Part I. Do 3 out of 4 questions (45 points).

1) Consider the model
\[ y_{it} = X_{it} \beta + u_i + \varepsilon_{it}, \]
where \( \varepsilon_{it} \sim iid \mathcal{N}(0, \sigma^2) \). Consider two cases:

a) \( u_i \sim iid \mathcal{N}(0, \sigma^2) \); and
b) \( u_i \) are parameters to estimate.

Write the ML estimator for \( \bar{X} \) for both cases, and compare the covariance matrices of your estimator under both cases.

2) Consider the model
\[ y_{ij} = X_{ij} \beta + Z_{ij} \gamma + u_{ij}, \]
where \( u_{ij} \sim iid \mathcal{N}(0, \sigma^2) \). The researcher observes \( \{y_{ij}, X_{ij}, Z_{ij}\}_{i=1}^{N} \). Suggest how to construct a Lagrange multiplier test statistic for \( H_0 : \gamma = 0 \) vs. \( H_A : \gamma \neq 0 \).

3) Consider the model
\[ y_{ij} = X_{ij} \beta + u_{ij}, \]
where \( u_{ij} \sim iid EV, j = 1, 2, \ldots, J \).

Let \( y_{ik} = j \) iff \( j \) is the \( k \)th best choice; i.e., \( y_i \) orders the choices in descending order of preference. The researcher observes \( \{y_i, X_{ij}\}_{i=1}^{N} \). What is the MLE of \( \beta \)?

4) Consider the model
\[ y_{ijk} = X_{ijk} \delta + Y_{ij} \lambda + Z_{ij} \theta + u_{ij} \]
where the errors are distributed so that
\[ P_{ijkl} = \frac{\exp \{Y_{ij} \lambda + Z_{ij} \theta + (1 - \alpha) H_{ij}\}}{\sum_l \exp \{Y_{il} \lambda + Z_{il} \theta + (1 - \alpha) H_{il}\}} \]
with
\[ H_{ij} = \ln \left[ \sum_k \exp \left\{ \frac{X_{ijk} \delta}{1 - \alpha} \right\} \right]. \]
Explain intuitively what type of variation in the data causes one to be able to identify $\alpha$.

Part II. Do 1 out of 2 questions (30 points).

1) Consider the model

\[
\begin{align*}
y_{it}^* &= X_{it}\beta + u_i + \varepsilon_{it}, \\
\varepsilon_{it} &\sim iid EV, \\
u_i &\sim iid N(0, \sigma^2), \\
y_{it} &= 1(y_{it}^* > 0).
\end{align*}
\]

The researcher observes \(\{[y_{it}, X_{it}]_{t=1}^{T}\}_{i=1}^{N}\). Write the likelihood contribution for observation \(i\), \(\{y_{it}, X_{it}\}_{t=1}^{T}\) as a single integral. Suggest in as much detail as you can how to test \(H_0 : \sigma^2 = 0\) vs. \(H_A : \sigma^2 > 0\).

2) Consider the model

\[
\begin{align*}
y_{i}^* &= X_{i}\beta + u_i, \\
u_i &\sim iid F \\
y_i &= 1(y_{i}^* > 0).
\end{align*}
\]

The researcher observes \(\{y_i, X_i\}_{i=1}^{N}\). Show that the MLE of $\beta$ is consistent. You may not just appeal to the fact the MLE’s are consistent. Explain intuitively why the MLE of $\beta$ has no finite expectation.