Implicit Contracts

1 Motivation

A. Explain sticky nominal wages
   B. Useful intuition in understanding bargaining between firms and workers

2 Problem Setup

Let $\theta$ index states of the world, and assume $\theta \sim G(\theta)$. Let $n$ be the pool of labor and $\rho(\theta)$ be the proportion of the labor pool (contingent on $\theta$) that is employed at a particular time. Output is

$$\theta f(\rho(\theta)n).$$

Note that $\theta$ directly affects output through productivity and indirectly through the firm’s choice of $\rho(\theta)$. Employed workers get paid $C_1(\theta)$, and unemployed workers get paid $C_2(\theta)$. Each worker has a utility function $U(C)$ if employed and $U(C + m)$ if unemployed (with $m > 0$), $U' > 0$, $U'' < 0$. Then the firm’s expected profit is

$$E\pi = \int [\theta f(\rho(\theta)n) - n\rho(\theta)C_1(\theta) - n(1 - \rho(\theta))C_2(\theta)]dG(\theta),$$

and the workers’ expected utility is

$$EU = \int [U(C_1(\theta))\rho(\theta) + U(C_2(\theta) + m)(1 - \rho(\theta))]dG(\theta).$$

We can think of the firm’s problem as either

$$\max E\pi \quad st \ u^* \leq EU$$

or equivalently as

$$\max EU \quad st \ \pi^* \leq E\pi;$$

both will give the same solution (with appropriate choice of $u^*$ and $\pi^*$). Consider the second problem:

$$\max EU = \int [U(C_1(\theta))\rho(\theta) + U(C_2(\theta) + m)(1 - \rho(\theta))]dG(\theta)$$

$$st \ \pi^* \leq \int [\theta f(\rho(\theta)n) - n\rho(\theta)C_1(\theta) - n(1 - \rho(\theta))C_2(\theta)]dG(\theta).$$

Since the firm can set all instruments contingent on $\theta$, optimization requires that the problem is solved for each value of $\theta$. Thus the problem is equivalent to

$$\max L(\theta) = [U(C_1(\theta))\rho(\theta) + U(C_2(\theta) + m)(1 - \rho(\theta))]$$

$$+ \lambda [\pi^*(\theta) - \theta f(\rho(\theta)n) + n\rho(\theta)C_1(\theta) + n(1 - \rho(\theta))C_2(\theta)]$$
with FOC’s
\[
\begin{align*}
\frac{\partial L}{\partial C_1 (\theta)} &= U' (C_1 (\theta)) \rho (\theta) + \lambda n \rho (\theta) = 0; \\
\frac{\partial L}{\partial C_2 (\theta)} &= U' (C_2 (\theta) + m) (1 - \rho (\theta)) + \lambda n (1 - \rho (\theta)) = 0; \\
\frac{\partial L}{\partial \rho (\theta)} &= U (C_1 (\theta)) - U (C_2 (\theta) + m) + \lambda n [C_1 (\theta) - C_2 (\theta) - \theta f' (\rho (\theta) n)] = 0.
\end{align*}
\]

The FOC’s for \( C_1 (\theta) \) and \( C_2 (\theta) \) imply that
\[
\begin{align*}
U' (C_1 (\theta)) + \lambda n &= 0; \\
U' (C_2 (\theta) + m) + \lambda n &= 0 \Rightarrow \\
U' (C_1 (\theta)) &= U' (C_2 (\theta) + m) \Rightarrow \\
C_1 (\theta) &= C_2 (\theta) + m.
\end{align*}
\]

Plugging this result into the FOC for \( \rho (\theta) \) implies
\[
m = \theta f' (\rho (\theta) n).
\]

Next, the FOC for \( C_1 (\theta) \) implies that
\[
U' (C_1 (\theta)) = -\lambda n
\]
which implies that \( C_1 (\theta) \) does not vary with \( \theta \). Given the result in equation (1), neither does \( C_2 (\theta) \). So the firm insures the worker completely and employs workers up to the point where the marginal product is equal to the marginal value of leisure. Does this imply sticky real wages? Does it imply sticky nominal wages?

What if \( C_2 (\theta) \) is restricted to be zero? Why might it be restricted? It is still true that the FOC for \( C_1 (\theta) \) implies that \( C_1 (\theta) \) does not depend on \( \theta \). So there are still sticky real wages.

B. Comparison of implicit contracts model to auction model

1) Auction leads to efficient production and inefficient risk sharing
2) Implicit contracts make labor employment more volatile than in an auction market

C. Implicit contracts models don’t really distinguish between layoffs and part-time work. One would have to add preferences of layoffs over part-time work to explain layoffs.

D. Incomplete information

Note: assume the firm is risk averse but not as risk averse as the worker. Now there is some risk sharing. If only the firm observes \( \theta \), then it has an incentive to lie and say \( \theta \) is lower than it really is. Requiring the firm to reduce output in bad \( \theta \) worlds gives the firm the right incentive to reveal \( \theta \) truthfully even though the worker is harmed.