Part I. Do 3 out of 4 questions [15 points each]

1) Joe has a utility function

\[ U(X_1, X_2, L) \]

where \( X_1 \) is quantity of potatoes consumed, \( X_2 \) is quantity of bananas consumed, and \( L \) is leisure. Joe maximizes utility subject to a budget constraint involving expenditures on potatoes and bananas and earnings from work. What are the first order conditions that determine how many potatoes and bananas Joe buys and how much leisure he takes?

2) A firm has a production function

\[ Q = AL^\alpha K^\beta \]

where \( \alpha + \beta < 1 \). What is the elasticity of demand for labor with respect to the wage in the short run?

3) Explain in words how a firm will react to an increase in the overtime rate.

4) A worker has the option to get some general training. The cost is \( c \). The benefit is that the worker’s wage increases by \( \Delta w \) each period in the future until the worker retires. Assume that everyone retires at age 65. Explain using math why a 30 year old is more likely to accept the training than a 55 year old man.

Part II. Do 1 out of 2 questions [30 points each]

1) There is a competitive industry with 5 firms, each with a production function

\[ Q = AL^\alpha K^\beta \]

where \( L \) is labor employed and \( K \) is capital. The demand curve for the industry is

\[ \hat{Q} = a + bp \]

where \( \hat{Q} \) is total output and \( p \) is the price of output. Let \( w \) be the wage and \( r \) the price of capital. What is the equilibrium demand for labor in the industry?

2) Use a graphical analysis to describe the income and substitution effect associated with the following events:
   a) An increase in the wage;
   b) An increase in the income tax rate for any income above $10,000;
   c) Winning the lottery.