and specify \(V\) as a polynomial function such that

\[
V(U_1, U_2) = \sum_{j=0}^{2-j} \sum_{k=0}^{2} \phi_{jk} U_1^j U_2^k
\]

over the domain

\[
b_{11} \leq U_1 \leq b_{12}, \quad b_{21} \leq U_2 \leq b_{22}
\]

with normalizations

\[
\phi_{00} = 0, \quad \phi_{10} = 1.
\]

The higher order terms – \(\phi_{11}, \phi_{20}, \phi_{02}\) – allow for limited transferability of utility in the form of changing marginal values resulting from one’s own or spouse’s marital surplus. Appendix 8.2 provides details on how to constrain the \(\phi\) coefficients in order to satisfy the restrictions required in Section 3.2. We allow \(\phi_{01}\) to vary across families with age of the husband and specify

\[
\phi_{01i} = \phi_{01} \exp \{\phi_{age} z_i^{age}\}
\]

\[\text{We considered less parametric specifications of } V \text{ using ideas in Gallant (1981,1982), Gallant and Golub (1984), Liu, Mroz, and Van der Klaauw (2010), Matzkin (1991), Mukarjee and Stern (1994), Stern (1996), and Engers, Hartmann, and Stern (2006). Each of these failed because they did not impose enough structure on } V \text{ to ensure that it behaved well.}\]
where \( x_{i}^{age} \) is the age of the husband.\(^{31}\)

Finally, since there is nothing in the model that allows us to specify the length of a period over which divorces predicted in the model might occur, we add one more parameter \( \tau \) that maps real-world time periods into model time periods. In particular, let \( \Psi_{rw} \) be the one-period probability of divorce in the real world and \( \Psi_{m} \) be the one-period probability of divorce in the model. Then we define \( \tau \) by

\[
1 - \Psi_{rw} = (1 - \Psi_{m})^{\tau},
\]

and \( \tau \) is identified by the ratio of the real-world marriage survival probability to the model marriage survival probability.

To estimate the model with caring preferences, we use the same methodology described in Section 4.1. We change the set of parameters to estimate to \( \Theta = \{ \beta, \Omega_\theta, t, \phi, \tau \} \) where \( \beta \) is the vector of coefficients on demographic terms \( X \) that affect observable happiness \( \theta \) in equation (8), \( \Omega_\theta \) is the covariance matrix of \( \theta \) in equation (10), \( t \) is the vector of threshold values dealing with the discreteness of reported \( \theta \) in equation (9), \( \phi \) is the vector of caring terms in equation (23), and \( \tau \) measures the time period length (equation (26)). Inclusion of \( \phi \) in \( \Theta \) changes the identification approach. In Section 4.1, divorce data were unnecessary for identification. In this section, the divorce data identify \( (\phi, \tau) \), while \( \{ \beta, \Omega_\theta, t \} \) are identified by the covariation and second moments of the happiness data in the NSFH. Estimates of \( \{ \beta, \Omega_\theta, t \} \) in the model without caring imply divorce probabilities. The \( (\phi, \tau) \) parameters involving caring are identified by the degree to which empirical divorce proportions by demographic groups differ from what is predicted by the model without caring.

### 6.2 Estimation Results for the Caring Model

Table 6 presents estimates from the caring model with divorce data. As in the no-caring model with divorce data, we restrict covariates to have the same effect on both spouses’ happiness \( \theta \), as we found no major differences in our Table 4 estimates.

The model with caring preferences fits the data better than the model without caring as the objective function is considerably greater. Many of the parameter estimates are quite similar to those from the basic model with no caring and no divorce data; as we noted earlier, adding divorce data without caring preferences in Table 4 changed the estimates substantially. Referring back to the identification argument we just made, this occurs because once we add divorce data in the earlier model, then the demographic terms \( X \) have to explain both happiness and divorce patterns; but, in the full version here, \( X \) variables must explain only happiness data, because caring terms explain divorce data. So, as before, we find that white couples and more educated couples have greater happiness from marriage, and the covariance in spouse’s happiness conditional on covariates is quite positive as well. Meanwhile, the coefficient on age has

\(^{31}\)We allow \( \phi_{01} \) to vary with age because some deeper analysis of the generalized divorce residuals (Gourieroux et al., 1987) suggest important age effects.
Table 6: Estimation Results with Divorce Data

<table>
<thead>
<tr>
<th>Explanatory Variables</th>
<th>Thresholds and Covariance Parameters</th>
<th>Preference Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variable</td>
<td>Estimate</td>
<td>Variable</td>
</tr>
<tr>
<td>Own Constant</td>
<td>1.088 **</td>
<td>( t_1 )</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.001)</td>
</tr>
<tr>
<td>Spouse constant</td>
<td>1.049 **</td>
<td>( t_2 )</td>
</tr>
<tr>
<td></td>
<td>(0.002)</td>
<td>(0.001)</td>
</tr>
<tr>
<td>Age/100</td>
<td>0.025 **</td>
<td>( t_3 )</td>
</tr>
<tr>
<td></td>
<td>(0.007)</td>
<td>(0.001)</td>
</tr>
<tr>
<td>White</td>
<td>0.213 **</td>
<td>( \text{Var}(B_h) )</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>(0.002)</td>
</tr>
<tr>
<td>Black</td>
<td>-0.264 **</td>
<td>( \text{Var}(B_w) )</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>(0.000)</td>
</tr>
<tr>
<td>ΔRace</td>
<td>-0.092 **</td>
<td>( \text{Corr}(B_h,B_w) )</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>(0.001)</td>
</tr>
<tr>
<td>HS diploma</td>
<td>0.052 **</td>
<td>( \phi_{10} )</td>
</tr>
<tr>
<td>College degree</td>
<td>0.171 **</td>
<td>( \phi_{01} )</td>
</tr>
<tr>
<td>ΔEducation</td>
<td>( \phi_{01} )</td>
<td>( \phi_{02} )</td>
</tr>
<tr>
<td>Objective fcn</td>
<td>-22747.4</td>
<td>( \phi_{01} )</td>
</tr>
</tbody>
</table>

Notes:
1. Numbers in parentheses are asymptotic standard errors.
2. One asterisk indicates significance at the 10% level, and two asterisks indicate significance at the 5% level.
3. See additional notes from Table 4.

fallen in Table 6, compared to Table 4, because we now let caring preferences vary with age.

Most important are the estimates of the degree of caring, represented by the \( \phi \) terms. The \( V(\cdot) \) functions are denominated in the same units as the \( U_i \) which tend to range between \((-2, 6)\). The first derivative of the value of marriage \( V(U_1, U_2) \) with respect to one’s own direct utility \( U_1 \) is governed by \( \phi_{10} \), which is normalized to 1, while the second derivative equals \( 2 \phi_{20} \); the estimate of \(-0.0071 \) for \( \phi_{20} \) indicates that the value of marriage declines extremely slowly in one’s direct utility. The derivatives of \( V(U_1, U_2) \) with respect to the spouse’s utility \( U_2 \) depend similarly on \( \phi_{01} \), \( \phi_{02} \), and \( \phi_{03} \); the parameter estimates of \( 2.445, -0.030 \), and \(-0.237 \), respectively, are statistically significant and imply that one cares for the spouse but at a somewhat declining rate in both the spouse’s utility and with age. Lastly, the estimated cross-partial term \( \phi_{11} \) is \( 0.0014 \), so the marginal value of own utility rises very slightly as spouse’s utility rises, and vice versa. The results imply very mild limits on the transferability of utility.

Next, we graph indifference curves in \( U_1 \) and \( U_2 \), based on the estimated \( \phi \) terms. Each curve in Figure 8 represents a value \( V_h \) to the husband from marriage, ranging from \(-1 \) to \( 3 \), with negative values indicating that he prefers divorce. By assumption, the wife’s indifference curves are the same. When
each spouse has a value of marriage of 2 (corresponding approximately to both being very happy in marriage) and both spouses are 25, the indifference curves are flatter, so one spouse requires quite a bit of extra utility from marriage if the other spouse receives less utility to stay on the same indifference curve. When both spouses are 50, the indifference curves are steeper, so one requires less extra utility for oneself if the spouse’s utility from marriage falls. The indifference curve is a little flatter when each spouse has a value of 1, rather than 2, at age 25. By our normalization, super-utility is 0 when both spouses have a value of 0, and the indifference curve at 0 has a steeper slope at age 50 than at age 25.

6.3 Interpretation of the “Caring” Estimates

6.3.1 Predicted Side Payments and Divorce Probabilities

In order to show how caring preferences and asymmetric information affect couples in our sample, we begin by graphing the smoothed estimated joint density of \((\theta_h, \theta_w)\), the publicly observable happiness of each spouse, in Figure 9. Using bin sizes of 0.5, the median values of \((\theta_h, \theta_w)\) are (2, 2). 15 percent of couples lie within 0.5 of (2, 2), and 31 percent lie within 1.0. Interestingly, for 36 percent of couples, one partner has \(\theta \leq 0\) and the other has \(\theta > 0\). It is those couples that would be most likely to divorce if no bargaining took place, making side payments crucial to those marriages.

Table 5 from Section 4.5 shows the average predicted divorce probability and side payment from the caring preferences model, while Figures 10 and 11 below show how they vary with values of \((\theta_h, \theta_w)\). The predicted mean divorce probability in Table 5 drops a great deal when allowing for caring preferences, from 0.321 in the model without divorce data to 0.141. Our estimated value of \(\tau\) of 0.07 suggests a time period over which these divorces are predicted to

\[\text{Figure 8: Indifference Curves}\]
occur of 14 years. This model fits the actual one-year divorce rate of 2.4 percent observed in the CPS quite well. Thus, caring preferences offset the inefficient bargaining otherwise generated by asymmetric information.

Moreover, predicted divorce probabilities now vary reasonably across households and if unobservables are varied within households. For households where the husband is 25 years old, in Figure 10 when $\theta_h = 1$ (so the husband is perceived as being somewhat happy in the marriage), the divorce probability is 18.1 percent (over $\tau^{-1} = 14.1$ years) if $\theta_w = 0$, and it falls to 6.2 percent when $\theta_w$ reaches 1 and 3.3 percent when $\theta_w$ reaches 2. When $\theta_h = 2.5$ (and so the husband is perceived as being quite happy), the divorce probability is 4.2 percent when $\theta_w$ is 0, and 2.9 percent when $\theta_w$ reaches 1. For households where the husband is 50 years old, predicted divorce probabilities are generally higher after conditioning on $(\theta_h, \theta_w)$.

Predicted side payments in Table 5 have a similar mean but more variation across households and are extremely sensitive to variation in the value of unobservables within households. Meanwhile, for a household with a 25 year old husband, in Figure 11, when $\theta_w = 1$, the side payment from the husband to the wife takes a value of about $-0.723$ for $\theta_h = 1$, while it takes a value of $-0.639$ when the household has a 50 year old husband.

6.3.2 Specification Tests

Next, we undertake a number of specification tests of the model. We first test to see whether it matters whether the husband or wife makes the side payment offer. When the wife makes the offer instead, the $\phi$ parameter estimates change by trivial amounts.\(^{33}\) Only the coefficients on White (0.599 $\rightarrow$ 0.854), College Degree ($-0.238 \rightarrow -0.442$), and $\Delta$Education (0.599 $\rightarrow$ 0.854) somewhat

\(^{33}\)The shifts were $\phi_{01}$ 1.192 $\rightarrow$ 1.18, $\phi_{02}$ -0.113 $\rightarrow$ -0.112, $\phi_{11}$ 0.00014 $\rightarrow$ 0.00022, $\phi_{20}$ -0.0009 $\rightarrow$ -0.0009.
Figure 10: Estimated Divorce Probabilities

Figure 11: Estimated Side Payments
These results are unsurprising, since the distributions of husbands’ and wives’ reported happiness from Table 1 and Figure 9 are quite similar. Similarly, indifference curves are almost exactly the same.

We also construct Lagrange Multiplier tests to evaluate whether omitted variables are systematically related to reported happiness or divorce. We find that religion, higher-order polynomials in age (a quadratic and cubic in husband’s age), and marriage duration significantly influence observable happiness but have much smaller direct effects on divorce (after controlling for happiness). We also find, as we did earlier, that children do not help explain reported happiness. Overall, the fact that this set of variables influences divorce mostly through reported happiness suggests that the model is not missing something important about determinants of divorce outside of the happiness variables (which we interpret to be inclusive of divorce costs) and the bargaining process. The effects of religion are somewhat surprising. We find that measures of religious intensity— if the husband is Catholic or Protestant (with other categories omitted), or if the husband reports having fundamentalist beliefs, or if both spouses have the same religion— have small but statistically significant negative effects on marital happiness, although Lehrer (2004) finds that religious intensity reduces the likelihood of divorce. The effect of marriage duration is also unexpected. The implications of the theory of investment in relationship-specific capital (Becker 1991) imply that duration should increase happiness and maybe also have a direct negative effect on divorce probabilities. We find a statistically significant negative LM statistic, implying that the effect of duration on observed marital surplus $\theta$ is negative; Brien, Lillard, and Stern (2006) found a similar result, and the raw data suggest, in particular, that husband’s reported happiness is declining in marital duration.\(^\text{35}\)

We construct another set of tests to determine whether some omitted variables affected the variance of private information $\sigma^2$ about happiness, which we assumed to be one in the model above. We define $\sigma^2_{z_i} = \exp(\psi_0 + \psi_1 z_i)$, and use a Lagrange Multiplier statistic to test whether a variable $z_i$ influences the variance $\sigma^2_{z_i}$, so $H_0 : \psi_1 = 0$ and $H_A : \psi_1 \neq 0$.\(^\text{36}\) We first try this test with $z_i$ as the duration of the marriage. As a couple gains experience, they may learn more about each other and $\sigma^2_{z_i}$ might fall, so $\psi_1 < 0$. We also try the test with $z_i$ as a dummy for whether the couple has a child under age 1, in which case they may be learning to deal with a new environment and $\sigma^2_{z_i}$ might rise.

\(^{34}\) Using the standard errors of the estimates from Table 6 (rather than the standard error of the difference of the estimates), we find that three of the changes are statistically significant. This use of standard errors probably biases $t$-statistics downwards because covariances of estimates would probably be negative. However, the point remains that only three estimates changed by any substantive amount and there were no significant changes in sign.

\(^{35}\) We felt uncomfortable including duration directly in our model because of endogeneity issues associated with pre-sample divorce. Controlling for this selection bias would strengthen these results.

\(^{36}\) The Lagrange-Multiplier statistic uses only the Wave 1 happiness data to avoid the problem that the penalty function associated with the divorce data is not part of the log likelihood function. We would like to thank participants of the Applied Micro Workshop at UCLA for suggesting these tests to us.
so $\psi_1 > 0$. In each case, we test $H_0 : \psi_1 = 0$ against $H_A : \psi_1 \neq 0$. For marriage duration, the $t$-statistic on $\psi_1$ is $-52.0$, implying that we should reject the null in favor of the alternative that, as marriage duration increases, the couple learns more about each other, and $\sigma^2_{\varepsilon_1}$ decreases.\(^{37}\) For the new child effect, the $t$-statistic is $7.03$, implying (at a 5% significance level) that we should reject the null in favor of the alternative that, when there is a new child, the couple needs to renegotiate under new conditions, and $\sigma^2_{\varepsilon_2}$ increases.\(^{38}\) Lastly, we construct Lagrange Multiplier tests and Wald tests of the effects of a few other variables on happiness and on divorce probabilities.

We then use the generalized residuals from the estimated model to test some aspects of our initial formulation of the bargaining problem.\(^{39}\) Recall that we treat the happiness responses as reflecting utility before side payments and that we assume a take-it-or-leave-it offer by one spouse to the other. One may be concerned that spouses’ answers about happiness are instead inclusive of the side payment, though, as we noted earlier, this raises a puzzle of why these couples are married (and remain married, on average, five years later in Wave 2). If the latter interpretation were true, then certain changes in reported happiness would be linked functionally, since $U_h = \theta_h + \varepsilon_h - p$, $U_w = \theta_w + \varepsilon_w + p$, and $p \equiv p(\theta_h + \varepsilon_h, \theta_w)$, and we can test whether these changes are observed in our data. Note that our assumption that the husband makes a take-it-or-leave-it offer is embodied in the definition of $p$, so this involves a joint test.

Recall the notation that $z_{ij}^* = \theta_{ij}$ is the latent value corresponding to a spouse’s answer about his partner’s happiness in couple $i$, and that $u_{ij}^* = \theta_{ij} + \varepsilon_{ij}$ is the latent value corresponding to spouse $j$’s bracketed answer about his own happiness. Then, reflecting our assumption that the answers to do not include the side payment $p$, $\theta_{ij} = E[u_{ij}^*]$, and, consequently,

$$\frac{\partial u_{ij}^*}{\partial z_{ik}^*} |_{z_{ij}^*} = \frac{\partial(\theta_{ij} + \varepsilon_{ij})}{\partial \theta_{ik}} |_{\theta_{ij} = 0}$$

for $k \neq j$. In other words, if a husband’s answer about his wife’s observable happiness changes, then his answer about his own happiness would not change, conditional on his wife’s answer about his happiness, because the private component of his happiness $\varepsilon_{ij}$ has not been revealed. On the other hand, if $z^*$

\(^{37}\)Of course, a model of learning with dynamic bargaining would be considerably more complicated and would also induce dynamic selection in which couples remain married. Another possible explanation for this effect, similar to results in Bowlsus and Seitz (2006), is that there may be unobserved heterogeneity in $\sigma^2_{\varepsilon_2}$. The couples with high values of $\sigma^2_{\varepsilon_2}$ are more likely to divorce, leading to the average value of $\sigma^2_{\varepsilon_2}$ declining with duration.

\(^{38}\)To check that the true effect of a new child was not on the mean of $\theta$, we also constructed a Lagrange Multiplier test associated with adding the new child dummy to $X_h \beta_h$ and $X_w \beta_w$ in equation (8). The $t$-statistic was $1.28$, implying that some of the effect may be directly on $\theta$.

\(^{39}\)This test was inspired by a discussion one of the authors had with Guillermo Caruana, Stephane Bonhomme, and Pedro Mira at CEMFI.
and $u^*$ include $p$, then

$$\frac{\partial z_{iw}^*}{\partial u_{ih}^*} \mid z_{iw}^* = \frac{\partial (\theta_{iw} + p_i)}{\partial (\theta_{iw} + \varepsilon_{iw} - p_i)} \mid (\theta_{ih} - p_i) > 0; \tag{28}$$

$$\frac{\partial z_{ih}^*}{\partial u_{iw}^*} \mid z_{ih}^* = \frac{\partial (\theta_{ih} - p_i)}{\partial (\theta_{iw} + \varepsilon_{iw} + p_i)} \mid (\theta_{iw} + p_i) = 0.$$

In this case, as the husband’s answer about himself changes, then conditional on his wife’s answer about him, his answer about his wife would increase, reflecting the greater side payment he would be making. The converse differs, however, as reflected in the second statement; changing the wife’s answer about herself does not alter her answer about her husband, conditional on her husband’s answer about her, because the husband is the first mover, making the offer of $p$ without direct knowledge of $\varepsilon_{iw}$.

The conditions in equation (27) versus those in equation (28) can be tested by first computing partial correlations of the generalized residuals of the dependent variables (Gourieroux et al., 1987) and then using the estimated average partial derivative described in Powell, Stock and Stoker (1989):

$$\hat{R} = - \frac{\sum_i y_i \sum_j \frac{\partial K(x_j - x_i)}{\partial x_{ik}}}{\sum_i \sum_j K(x_j - x_i)}$$

where $(y_i, x_i)$ is the vector of dependent variables and explanatory variables corresponding to the null hypotheses and $K(\cdot)$ is a bivariate kernel function. For example, to estimate the average partial derivative implied by equation (27) for $j = h$, we set $x_{i1}$ equal to $z_{iw}^*$ and $x_{i2}$ equal to $z_{ih}^*$.

The set of dependent variables and explanatory variables for each test is listed in the first columns of Table 7. The first two rows show tests of the condition in equation (27), indicating that changes in some happiness reports should not alter other reports because they do not include the side payment, according to our assumptions about the happiness reports; the alternative hypothesis is that the derivatives are non-zero. The second two rows show tests of the condition in (28), indicating that changes in happiness reports should be correlated through the side payment, according to the alternative interpretation of the happiness reports which we have mentioned here; now, the null hypothesis is that they are not correlated, and hence the average derivatives are zero, and the alternative is that they are positive.\(^{10}\)

\(^{10}\)While the average derivative in the second condition in equation (28) should equal zero when side payments are included in the answers, plausible alternatives would lead to this derivative being positive, as it indicates that a higher value of the wife’s unobservable happiness leads the husband to make (or her to bargain for) a higher offer of $p$.\[^{37}\]
Table 7: Specification Test Results

<table>
<thead>
<tr>
<th>Avg Derivative</th>
<th>$y_i$</th>
<th>$x_{i1}$</th>
<th>$x_{i2}$</th>
<th>$H_0$</th>
<th>$H_A$</th>
<th>Estimate</th>
<th>Std. Err.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{\partial u_{ih}}{\partial z_{ih}}</td>
<td>z_{ih}^*$</td>
<td>$u_{ih}^*$</td>
<td>$z_{iw}^*$</td>
<td>$z_{ih}^*$ = 0</td>
<td>$\neq$ 0</td>
<td>0.093</td>
<td>0.299</td>
</tr>
<tr>
<td>$\frac{\partial u_{iw}}{\partial z_{iw}}</td>
<td>z_{iw}^*$</td>
<td>$u_{iw}^*$</td>
<td>$z_{ih}^*$</td>
<td>$z_{iw}^*$ = 0</td>
<td>$\neq$ 0</td>
<td>0.011</td>
<td>0.143</td>
</tr>
<tr>
<td>$\frac{\partial z_{ih}}{\partial z_{iw}}</td>
<td>z_{ih}^*$</td>
<td>$z_{iw}^*$</td>
<td>$u_{ih}^*$</td>
<td>$z_{ih}^*$ = 0</td>
<td>$&gt; 0$</td>
<td>-0.598**</td>
<td>0.291</td>
</tr>
<tr>
<td>$\frac{\partial z_{ih}}{\partial z_{iw}}</td>
<td>z_{iw}^*$</td>
<td>$z_{ih}^*$</td>
<td>$u_{iw}^*$</td>
<td>$z_{iw}^*$ = 0</td>
<td>$&gt; 0$</td>
<td>-0.832*</td>
<td>0.458</td>
</tr>
</tbody>
</table>

The specification tests are reported in the final columns of Table 7. The results for the first two estimated average derivatives provide strong support for assuming that $\frac{\partial u_{ih}}{\partial z_{ih}} | z_{ih}^* = 0$, as we did originally. These partial derivatives are not statistically different from zero, and the point estimates are in fact quite close to zero. The last two estimated average derivatives are a bit more puzzling. These are one-sided tests because only positive derivatives are predicted under the alternative. In fact, both are statistically significant (student $t = -1.82, 2.05$). But, both are negative, thus not rejecting $H_0$ and implying that survey responses are made prior to the side payment $p$. It is not clear how to interpret negative estimates, i.e., what model of bargaining and assumption about response timing would result in negative estimates.

6.4 Policy Analysis

We finish by considering two types of policy analysis. First, we consider the case of a social planner who evaluates marriages on a case by case basis, and we compute welfare under different information scenarios. After that, we consider the much simpler policy of altering the cost of divorce $C$.

6.4.1 Impact of a Social Planner

It turns out that couples on their own, even with their limited information, do almost as well as a social planner with perfect information. In contrast, a social planner with limited information does considerably worse, as evaluated in terms of in average welfare and average divorce probabilities. Average divorce probabilities are shown in Figure 12 as a function of the husband’s information $(\theta_h + \theta_w + \varepsilon_h)$ and for different caring and planner scenarios. Using the caring estimates from Table 6, we consider four cases:

1. a couple has asymmetric information and cares for each other;

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2. an omniscient planner, knowing \((\theta_h + \varepsilon_h, \theta_w + \varepsilon_w)\), maximizes \(V_h(U_h, U_w) + V_w(U_w, U_h)\), the sum of welfare with caring preferences, over choices of \(p\);

3. a limited planner, knowing only \((\theta_h, \theta_w)\), maximizes the sum of welfare with caring preferences, over choices of \(p\), as follows:

\[
\int_{(\varepsilon_h, \varepsilon_w): V_h(U_h, U_w) \geq 0, V_w(U_w, U_h) \geq 0} [V_h(U_h, U_w) + V_w(U_w, U_h)] dF(\varepsilon_h) dF(\varepsilon_w),
\]

4. a “Becker” planner, knowing \((\theta_h + \varepsilon_h, \theta_w + \varepsilon_w)\), picks \(p\) so that a divorce occurs iff \(U_h + U_w < 0\).

Figure 12 reveals several interesting features. First, an omniscient planner with caring and a “Becker” planner yield identical divorce probabilities. This occurs because each of these planners wants to keep marriages intact if and only if the “Becker” condition is satisfied. Second, it is noteworthy that caring couples with limited information perform significantly better than the limited social planner. In Figure 12, the “no planner” curve is relatively closer to the “omniscient planner” curve, but the “limited planner” curve is usually farther away. In particular, the limit planner makes frequent mistakes keeping couples together when they have low values of \((\theta, \varepsilon)\). On the other hand, couples on their own do worse in some cases when the husband’s information indicates a relatively high level of happiness.

### 6.4.2 Impact of Changing the Divorce Cost

Earlier, we discussed the theoretical implications of divorce costs in a model without caring. Since we assume that reported happiness in marriage, as reflected in \(\theta_i, \varepsilon_i\), etc., incorporates losses associated with divorce, we can explicitly separate out the cost \(C\).\(^{43}\) The welfare effects that arise if the government

\(^{43}\)Keep in mind that, although we find that the caring couple does virtually as well as the omniscient planner in maximizing social welfare, the analysis is ceteris paribus, including divorce costs that are embedded in reports of relative happiness in marriage.
imposes a divorce cost may be positive or negative, depending on the magnitude of the asymmetric information problem, as we showed in our numerical example earlier. Couples gain when the value of $\theta_h + \theta_w$ is large enough that a) the probability of divorce is relatively small and b) the loss associated with asymmetric information is relatively large. Figure 9 showed that the density of $(\theta_h, \theta_w)$ is concentrated in such regions. For those couples where $\theta_h + \theta_w$ is relatively small, the imposition of the divorce cost just adds an extra cost to the impending divorce and thus reduces welfare. These outcomes depend, moreover, on how the cost of divorce is split; the expected welfare gains from $C$ rise as $\gamma$, denoting the husband’s share of $C$, approaches 1.

Figure 13 shows the derivative of total welfare $V_h + V_w$ with respect to the divorce cost $C$, evaluated at $\theta_w = 1.92$ (which is roughly the median, with about 30 percent more couples lying within 0.5 of that value), and for different values of $\theta_h$ and $\gamma$. When $\theta_h$ is large (the top line in the graph), the welfare gain of increasing $C$ is always positive, but it declines with $\gamma$ from a maximum of a little over 0.6 to 0. When $\theta_h$ is around its median value (the next line), the welfare gain is almost always positive but smaller than before (at 0.2 or less), and again declining with $\gamma$. Given our parameter estimates, the welfare gains are decreasing in absolute value as $\gamma$ increases because $|\partial p/\partial C|$ is declining in $\gamma$, thus making welfare gains less volatile as $\gamma$ increases. Note that these results look quite different from the numerical example in Section 3.4 with no caring preferences. However, as before, the results show that typical couples in our sample benefit from the government imposing a divorce cost – though the gains are small regardless of $\gamma$, which is consistent with our finding that divorce probabilities are quite close to what the omniscient planner in Figure 12 would choose. Lastly, when the husband is not very happy, then the couple typically suffers a welfare loss from costly divorce.

7 Conclusions

In this study we have found direct evidence that couples bargain and that couples care about each other. Furthermore, we have found that couples do not have perfect information about each other, and this asymmetric information would lead to a quite high divorce rate in the absence of caring. However, caring couples with limited information divorce at almost the rate that an omniscient planner would choose. In contrast, a limited planner does a very poor job when deciding on divorce. Thus, we have shown the importance of two key features of marriage – asymmetric information and interdependent utility – which are of great interest but are difficult to identify in most studies of interpersonal relationships. While our evidence justifies incorporating “love” into economic theory, it also shows important limits on love (perhaps retaining a

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44It is not clear that couples could replicate the divorce cost on their own through an ex ante contract, since any such commitment may not be legally binding. Perhaps, though, allowing a “covenant marriage” with higher divorce costs, as implemented recently in Louisiana and a few other states, is an attempt at providing such a legally binding commitment.
measure of victory for cynical economists?). On the other hand, our results suggest very mild limits on the transferability of utility within households.

Interesting extensions to our empirical framework may be possible using additional data from the NSFH. As an example, information on specific ways in which people expect to be happier or unhappier if they separated—in their social life, standard of living, etc.—along with actual outcomes following divorce could be used to investigate the determinants of threat points. Information on time spent on chores and other aspects of domestic life could be used to analyze the nature of side payments. Research in these areas can shed additional light on the nature of bargaining in marriages.

8 Appendix

8.1 Proofs

Proof. (Proposition 2, husband’s monotonicity) Given equation (19) and condition (2),

\[ V_h^* (\varepsilon_h, p) = \frac{\int_{\varepsilon_w^*}^{\infty} V_h (\theta_h - p - \varepsilon_h, \theta_w + p + \varepsilon_w) f_w (\varepsilon_w) d\varepsilon_w}{1 - F_w (\varepsilon_w^*)}, \]

and

\[ \frac{\partial V_h^* (\varepsilon_h, p)}{\partial \varepsilon_h} = \frac{\partial}{\partial \varepsilon_h} \int_{\varepsilon_w^*}^{\infty} V_h (\theta_h - p + \varepsilon_h, \theta_w + p + \varepsilon_w) f_w (\varepsilon_w) \frac{1}{1 - F_w (\varepsilon_w^*)} d\varepsilon_w \]

\[ = \int_{\varepsilon_w^*}^{\infty} V_h (\theta_h - p + \varepsilon_h, \theta_w + p + \varepsilon_w) f_w (\varepsilon_w) \frac{1}{1 - F_w (\varepsilon_w^*)} d\varepsilon_w + \int_{\varepsilon_w^*}^{\infty} V_h (\theta_h - p + \varepsilon_h, \theta_w + p + \varepsilon_w) \frac{\partial}{\partial \varepsilon_h} \frac{f_w (\varepsilon_w)}{1 - F_w (\varepsilon_w^*)} d\varepsilon_w \]

\[ = \int_{\varepsilon_w^*}^{\infty} V_h (\theta_h - p + \varepsilon_h, \theta_w + p + \varepsilon_w) f_w (\varepsilon_w) \frac{1}{1 - F_w (\varepsilon_w^*)} d\varepsilon_w + c > 0. \]
Proof. (Proposition 3, husband’s reservation values) This follows directly from Proposition 2.

Proof. (Proposition 4, effect of $p$ on husband’s reservation values)

$$\frac{d\varepsilon^*_h(p)}{dp} = -\frac{\partial V^*_h(\varepsilon^*_h, p) / \partial p}{\partial V^*_h(\varepsilon^*_h, p) / \partial \varepsilon^*_h}.$$  

The denominator is positive from Proposition 2. The numerator is negative in the range of interest; otherwise the husband could make himself and his wife happier in expected value by increasing $p$.

Proof. (Proposition 5, second order condition) Given condition (2), $(1 - F_w(\varepsilon^*_w)) V^*_h \to 0$ as $p \to -\infty$. Also, $V^*_h \to -\infty$ as $p \to \infty$ because of equation (16). $V^*_h$ is continuous and differentiable in $p$ because $V_h$ is continuous and differentiable in $p$ and, by condition (2), $\varepsilon^*_w$ is continuous and differentiable in $p$. Given that $\exists p: V^*_h > 0$, $\exists p^*$ that maximizes $V^*_h$. Since such a point is an interior maximum of a continuous and differentiable condition, it must satisfy equations (21) and (22).

Proof. (Proposition 6, comparative statics for optimal offer) The derivative of each term has the same sign as the derivative of the first order condition in equation (21) (given that the SOC is satisfied). Thus

$$\frac{\partial p^*}{\partial \theta_h} = \frac{\partial}{\partial \theta_h} \left[ \frac{\partial \log V^*_h(\varepsilon^*_h)}{\partial p} + \frac{\partial \log [1 - F_w(\varepsilon^*_w(p))]}{\partial p} \right]$$

At the optimum, $\frac{\partial \log V^*_h(\varepsilon^*_h)}{\partial \theta_w} < 0$ (otherwise the husband should reduce his side payment offer), and

$$\frac{\partial}{\partial \theta_h} \left[ \frac{\partial \log V^*_h(\varepsilon^*_h)}{\partial \theta_w} - \frac{\partial \log V^*_h(\varepsilon^*_h)}{\partial \theta_h} \right] > 0$$

as long as $V_h$ and $V_w$ have nonpositive second derivatives (along with an Envelope theorem). Similarly,

$$\frac{\partial \log [1 - F_w(\varepsilon^*_w(p))]}{\partial \theta_w} - \frac{\partial \log [1 - F_w(\varepsilon^*_w(p))]}{\partial \theta_h}$$

$$= -\frac{f_w(\varepsilon^*_w(p))}{1 - F_w(\varepsilon^*_w(p))} \left[ \frac{\partial \varepsilon^*_w(p)}{\partial \theta_w} - \frac{\partial \varepsilon^*_w(p)}{\partial \theta_h} \right] > 0;$$

$$\frac{\partial}{\partial \theta_h} \left[ \frac{\partial \log [1 - F_w(\varepsilon^*_w(p))]}{\partial \theta_w} - \frac{\partial \log [1 - F_w(\varepsilon^*_w(p))]}{\partial \theta_h} \right]$$

$$= -\frac{f_w(\varepsilon^*_w(p))}{1 - F_w(\varepsilon^*_w(p))} \frac{\partial}{\partial \theta_h} \left[ \frac{\partial \varepsilon^*_w(p)}{\partial \theta_w} - \frac{\partial \varepsilon^*_w(p)}{\partial \theta_h} \right] > 0.$$
Thus, $\frac{\partial p^*(\varepsilon_h)}{\partial \varepsilon_h} > 0$. By a similar argument, $\frac{\partial p^*(\varepsilon_h)}{\partial \varepsilon_w} < 0$. Also,

$$\frac{\partial}{\partial \varepsilon_h} \left[ \frac{\partial \log V_h^*(\varepsilon_h)}{\partial \varepsilon_w} - \frac{\partial \log V_h^*(\varepsilon_h)}{\partial \varepsilon_h} \right] > 0,$$

and

$$\frac{\partial}{\partial \varepsilon_h} \left[ \frac{\partial \log [1 - F_w(\varepsilon_w^*(p))]}{\partial \varepsilon_w} - \frac{\partial \log [1 - F_w(\varepsilon_w^*(p))]}{\partial \varepsilon_h} \right] = 0$$

because $\varepsilon_w^*(p)$ does not depend on $\varepsilon_h$. Thus, $\frac{\partial p^*(\varepsilon_h)}{\partial \varepsilon_h} > 0$. ■

**Proof.** (Proposition 7, information in $p$) Since $\frac{\partial p^*(\varepsilon_h)}{\partial \varepsilon_h} > 0$ from Proposition 6, the result follows. ■

**Proof.** (Proposition 8, wife’s monotonicity) If $p^*(\varepsilon_h) \Rightarrow \varepsilon_h$, then $\frac{\partial V_w^*(\varepsilon_w, p)}{\partial \varepsilon_w} \geq c > 0$. ■

**Proof.** (Proposition 9, reservation values) This follows directly from Proposition 8. ■

**Proof.** (Proposition 10, effect of $p$ on reservation values)

$$\frac{d\varepsilon_w^*(p)}{dp} = - \frac{\partial V_w^*(\varepsilon_w^*, p)}{\partial \varepsilon_w} \frac{\partial p^*(\varepsilon_w^*, p)}{\partial \varepsilon_w} = \frac{V_{w2} - V_{w1}}{V_{w2}}.$$

The denominator is positive from Proposition 8. The numerator is positive in the range of interest; otherwise the husband could make himself and his wife happier in expected value by increasing $p$. ■

**Proof.** (Proposition 11, equilibrium) The proof follows from a series of lemmas. Let $\mathfrak{S}$ be the set of bivariate distribution functions, $\mathfrak{S}_w$ the set of value functions for the wife $V_w$, and $\mathfrak{S}_h$ the set of value functions for the husband $V_h$. Consider the set of functions $\mathfrak{S}_w^*$ each member $V_w^*(\varepsilon_w, p)$ satisfying conditions (1)-(3). Let $C_2 = \{v(x_1, x_2): v(x_1, x_2) \text{ is continuous and } |v(x_1, x_2) f_h(x_1) f_w(x_2)| \leq B < \infty \text{ for all } -\infty < x_1 < \infty, -\infty < x_2 < \infty\}$.\(^{45}\) $C_2$ is a Banach space for all $B < \infty$ Define the norm of $v(\cdot, \cdot)$ to be

$$\|v(x_1, x_2)\| = \max_x |v(x_1, x_2) f_h(x_1) f_w(x_2)|.$$  \hspace{1cm} (29)

It is straightforward to show that this norm satisfies all of the conditions of a norm. ■

**Lemma 13** $\exists B < \infty: \mathfrak{S}_w^* \subset C_2$.

**Proof.** Let $v \in \mathfrak{S}_w^*$. Then $|v(x_1, x_2) f_h(x_1) f_w(x_2)| \leq B$ for some $B < \infty$ because of equation (18). This implies that $v \in C_2$ $\Rightarrow$ $\mathfrak{S}_w^* \subset C_2$. ■

\(^{45}\)Throughout this proof, we use the result that $\sup_{\varepsilon_h} ||p(\varepsilon_h)/\varepsilon_h|| < \infty$. This follows because the husband is never going to provide a sidepayment resulting in a negative value for him, causing $|p(\varepsilon_h)/\varepsilon_h| < \infty$ for $\varepsilon_h > 0$, and he is limited by his wife’s participation choice and the vanishing of her $f_w$ in the tails, causing $|p(\varepsilon_h)/\varepsilon_h| < \infty$ for $\varepsilon_h < 0$. 43
Lemma 14 \( \mathcal{N}_w^* \) is convex and compact.

**Proof.** Let \( v_1 \) and \( v_2 \) be elements of \( \mathcal{N}_w^* \). Define

\[
v_\lambda = \lambda v_1 + (1 - \lambda) v_2 \quad \text{for} \quad 0 < \lambda < 1.
\]

It is straightforward to show that \( v_\lambda \) is continuous and \( v_\lambda \) satisfies conditions (1)-(3). Thus, \( v_\lambda \in \mathcal{N}_w^* \Rightarrow \mathcal{N}_w^* \) is convex. It is straightforward to show that \( \mathcal{N}_w^* \) is bounded, closed, and equicontinuous. Given equation (29), \( \mathcal{N}_w^* \) vanishes uniformly at \( \infty \). Thus, \( \mathcal{N}_w^* \) is compact by Ascoli’s Theorem. ■

**Proof.** (continuation of Proposition 11) Let \( \mathcal{N}_h^* \) be the set of \( V_h^* (\varepsilon_h, p) \) satisfying Proposition 2; by analogous arguments \( \mathcal{N}_h^* \subset C_2' = \{ v (x_1, x_2) : v (x_1, x_2) \text{ is continuous and } |v (x_1, x_2) f_h (x_1) f_w (x_2)| \leq B < \infty \text{ for all } -\infty < x_1 < \infty, -\infty < x_2 < \infty \} \), \( C_2' \) is a Banach space for all \( B < \infty \), \( \exists B < \infty : \mathcal{N}_h^* \subset C_2' \), and \( \mathcal{N}_h^* \) is convex and compact. Define \( \Gamma_h : \mathcal{N}_h \times \mathcal{N} \times C_1 \rightarrow \mathcal{N}_h^* \) as the functional that determines \( V_h^* \) as a function of \( V_h \), \( F_e \), and \( \varepsilon_w^* \) in equation (19), and define \( \Gamma_w : \mathcal{N}_w \times \mathcal{N} \times C_1 \rightarrow \mathcal{N}_w^* \) as the functional that determines \( V_w^* \) as a function of \( V_w \), \( F_e \), and \( p \) in equation (20). Let \( \Gamma_p : \mathcal{N}_h^* \times C_1 \rightarrow \mathcal{N}_h^* \) be the functional that determines the husband’s optimal side payment offer as a function of his own \( V_h^* \), his wife’s reservation value \( \varepsilon_w^* \), and the distribution of his wife’s \( \varepsilon_w \) implied by equation (21). Define \( \Gamma_r : \mathcal{N}_w^* \times C_1 \rightarrow C_1 \) as the functional that determines the wife’s optimal reservation value as a function of her \( V_w^* \) and her husband’s side payment offer \( p \) implied by Proposition 9. \( \Gamma_h, \Gamma_w, \Gamma_p, \) and \( \Gamma_r \) are all continuous. Define

\[
\Gamma (\varepsilon_w^*, V_h, V_w, F_e) = \Gamma_r [V_h^*, p] = \Gamma_r [\Gamma_w (V_w, F_e, p), p] = \Gamma_r [\Gamma_w (V_w, F_e, \Gamma_p (V_h^*, \varepsilon_w^*, F_e)), \Gamma_p (V_h^*, \varepsilon_w^*, F_e)] = \Gamma_r [\Gamma_w (V_w, F_e, \Gamma_p (\Gamma_h (V_h, F_e, \varepsilon_w^*), \varepsilon_w^*, F_e)), \Gamma_p (\Gamma_h (V_h, F_e, \varepsilon_w^*), \varepsilon_w^*, F_e)].
\]

Since \( \Gamma_h, \Gamma_w, \Gamma_p, \) and \( \Gamma_r \) are all continuous, so is \( \Gamma \). Given these results, \( \Gamma \) satisfies the conditions for the Schauder fixed point theorem to apply. ■

To wrap up, we will mention some comparative statics of the equilibrium. We can prove that the probability of divorce falls with each spouse’s observable and unobservable happiness.

**Proof.** (Proposition 12, comparative statics for divorce probabilities) The

\[
\Pr [V_w^* (\varepsilon_w, p) \geq 0] = \int \Pr [\varepsilon_w > \varepsilon_w^* (p^* (\varepsilon_h))] \, d\varepsilon_h.
\]

and the

\[
\Pr [V_w^* (\varepsilon_w, p^* (\varepsilon_h)) \geq 0 | \varepsilon_h] = 1 - F_w (\varepsilon_w^* (p^* (\varepsilon_h))).
\]

Thus,

\[
\frac{\partial}{\partial \varepsilon_h} \Pr [V_w^* (\varepsilon_w, p) \geq 0 | \varepsilon_h] = -f_w (\varepsilon_w^* (p^* (\varepsilon_h))) \frac{\partial \varepsilon_w^*}{\partial p^*} \frac{\partial p^*}{\partial \varepsilon_h}.
\]
$$\frac{\partial \varepsilon^*_w}{\partial p} < 0$$ by Proposition 10, and \(\frac{\partial \varepsilon^*_w}{\partial \theta_h} > 0\) by Proposition 6.

Next,

$$\frac{\partial}{\partial \theta_h} \Pr [V^*_w (\varepsilon_w, p) \geq 0] = \frac{\partial}{\partial \theta_h} \int \Pr [\varepsilon_w > \varepsilon^*_w (p^* (\varepsilon_h)) \mid \varepsilon_h] f_h (\varepsilon_h) \, d\varepsilon_h$$

$$= - \int f_w (\varepsilon^*_w (p^* (\varepsilon_h))) \frac{\partial \varepsilon^*_w}{\partial p} \frac{\partial p^*}{\partial \theta_h} f_h (\varepsilon_h) \, d\varepsilon_h > 0$$

because \(\frac{\partial p^*}{\partial \theta_h} > 0\) from Proposition 6.

Finally,

$$\frac{\partial}{\partial \theta_w} \Pr [V^*_w (\varepsilon_w, p) \geq 0] = \frac{\partial}{\partial \theta_w} \int \Pr [\varepsilon_w > \varepsilon^*_w (p^* (\varepsilon_h)) \mid \varepsilon_h] f_h (\varepsilon_h) \, d\varepsilon_h$$

$$= - \int f_w (\varepsilon^*_w (p^* (\varepsilon_h))) \left[ \frac{\partial \varepsilon^*_w}{\partial p} \frac{\partial p^*}{\partial \theta_w} + \frac{\partial \varepsilon^*_w}{\partial \theta_w} \right] f_h (\varepsilon_h) \, d\varepsilon_h$$

At the optimum, \(\frac{\partial \varepsilon^*_w}{\partial p} \frac{\partial p^*}{\partial \theta_w} + \frac{\partial \varepsilon^*_w}{\partial \theta_w} < 0\).

### 8.2 Caring Preferences Specification

For \(V\) to be increasing in both arguments, we require that

$$V_1 (U_1, U_2) > 0 \quad (30)$$

$$\Rightarrow \sum_{i=1}^{2-i} \sum_{j=0}^{2-i} i \phi_{ij} U_1^{i-1} U_2^j > 0;$$

$$V_2 (U_1, U_2) \geq 0 \quad (31)$$

$$\Rightarrow \sum_{i=0}^{1} \sum_{j=1}^{2-i} j \phi_{ij} U_1^i U_2^{j-1} > 0;$$

next, for the function to be concave in both arguments, we require that

$$V_{11} (U_1, U_2) \leq 0 \quad (32)$$

$$\Rightarrow \phi_{20} \leq 0;$$

$$V_{22} (U_1, U_2) \leq 0 \quad (33)$$

$$\Rightarrow \phi_{02} \leq 0;$$

and, finally, meeting equation (17) from earlier requires that

$$V_{12} (U_1, U_2) \geq \max [V_{11} (U_1, U_2), V_{22} (U_1, U_2)] \quad (34)$$

$$\Rightarrow \phi_{11} \geq 2\phi_{20}, \phi_{11} \geq 2\phi_{02}.$$
Condition (30) further implies that

\[ 0 < 1 + \phi_{11} U_2 + 2\phi_{20} U_1 \quad \forall b_{11} \leq U_1 \leq b_{12}, b_{21} \leq U_2 \leq b_{22} \]

\[ \Rightarrow \phi_{11} U_2 > -1 - 2\phi_{20} U_1 \quad \forall b_{11} \leq U_1 \leq b_{12}, b_{21} \leq U_2 \leq b_{22} \]

\[ \Rightarrow \phi_{11} U_2 > -1 - 2\phi_{20} b_{12} \quad \forall b_{21} \leq U_2 \leq b_{22} . \]

\[ \Rightarrow \phi_{11} > \frac{1 - 2\phi_{20} b_{12}}{b_{21}} \quad \text{if } \phi_{11} < 0 \]

\[ \phi_{11} < \frac{1 - 2\phi_{20} b_{12}}{b_{21}} \quad \text{if } \phi_{11} > 0 \quad . \quad (35) \]

If \( \phi_{11} < 0 \), then \( \phi_{11} U_2 \) is minimized at \( U_2 = b_{22} > 0 \), and, if \( \phi_{11} > 0 \), then \( \phi_{11} U_2 \) is minimized at \( U_2 = b_{21} < 0 \). This implies that, if we further assume

\[ b_{22} = b_{12} = -b_{11} = -b_{21} = b, \quad (36) \]

then equation (35) simplifies to

\[ \phi_{11} > \frac{1}{2} - 2\phi_{20} \quad \text{if } \phi_{11} < 0 \]

\[ \phi_{11} < \frac{1}{2} + 2\phi_{20} \quad \text{if } \phi_{11} > 0 \]

\[ 0 > \phi_{11} > -\frac{1}{2} - 2\phi_{20} \quad \text{if } -\frac{1}{2} - 2\phi_{20} < 0 \]

\[ 0 < \phi_{11} < \frac{1}{2} + 2\phi_{20} \quad \text{if } \frac{1}{2} + 2\phi_{20} > 0 \quad . \quad (37) \]

Note that equations (34) and (37) always have a continuum of solutions iff

\[ \frac{1}{b} > -2\phi_{20} \]

and that neither is always dominant. Conditions (32), (33), (34), and (37) are a finite set of restrictions on \((\phi_{20}, \phi_{11}, \phi_{02})\) that are easy to impose. We first impose equations (32) and (33).\(^{46}\) Then, we determine which restriction on \(\phi_{11}\) is binding and impose it.\(^{47}\) With the parameters \((\phi_{00}, \phi_{10})\) normalized to \((0, 1)\), this leaves just \(\phi_{01}\) to satisfy equations (30) and (31) over the domain in equation (24).

We can solve condition (31) for \(\phi_{01}\) to get

\[ 0 < \phi_{01} + 2\phi_{02} U_2 + \phi_{11} U_1 \quad \forall b_{11} \leq U_1 \leq b_{12}, b_{21} \leq U_2 \leq b_{22} \]

\[ \Rightarrow \phi_{01} > -2\phi_{02} U_2 - \phi_{11} U_1 \quad \forall b_{11} \leq U_1 \leq b_{12}, b_{21} \leq U_2 \leq b_{22} \]

\[ \Rightarrow \phi_{01} > -2\phi_{02} b_{22} - \phi_{11} b_{12} \quad \text{if } \phi_{11} > 0 \]

\[ \phi_{01} > -2\phi_{02} b_{22} + \phi_{11} b_{11} \quad \text{if } \phi_{11} < 0 \]

which simplifies to

\[ \phi_{01} > -2\phi_{02} b + |\phi_{11}| b \]

---

\(^{46}\)These can be imposed by estimating \(-\log \phi_{ii}\) without restrictions for \(i = 1, 2\).

\(^{47}\)This can be imposed by setting

\[ \phi_{11} = \kappa_1 + (\kappa_2 - \kappa_1) \frac{e^{\alpha}}{1 + e^{\alpha}}, \]

where \(\alpha\) is a free parameter and \(\kappa_2, \kappa_1\) are the bounds on \(\phi_{11}\) implied by (34) and (37).
if equation (36) holds.\textsuperscript{48}

We still must decide how to define the polynomial outside of the range in (24). Even outside of this range, we would like the function to satisfy monotonicity and concavity restrictions. Consider the following case: \(b_{11} \leq u_1 \leq b_{12}, b_{22} < u_2\). Define

\[
V_{22}^* (U_1, U_2) = v_{22} (U_1, b_{22}) = 2\phi_{02}
\]
as the second partial derivative of \(V (U_1, U_2)\) which implies that the first derivative is

\[
V_2^* (U_1, U_2) = V_2 (U_1, b_{22}) + 2\phi_{02} (U_2 - b_{22}).
\]

If \(\phi_{02} < 0\), the first derivative will eventually turn negative, violating monotonicity.\textsuperscript{49} Thus we adjust the derivative to

\[
V_2^* (U_1, U_2) = \max \left[ V_2 (U_1, b_{22}) + 2\phi_{02} (U_2 - b_{22}), 0 \right].
\]

The point where \(V_2 (U_1, b_{22}) + 2\phi_{02} (U_2 - b_{22}) = 0\) occurs where

\[
0 = \sum_{i=0}^{2-i} \sum_{j=1}^{2-i} j\phi_{ij} U_i^j b_{22}^{j-1} + 2\phi_{02} (U_2 - b_{22})
\]

\[
\Rightarrow U_2^* = \frac{2\phi_{02} b_{22} - \sum_{i=0}^{2} \sum_{j=1}^{2-i} j\phi_{ij} U_i^j b_{22}^{j-1}}{2\phi_{02}} > b_{22}.
\]

This implies that

\[
V^* (U_1, U_2) = V (U_1, b_{22}) + \int_{b_{22}}^{u_{22}} \max \left[ V_2 (U_1, b_{22}) + 2\phi_{02} (u - b_{22}), 0 \right] du
\]

\[
= V (U_1, b_{22}) + \int_{b_{22}}^{\min (U_2^*, U_2)} V_2 (U_1, b_{22}) + 2\phi_{02} (u - b_{22}) du
\]

\[
= V (U_1, b_{22}) + V_2 (U_1, b_{22}) \left[ \min (U_2^*, U_2) - b_{22} \right]
\]

\[
+ \phi_{02} \left[ \min (U_2, U_2^*) - b_{22} \right]^2.
\]

We make similar adjustments for all other cases outside the region where conditions (30) through (37) hold.

\subsection*{8.3 Estimation}

Define the objective function as

\[
\mathcal{L} = \sum_i L_i (\Theta) - \lambda e (\Theta)^T \Omega_e^{-1} e (\Theta)
\]

\textsuperscript{48}This restriction can be imposed in a way similar to those for \(\phi_{02}\) and \(\phi_{20}\).

\textsuperscript{49}If \(\phi_{02} = 0\), then no adjustment is necessary.
with first derivative
\[
\frac{\partial L}{\partial \Theta}(\hat{\Theta}) = \sum_i L_{i\Theta}(\hat{\Theta}) - 2\lambda e_{\Theta}(\Theta^*)' \Omega_{\varepsilon}^{-1} e(\hat{\Theta}) \\
= \sum_i L_{i\Theta}(\Theta^*) + \sum_i L_{i\Theta\Theta'}(\Theta^*) (\hat{\Theta} - \Theta^*) \\
- 2\lambda e_{\Theta}(\Theta^*)' \Omega_{\varepsilon}^{-1} e(\Theta^*) - 2\lambda e_{\Theta}(\Theta^*)' \Omega_{\varepsilon}^{-1} e_{\Theta}(\Theta^*) (\hat{\Theta} - \Theta^*) \\
= \sum_i L_{i\Theta}(\Theta^*) - 2\lambda e_{\Theta}(\Theta^*)' \Omega_{\varepsilon}^{-1} e(\Theta^*) \\
+ \left[ \sum_i L_{i\Theta\Theta'}(\Theta^*) - 2\lambda e_{\Theta}(\Theta^*)' \Omega_{\varepsilon}^{-1} e_{\Theta}(\Theta^*) \right] (\hat{\Theta} - \Theta^*) ,
\]
where \( \hat{\Theta} \) is the value of \( \Theta \) where \( L \) is maximized and \( \Theta^* \) is the true value of \( \Theta \). Then
\[
(\hat{\Theta} - \Theta^*) = \left[ \sum_i L_{i\Theta\Theta'}(\Theta^*) - 2\lambda e_{\Theta}(\Theta^*)' \Omega_{\varepsilon}^{-1} e_{\Theta}(\Theta^*) \right]^{-1} \cdot \left[ \sum_i \left\{ L_{i\Theta}(\hat{\Theta}) - 2\lambda e_{\Theta}(\Theta^*)' \Omega_{\varepsilon}^{-1} e(\hat{\Theta}) \} \right] \\
= \left[ \sum_i L_{i\Theta\Theta'}(\Theta^*) - 2\lambda e_{\Theta}(\Theta^*)' \Omega_{\varepsilon}^{-1} e_{\Theta}(\Theta^*) \right]^{-1} \left[ \sum_i \left\{ L_{i\Theta}(\hat{\Theta}) - L_{\Theta}(\hat{\Theta}) \} \right] \\
\]
where
\[
L_{\Theta}(\hat{\Theta}) = 2\lambda e_{\Theta}(\Theta^*)' \Omega_{\varepsilon}^{-1} e(\hat{\Theta}) .
\]
This implies that the asymptotic covariance matrix is
\[
C(\hat{\Theta} - \Theta^*) = \left[ \sum_i L_{i\Theta\Theta'}(\Theta^*) - 2\lambda e_{\Theta}(\Theta^*)' \Omega_{\varepsilon}^{-1} e_{\Theta}(\Theta^*) \right]^{-1} \cdot \left[ \sum_i \left\{ L_{i\Theta}(\hat{\Theta}) - L_{\Theta}(\hat{\Theta}) \} \right] \\
\cdot \left[ \sum_i \left\{ L_{i\Theta}(\hat{\Theta}) - L_{\Theta}(\hat{\Theta}) \} \right]^' \\
\cdot \left[ \sum_i L_{i\Theta\Theta'}(\Theta^*) - 2\lambda e_{\Theta}(\Theta^*)' \Omega_{\varepsilon}^{-1} e_{\Theta}(\Theta^*) \right]^{-1}.
\]
8.4 Specification Tests

8.4.1 Test of Child Effects on Divorce

Partition the CPS sample into cells $jk$ where all of the cells with common index $j$ have common values for explanatory variables and all of the cells with common index $k$ have common child characteristics. Assume that $i = 1, 2, ..., n_{jk}$, $k = 1, 2, ..., K$ and $j = 1, 2, ..., J$. Let $Y_{ijk}$ be an indicator for whether CPS sample person $i$ from cell $jk$ divorces, and assume that $Y_{ijk} \sim Bernoulli(\pi_{ijk})$. Define $S_{jk}$ as the subset of NSFH sample observations with observed characteristics consistent with cell $jk$. Assume that

$$\pi_{ijk} = \pi_{ijk}^*(\theta_i) + \pi_j^* + \pi_{jk}^*$$

where $\pi_{ijk}^*(\theta_{ij})$ captures variation in $\pi_{ijk}$ over $i$ within $jk$ caused by variation in $\theta_{ij}$, $\pi_j^*$ is any other effect of $j$ on divorce not working through $\theta_{ij}$, and $\pi_{jk}^*$ is any other effect of $jk$ on divorce not working through $\theta_{ij}$. With no loss in generality, we can restrict $\sum_k \pi_{jk}^* = 0$. Consider $H_0 : \pi_{j**} = 0 \forall jk$ against the general alternative. Define

$$\hat{\pi}_{jk}^{***} = \left[ \hat{\pi}_{jk} - \pi_{jk} \left( \hat{\theta} \right) \right] - \frac{1}{K} \sum_{k=1}^{K} \left[ \hat{\pi}_{jk} - \pi_{jk} \left( \hat{\theta} \right) \right]$$

$$= \left[ \hat{\pi}_{jk} - \frac{1}{K} \sum_{k=1}^{K} \hat{\pi}_{jk} \right] - \left[ \pi_{jk} \left( \hat{\theta} \right) - \frac{1}{K} \sum_{k=1}^{K} \pi_{jk} \left( \hat{\theta} \right) \right]$$

$$= \Delta \hat{\pi}_{jk} - \Delta \pi_{jk} \left( \hat{\theta} \right)$$

where $\hat{\pi}_{jk}$ is the CPS sample divorce proportion, $\pi_{jk} \left( \hat{\theta} \right)$ is the average probability of divorce for those NSFH sample observations in $S_{jk}$ conditional on $\hat{\theta}$. Define $\pi_{j**}^* = (\pi_{j1}^{***}, \pi_{j2}^{***}, ..., \pi_{JK}^{***})'$. Then, under $H_0$,

$$\sqrt{n^*\pi_{j**}^*} \sim indN(0, \Upsilon_j)$$

with

$$\Upsilon_j = \Upsilon_{j1} + \Upsilon_{j2},$$

$$\Upsilon_{j1} = n^* \frac{K - 1}{K} \begin{pmatrix} \frac{1}{K} (b_1 + b_2) & \frac{1}{K} (b_1 + b_2) & \cdots & \frac{1}{K} (b_1 + b_K) \\ \frac{1}{K} (b_2 + b_2) & \frac{1}{K} (b_2 + b_2) & \cdots & \frac{1}{K} (b_2 + b_K) \\ \vdots & \vdots & \ddots & \vdots \\ \frac{1}{K} (b_1 + b_K) & \frac{1}{K} (b_1 + b_K) & \cdots & \frac{1}{K} (b_1 + b_K) \\ \frac{1}{K} (b_1 + b_K) & \frac{1}{K} (b_1 + b_K) & \cdots & \frac{1}{K} (b_1 + b_K) \end{pmatrix}$$

$$\Upsilon_{j2} = \frac{n^*}{n} \begin{pmatrix} C_{11} & C_{12} & \cdots & C_{1K} \\ C_{21} & C_{22} & \cdots & C_{2K} \\ \vdots & \vdots & \ddots & \vdots \\ C_{K1} & C_{K2} & \cdots & C_{KK} \end{pmatrix}$$
where
\[
b_k = \frac{1}{n_{jk}} \sum_{i \in S_{jk}} \pi_{ijk} (1 - \pi_{ijk}),
\]
\[
C_{km} = \frac{\partial \Delta \tilde{\pi}_{jk}'}{\partial \Theta} \mathbb{C} \left( \tilde{\Theta} \right) \frac{\partial \Delta \tilde{\pi}_{jm}}{\partial \Theta}
\]
\[
= \frac{\partial \Delta \tilde{\pi}_{jk}'}{\partial \Theta} \mathbb{R}^t \mathbb{C} \left( \tilde{\Theta} \right) \mathbb{R} \frac{\partial \Delta \tilde{\pi}_{jk}}{\partial \Theta},
\]
\(\mathbb{C} \left( \tilde{\Theta} \right)\) is the asymptotic covariance matrix of \(\tilde{\Theta}\), and
\[
R = \frac{1}{K} \begin{pmatrix}
K - 1 & -1 & \cdots & -1 \\
-1 & K - 1 & \cdots & -1 \\
\vdots & \vdots & \ddots & \vdots \\
-1 & -1 & \cdots & K - 1
\end{pmatrix}.
\]

Note that, by construction, \(\sum_k \hat{\pi}_{jk}^{***} = 0\). Thus, we should exclude one element of \(\hat{\pi}_{jk}^{***}\), and we lose one degree of freedom in a \(\chi^2\) test.

### 8.4.2 Test of Data Interpretation

The text describes our joint test of the assumption that spouses report their happiness before considering the side payment \(p\) and that the husband makes the take-it-or-leave-it offer of \(p\). In order to implement these tests we need only compute partial correlations of the generalized residuals of the dependent variables (Gourieroux et al., 1987). In particular, the generalized residuals of \(z_i^*\) are simulated as \(E (z_i^* | X_{ij}, z_{ij})\), and the generalized residuals of \(u_i^*\) are
\[
E (u_i^* | z_i^*, u_{ij}) = \frac{\phi \left( t_{u_{ij}} - z_i^* \right) - \phi \left( t_{u_{ij}+1} - z_i^* \right)}{\Phi \left( t_{u_{ij}+1} - z_i^* \right) - \Phi \left( t_{u_{ij}} - z_i^* \right)},
\]
conditional on the simulated values of \(z_i^*\). The variance of the generalized residuals for \(z_i^*\) are simulated, and the variance of the generalized residuals for \(u_i^*\) are simulated as
\[
Var \left( u_{ij}^* \right) = Var \left[ E \left( u_{ij}^* | z_{ij}^*, u_{ij} \right) \right] + \int Var \left[ u_{ij}^* | z_{ij}^*, u_{ij} \right] dF \left( z_{ij}^* | z_{ij}, X_{ij} \right)
\]
where the integrand in the second term is

\[
\text{Var} \left[ u_{ij}^* | z_{ij}^*, u_{ij} \right] = \text{Var} \left[ u_{ij}^* | t_{uij}^* \leq u_{ij}^* \leq t_{uij}^* + 1 \right]
\]

\[
= 1 + \frac{\frac{t_{uij}^* - z_{ij}^*}{\sigma_x} \phi \left( \frac{t_{uij}^* - z_{ij}^*}{\sigma_x} \right) - \frac{t_{uij}^* + 1 - z_{ij}^*}{\sigma_x} \phi \left( \frac{t_{uij}^* + 1 - z_{ij}^*}{\sigma_x} \right)}{\Phi \left( \frac{t_{uij}^* + 1 - z_{ij}^*}{\sigma_x} \right) - \Phi \left( \frac{t_{uij}^* - z_{ij}^*}{\sigma_x} \right)}
\]

\[
- \left( \frac{\phi \left( \frac{t_{uij}^* - z_{ij}^*}{\sigma_x} \right) - \phi \left( \frac{t_{uij}^* + 1 - z_{ij}^*}{\sigma_x} \right)}{\Phi \left( \frac{t_{uij}^* + 1 - z_{ij}^*}{\sigma_x} \right) - \Phi \left( \frac{t_{uij}^* - z_{ij}^*}{\sigma_x} \right)} \right)^2.
\]

Once we have simulated generalized residuals, we test the null hypotheses associated with equations (27) and (28) using the estimated average partial derivative described in Powell, Stock and Stoker (1989):\(^{50}\)

\[
\hat{R} = -\sum_i y_i \sum_j \frac{\partial K(x_j - x_i)}{\partial x_i} \sum_j \frac{\partial K(x_j - x_i)}{\partial x_i}
\]

where \((y_i, x_i)\) is the vector of dependent variables and explanatory variables corresponding to the null hypotheses and \(K(\cdot)\) is a bivariate kernel function.\(^{51}\) The set of dependent variables and explanatory variables for each test was listed in Table 7 in the text. The asymptotic variance of the estimate is:\(^{52}\)

\[
\text{Var} \hat{R} = \text{Var} \left[ -\sum_i y_i \sum_j \frac{\partial K(x_j - x_i)}{\partial x_i} \sum_j \frac{\partial K(x_j - x_i)}{\partial x_i} \right]
\]

\[
= \left[ \sum_i \sum_j K(x_j - x_i) \right]^{-2} \left( \sum_i \frac{\partial K(x_j - x_i)}{\partial x_i} \right)^2 \text{Var} (y_i)
\]

where \(\text{Var} (y_i)\) is simulated.

9 References

References


\(^{50}\)This estimate follows from constructing the relevant partial derivative and then using integration by parts with an appropriate boundary condition.

\(^{51}\)We use a bivariate normal density function truncated at 4 with bandwidths chosen as proportions of the standard deviation of the explanatory variables.

\(^{52}\)We use this estimate of the variance, rather than the one provided in Powell, Stock, and Stoker (1989) because our dependent variables exhibit heteroskedasticity (implied by equation (38), and Powell, Stock, and Stoker (1989) assume homoskedasticity.


