Family Household Behavior

1. A Basic Model of Family Household Behavior

1.1. Introduction

Consider some examples of how time and goods are used to produce utility:
1) Raw potatoes and preparation time are used to make mashed potatoes which directly enters the utility function;
2) Clothing, books, lots of diapers, and other market goods are used along with time spent with each parent (maybe there is some substitution for market care) to produce good children (whips are important also) which directly enters the utility function;
3) Vacation package and time spent on vacation are used to produce quality vacations which directly enters the utility function.
The family must decide how to allocate the time of each family member between each family activity and work. Why does the family want to spend any time at work?

1.2. A Simple Graphical Example

Consider an individual that spends her time either preparing food or working. She can substitute preparation time for higher quality food. We can draw a picture (Figure 3.1) representing the tradeoffs available between time spent preparing food and the value of food used. Each indifference curve represents the set of points providing the same level of utility. Why do the curves look like this? There is also a budget constraint with a slope equal to the negative of the wage. We can put the two together and then do comparative statics:
1) What happens when the wage increases?
2) What happens when nonwage income increases?
3) What happens when there is new, better technology for producing food?

1.3. An Analytical Example

Let household utility be $U = U\{Z_1, Z_2\}$ where Z_1 and Z_2 are the two final goods that directly affect utility. Let

$$\begin{aligned} Z_1 &= f(X_1, t_{1h} + t_{1w}); \\ Z_2 &= g(X_2, \min[\, t_{2h}, t_{2w}]) \end{aligned}$$

where X_1 and X_2 are raw goods used in the production of Z_1 and Z_2 respectively, t_{1h} and t_{1w} are time spent by the husband and wife respective in the production of Z_1, and t_{2h} and t_{2w} are time spent by the husband and wife respective in the production of Z_2. The time constraint on each family member is

$$\begin{aligned} t_{1h} + t_{2h} + h_h &\leq 1 \text{ (husband); and} \\ t_{1w} + t_{2w} + h_w &\leq 1 \text{ (wife)} \end{aligned} \tag{1.1}$$

where h_h and h_w are the hours worked in the market by the husband and wife respectively. The husband's market wage is w_h, the wife's market wage is w_w, and the families nonwage income is Y. The prices of X_1 and X_2 are p_1 and p_2 respectively. Thus, the family's budget constraint is

$$Y + w_h h_h + w_w h_w \geq p_1 X_1 + p_2 X_2. \tag{1.2}$$

The family's optimization problem is to maximize

$$U \{ f(X_1, t_{1h} + t_{1w}), g(X_2, \min[\, t_{2h}, t_{2w}]) \}$$

subject to equations (1.1) and (1.2). To solve the problem, first substitute for h_h and h_w in the budget constraint (relying on equation (1.1) being binding):

$$Y + w_h (1 - t_{1h} - t_{2h}) + w_w (1 - t_{1w} - t_{2w}) \geq p_1 X_1 + p_2 X_2.$$

Next, set up a Lagrangian equation:

$$\begin{aligned} L \{ t_{1h}, t_{2h}, t_{1w}, t_{2w}, X_1, X_2 \} &= U \{ f(X_1, t_{1h} + t_{1w}), g(X_2, \min[\, t_{2h}, t_{2w}]) \} + \\ &\lambda [Y + w_h (1 - t_{1h} - t_{2h}) + w_w (1 - t_{1w} - t_{2w}) - p_1 X_1 - p_2 X_2] \end{aligned}$$

The first order conditions (FOC's) are

$$\begin{aligned} \frac{\partial L}{\partial t_{1h}} &= U_1 f_2 - \lambda w_h \leq 0; \\ \frac{\partial L}{\partial t_{2h}} &= U_2 g_2 - \lambda w_h \leq 0; \\ \frac{\partial L}{\partial t_{1w}} &= U_1 f_2 - \lambda w_w \leq 0; \\ \frac{\partial L}{\partial t_{2w}} &= U_2 g_3 - \lambda w_w \leq 0; \end{aligned}$$

$$\begin{aligned}
\frac{\partial L}{\partial X_1} &= U_1 f_1 - \lambda p_1 \leq 0; \\
\frac{\partial L}{\partial X_2} &= U_2 g_1 - \lambda p_2 \leq 0; \text{ and} \\
\frac{\partial L}{\partial \lambda} &= Y + w_h\left(1 - t_{1h} - t_{2h}\right) + w_w\left(1 - t_{1w} - t_{2w}\right) - p_1 X_1 - p_2 X_2.
\end{aligned}$$

Note that there is a g_3 in the FOC for t_{2w}. This will denote the fact that

$$\begin{aligned}
\begin{array}{l}\partial g/\partial\ t_{2h} = 0 \\ \partial g/\partial t_{2w} = g_2\end{array} \text{ if } t_{2h} &> t_{2w}; \\
\begin{array}{l}\partial g/\partial t_{2h} = g_2 \\ \partial g/\partial t_{2w} = 0\end{array} \text{ if } t_{2w} &> t_{2h}; \text{ and} \\
\frac{\partial g}{\partial t_{2h}} &= \frac{\partial g}{\partial t_{2w}} = 0 \uparrow \ (g_2 \downarrow).
\end{aligned}$$

Thus g_2 is used to denote $\partial g/\partial t_{2h}$, and g_3 is used to denote $\partial g/\partial t_{2w}$.

The characteristics of behavior can be examined by combining FOC's. For example, the conditions for time to be spent efficiently by each family member between the two household activities is

$$\begin{aligned}
\frac{U_1 f_2}{w_h} &= \lambda = \frac{U_2 g_2}{w_h}; \\
\frac{U_1 f_2}{w_w} &= \lambda = \frac{U_2 g_3}{w_w}.
\end{aligned}$$

The conditions for time to be allocated efficiently between husband and wife are

$$\begin{aligned}
\frac{U_1 f_2}{w_h} &= \frac{U_1 f_2}{w_w}; \\
\frac{U_2 g_2}{w_h} &= \frac{U_2 g_3}{w_w}.
\end{aligned}$$

What are the implications of these equations? In particular what do they say about time spent by each family member in activity 2? The conditions for money to be spent efficiently between X_1 and X_2 is

$$\frac{U_1 f_1}{p_1} = \frac{U_2 g_1}{p_2}.$$

The conditions for time to be spent efficiently by each family member between work and household production is

$$\frac{U_1 f_1}{p_1} = \frac{U_2 g_1}{p_2} = \frac{U_1 f_2}{w_h} = \frac{U_1 f_2}{w_w} = ... \tag{1.3}$$

Under what conditions do both family members work? Under what conditions does one family member not work? Does this mean that one is a better worker (or household production worker) than the other?

We can also look at comparative statics. First, consider what happens as Y increases. The family can buy more goods, work less, or do some combination of the two. They allocate the extra income according to equation (1.3). Consider what happens when w_h or p_1 increases. What happens when the wife becomes more efficient in the production of Z_1. What are the implications of this model for addressing:

1) Why do we observe husbands working more frequently than wives?

2) What does the model say about life cycle issues such as when people retire and how they save?

The model's comparative statics are complicated by the fact that, for some household production activities, the husband's and wife's time are substitutes (Z_1) and, for others, they are complements (Z_2).

2. Other Models of Decision-Making

In the model presented above, the family acted as a single decision maker. In fact, each family member might have different preferences, and there might be some conflict or bargaining over how allocations are made.

2.1. Long-term Care Models

Consider the family decision-making process in a series of papers by me and coauthors on long-term care decisions. Consider a family with n children and a parent where the parent needs care. Index the family members by i ($i = 0$ is the parent, and $1 \leq i \leq n$ is the ith child). There are $n + 2$ alternatives available to the family indexed by j. They are having the parent live independently ($j = 0$), having the parent live in a nursing home ($j = n + 1$), or having the parent cared for by the ith child ($j = i$). Let V_{ij} be the value to family member i of the family choosing alternative j. For example, consider the Jones family represented by

	Alternatives			
Family Member	Live Independently	Joe	Sally	Nursing Home
Mom	0	5	5	-10
Joe	0	-3	4	-10
Sally	0	4	-2	-10

Joe gets utility 4 if Sally cares for Mom, but Sally gets utility -2 if she cares for Mom. Note that the value of Mom living independently is 0 for each family member. This is just a normalizing assumption; each of the other values should be interpreted as the value of that choice relative to the value of Mom living independently. How does the family decide how to care for Mom? The model that comes closest to the joint decision-making model described above is to maximize the sum of utilities and then make sidepayments so that everyone is happy. In the Jones family, $\sum_i V_{ij}$ equals 0, 6, 7, and -30, respectively. Thus the family should choose Sally as the caregiver and provide Sally with a side payment of at least 4 (to make her indifferent between caring for Mom and having Mom and Joe choose the next best alternative). But it is easy to construct examples of families where one or more family members has no incentive to participate in the joint decision. In the Jones family, for example, if Joe refuses to make any sidepayment, i.e., refuses to participate in any family decision, then Sally and Mom will face three choices with total values (over Sally and Mom) of 0, 3, and -20. In fact, Sally will agree to provide care even if Joe makes no side-payment. So there is room for strategic playing on Joe's part (and maybe on other family members' parts). Consider a game where each child decides whether to offer Mom care and then Mom chooses the best available alternative. Where is there room for strategy? What happens in the Jones family. Consider a different game where each child decides whether to participate and commit to the outcome of a family decision-making meeting. Assume that those at the meeting behave as a single decision-maker. Where is there room for strategy? What happens in the Jones family? How do you think families make decisions? How do dynamics enter into the model?