Quasi-Fixed Labor Costs

1 Non Wage Labor Costs

A. Hiring and training costs
  1) Explicit monetary costs of hiring and training
  2) Opportunity costs of trainee's time
  3) Opportunity cost of trainer's time and equipment used in training

B. Fringe benefits
  1) Legally required
    a) Social security
    b) Unemployment insurance
    c) Worker's compensation
  2) Not required
    a) Vacation
    b) Health and life insurance

C. Quasi-fixed costs vary with worker rather than with manhours. Thus, they provide for a significant tradeoff between hours and workers.

2 Employment/Hours Tradeoff

Let

$$Q = f(MH, H)$$

where Q = output, M = # employees, and H = # hours/employee. It is assumed that

$$f_2 < 0 \ \ \forall H > H^*.$$

Cost is

$$C = MHw + Mz.$$

We want to minimize the cost of producing $\overline{Q}$:

$$\begin{aligned} \min_{M,H} C &= MHw + Mz. \\ st \ \ f(MH, H) &\geq \overline{Q}. \end{aligned}$$

We set up a Lagrangian equation:

$$L = MHw + Mz + \lambda \left[\overline{Q} - \ f(MH, H) \right].$$

First order conditions are

$$\begin{aligned}\frac{\partial L}{\partial M} &= Hw + z - \lambda f_1 H = 0; \\ \frac{\partial L}{\partial H} &= Mw - \lambda f_1 M - \lambda f_2 = 0; \\ \frac{\partial L}{\partial \lambda} &= \overline{Q} - \ f(MH, H) = 0.\end{aligned}$$

Note that $\frac{\partial L}{\partial \lambda}$ gives you back the output constraint. Solve for λ:

$$\frac{Hw + z}{f_1 H} = \lambda = \frac{Mw}{f_1 M + f_2}. \tag{1}$$

$Hw + z$ is the marginal cost of an extra employee. Mw is the marginal cost of an extra hour/employee. $f_1 H$ is the marginal product of an extra employee. $f_1 M + f_2$ is the marginal product of an extra hour/employee. So the general rule is

$$\frac{MC_M}{MP_M} = \frac{MC_H}{MP_H}.$$

What if $f_2 = 0$? Then

$$\begin{aligned}\frac{Hw + z}{f_1 H} &= \frac{Mw}{f_1 M} \\ &\Rightarrow w + \frac{z}{H} = w\end{aligned}$$

which can not occur if $z > 0$. So LHS of equation (1) is always greater than RHS. Thus, we set M as small as possible and H so that $\overline{Q} = f(MH, H)$. This occurs because there is a cost associated with M but no cost (or loss in output) associated with H.

Now add overtime:

$$C = \begin{cases} MHw + Mz & \text{if } H \leq 8 \\ MHw + Mz + aM(H-8)w & \text{if } H \geq 8 \end{cases}.$$

If optimal H is less than 8, then overtime is irrelevant. If, at $H = 8$,

$$\frac{Hw + z}{f_1 H} > \frac{Mw}{f_1 M + f_2},$$

then optimal H is greater than or equal to 8. The condition for an optimum is

$$\frac{Hw + z + a(H-8)w}{f_1 H} = \frac{Mw + Maw}{f_1 M + f_2}.$$

When $a \uparrow$, LHS increases by

$$\frac{(H-8)w}{f_1 H},$$

and RHS increases by

$$\frac{Mw}{f_1 M + f_2}.$$

Which is more? Compare LHS to RHS. Consider multiplying both by $(1+a)$:

$$\underset{\text{LHS}}{\frac{(1+a)(H-8)w}{f_1 H}} \qquad \underset{\text{RHS}}{\frac{(1+a)Mw}{f_1 M+f_2}}$$

Rewrite the LHS as

$$\begin{aligned} & \frac{Hw + z - z + a(H-8)w - 8w}{f_1 H} \\ = & \frac{(1+a)Mw}{f_1 M + f_2} - \frac{8w+z}{f_1 H} < \frac{(1+a)Mw}{f_1 M + f_2}. \end{aligned}$$

Therefore, the LHS increases less than the RHS, $\Rightarrow \frac{MC_M}{MP_M} < \frac{MC_H}{MP_H} \Rightarrow M \uparrow, H \downarrow$. Therefore, overtime "spreads the work" but only if $H > 8$.

Example: Assume

$$f(MH, H) = MH^{\alpha} = \frac{MH}{H^{1-\alpha}}.$$

Then the Lagrangean equation is

$$L = MHw + Mz + aM(H-8)w + \lambda\left[\overline{Q} - f(MH, H)\right].$$

First order conditions are

$$\begin{aligned} \frac{\partial L}{\partial M} &= Hw + z + a(H-8)w - \lambda f_1 H \\ &= Hw + z + a(H-8)w - \lambda H^{\alpha} = 0; \\ \frac{\partial L}{\partial H} &= Mw + aMw - \lambda \alpha M H^{\alpha-1} = 0; \\ \frac{\partial L}{\partial \lambda} &= \overline{Q} - f(MH, H) = 0. \end{aligned}$$

Solving for λ, we get

$$\begin{aligned} \frac{Hw + z + a(H-8)w}{H^{\alpha}} &= \frac{Mw + aMw}{\alpha M H^{\alpha-1}} \\ &\Rightarrow \alpha\left[Hw + z + a(H-8)w\right] = Hw + Haw \\ &\Rightarrow (\alpha-1)Hw(1+a) = \alpha\left[8aw - z\right] \\ &\Rightarrow H = \frac{8aw - z}{w(1+a)\left(\frac{\alpha-1}{\alpha}\right)} \end{aligned}$$

which implies that a necessary condition is that $z > 8aw$. What is the intuition of this? How do you solve for M? What happens if a changes?

3 Labor Investment and Demand for Labor

A. Digression on Present Value

1) Explain tradeoff of a dollar today vs a dollar in a year
2) Value of $\$x$ in n years: $x/(1+r)^n$
3) Value of a stream of income, $x_t, t = 0, 1, .., T$:

$$\sum_{t=0}^{T} \frac{x_t}{(1+r)^t}.$$

4) Value of a dollar one year from now compounded quarterly:

$$\left(1+\frac{r}{4}\right)^4.$$

5) Value of a dollar one year from now compounded n times:

$$\left(1+\frac{r}{n}\right)^n.$$

6) Value of a dollar one year from now compounded continuously:

$$\lim_{n\to\infty}\left(1+\frac{r}{n}\right)^n = e^{-r}.$$

7) Value of a continuous stream:

$$\int_0^T e^{-rt}x(t)\,dt.$$

B. Training in General

Let T = training and hiring costs at hiring time
w = wage per period
f = output of each worker
p = probability a worker leaves$\Rightarrow 1-p$ = probability a worker does not leave

At time 0, profit earned on the worker is $-T$. At time 1, profit is $f-w$ conditional on staying, and probability of staying is $1-p$. So expected profit is $(1-p)(f-w)$. At time 2, profit is $f-w$ conditional on staying, and probability of staying is $(1-p)^2$. So expected profit is $(1-p)^2(f-w)$. In general, at time t, profit is $f-w$ conditional on staying, and probability of staying is $(1-p)^t$. So expected profit is $(1-p)^t(f-w)$. What is the lifetime value of an employee?

$$\begin{aligned} & -T+\frac{(1-p)(f-w)}{(1+r)}+\frac{(1-p)^2(f-w)}{(1+r)^2}+... \qquad (2)\\ = & -T+(f-w)\sum_{t=1}^{\infty}\frac{(1-p)^t}{(1+r)^t} \\ = & -T+(f-w)\sum_{t=1}^{\infty}\left(\frac{1-p}{1+r}\right)^t. \end{aligned}$$

Digression on geometric declining series: Consider the sum

$$S = \sum_{t=1}^{T} \beta^t.$$

Then

$$\beta S = \sum_{t=1}^{T} \beta^{t+1} = \sum_{t=2}^{T+1} \beta^t,$$

and

$$\begin{aligned} S - \beta S &= \beta - \beta^{T+1} \\ &\Rightarrow \quad S = \frac{\beta - \beta^{T+1}}{1-\beta}. \end{aligned} \tag{3}$$

Using the result in equation (3), equation (2) becomes

$$-T + (f-w)\sum_{t=1}^{\infty}\left(\frac{1-p}{1+r}\right)^t = -T + (f-w)\frac{\frac{1-p}{1+r}}{1-\frac{1-p}{1+r}} = -T + (f-w)\frac{1-p}{r+p}.$$

The worker is hired iff this term is positive. Note that the condition requires that $f > w$ to make up for initial hiring costs.

C. General vs. Specific Training

1) Provide definitions
   a) General: good at any firm
   b) Specific: good only at particular firm
2) Examples of each
3) Who bears the cost?
   a) General: worker
   b) Specific: shared
   Explain why
4) Implications
   a) Firms are slow to lay off workers with lots of specific training
   b) Labor hording
   c) Investment and minimum wage