Demand

1 Basic Model

A. Assumptions

1) Profit maximization

\[ \pi = pQ - wL - rK \]

2) Production function

\[ Q = f(L, K) \]

\[ f(0, 0) = 0 \]

\[ f_1 \geq 0, \quad f_2 \geq 0 \]

3) w is total compensation for labor

4) Competitive product and labor market

B. Short-run demand by a firm

1)

\[ \pi = pQ - wL - rK \]

\[ pf(L, K) - wL - rK \]

Competitive markets imply that \( p, w, \) and \( r \) are fixed.

Short run implies that \( K \) is fixed.

So the problem is

\[ \max_L \pi. \]

\[ \frac{\partial \pi}{\partial L} = p \frac{\partial f(L, K)}{\partial L} - w = 0. \tag{1} \]

\( \frac{\partial f(L, K)}{\partial L} \) is the marginal product of labor. \( p \frac{\partial f(L, K)}{\partial L} \) is the value marginal product of labor (VMPL). Equation (1) says that VMPL = w. Why is this a necessary condition for profit maximization? If VMPL > w, then a small increase in employment increases revenue by VMPL and increases cost by w. Thus profits increase by VMPL - w > 0. [similar argument for why VMPL < w is inconsistent with profit maximization].

2) Objections

a) Employers don’t understand VMPL = w, so they don’t do it.

Response: Friedman’s pool player’s example.

3) What happens to demand as wage rises?

a) Geometry (assume \( \frac{\partial^2 L}{\partial W^2} < 0 \))

See Figure 1
At $w_1$, profits are maximized where $\frac{\partial f}{\partial L} = w_1$. This occurs at $L_1$. At $w_2$, profits are maximized at $L_2$. Note that $L_2 < L_1$ because $\frac{\partial^2 f}{\partial L^2} < 0$. Therefore, demand is downward sloping.

b) Algebra:

$$p \frac{\partial f}{\partial L} = w.$$ 

To find $\frac{\partial L}{\partial w}$, take full derivative:

$$p \frac{\partial^2 f}{\partial L^2} \partial L = \partial w \Rightarrow \frac{\partial L}{\partial w} = \left(p \frac{\partial^2 f}{\partial L^2}\right)^{-1} < 0$$

C. Market demand curve is just the sum of individual firm demand curves. Since each firm demand curve is negatively sloped, so is the industry demand curve.

D. Application: Payroll tax incidence: See Figure 2

With no tax, equilibrium is at $(L_0, w_0)$.

Now institute a tax of $t$ per employee paid by employer. This means that total cost to employee is $w + t$. Demand curve shifts down by $t$. Labor falls to $L_n$, and wage falls to $w_n$. But $t > w_0 - w_n$. Why? What is necessary for employer to bear the whole incidence of the tax?

E. Modified models

1) Monopoly in the product market implies

$$p = D(Q)$$

with $D'(Q) < 0$. Now

$$\pi = pQ - wL - rK = D[f(K, L)]f(K, L) - wL - rK.$$ 

Note that

Revenues $= [f(K, L)]f(K, L) = pQ$;

Cost $= wL + rK$.

$$\frac{\partial \pi}{\partial L} = \left[D'(Q) \frac{\partial f}{\partial L}Q + D(Q) \frac{\partial f}{\partial L}\right] - w. \quad (2)$$

The first term is negative because $D'(Q) < 0$; the second term is the VMPL > 0. Equation (2) implies that, at the optimum, VMPL > $w$. The first term represents the reduction in price when $Q$ rises. Prices fall for all units sold.

2) Monopoly in the labor market implies

$$w = S(L)$$
with $S'(L) > 0$. Now

$$
\pi = pQ - wL - rK
= pQ - S(L) - rK.
$$

$$
\frac{\partial \pi}{\partial L} = p \frac{\partial f}{\partial L} - S'(L) - w = 0.
$$

Again, because $S'(L) > 0$, equation (3) implies that $VMP_L > w$. The middle term, $S'(L) L$, represents the increase in wages necessary to pay all employees to hire more employees.

F. Long-run Demand

Now $K$ can be adjusted. So

$$
\pi = pQ - wL - rK
= pf(K, L) - wL - rK.
$$

$$
\frac{\partial \pi}{\partial L} = p \frac{\partial f}{\partial L} - w = 0;
\frac{\partial \pi}{\partial K} = p \frac{\partial f}{\partial K} - r = 0
$$

which implies that

$$
\frac{\partial f}{\partial L}/w = \frac{\partial f}{\partial K}/r.
$$

Intuition: Assume that $\frac{\partial f}{\partial K}/r < \frac{\partial f}{\partial L}/w$ at a place where the firm chose to produce. Then the firm could save $1 in cost by reducing $K$ by $\frac{1}{r}$. This would cause output to decrease by $\frac{1}{w} \frac{\partial f}{\partial K}$. Now it could take the $1 in cost savings and spend it on $\frac{1}{w}$ extra workers. This would cause output to increase by $\frac{1}{w} \frac{\partial f}{\partial w}$. Since, by assumption, $\frac{\partial f}{\partial K}/r < \frac{\partial f}{\partial L}/w$, revenues increase; costs stay the same; so profits increase. Thus, it could not have been optimal to choose a place where $\frac{\partial f}{\partial K}/r < \frac{\partial f}{\partial L}/w$.

Equation (4) tells us the proper allocation of inputs given $Q$; we still need to maximize profits over $Q$. This analysis easily generalizes for more than two inputs.

G. Minimum Wage Analysis: See Figures 3 and 4

2 Elasticities

A. Definitions

1) Own wage elasticity:

$$
\frac{w \frac{\partial L}{L}}{\frac{\partial w}{w}}
$$
2) Cross wage elasticity:

\[ \frac{w_i \partial L_j}{L_j \partial w_i} \]

3) If cross wage elasticity is positive, then gross substitutes; if negative, then gross compliments.

B. Hicks-Marshall Laws of Derived Demand: Own wage elasticity is high when:

1) Price elasticity of demand is high;
2) Other categories of inputs can easily be substituted;
3) When supply of other inputs are highly elastic;
4) When cost of input is large share of total cost of production.

Explain and provide examples