Background Material Review

1 Statistics

1.1 Topics to Mention

1. Density
2. Distribution
3. Expected Value
4. Variance
5. Covariance
6. Multivariate normality (geometry, properties)
7. Independence

1.2 Topics to Discuss

1. $$X \sim N\left(0, \sigma^2\right) \Rightarrow \frac{X^2}{\sigma^2} \sim \chi_1^2$$

2. $$X_i \sim iid \chi_{k_i}^2 \Rightarrow \sum_i X_i \sim \chi_n^2 \text{ with } n = \sum_i k_i$$

3. $$\left.\begin{array}{l} U \sim N(0,1) \\ V \sim \chi_m^2 \end{array}\right\} \text{independent} \Rightarrow \frac{U}{\sqrt{V/m}} \sim t_m$$

   Application:

$$\begin{aligned} X_i &\sim iidN\left(\mu, \sigma^2\right) \\ &\Rightarrow Z = \frac{\overline{X} - \mu}{\sigma/\sqrt{n}} \sim N(0,1) \\ s^2 &= \frac{1}{n-1} \sum_i \left(X_i - \overline{X}\right)^2 \\ W &= \frac{(n-1)\, s^2}{\sigma^2} \sim \chi_{n-1}^2 \\ &\quad Z, W \text{ independent} \\ &\Rightarrow \frac{Z}{\sqrt{W/(n-1)}} = \frac{\sqrt{n}\left(\overline{X} - \mu\right)/\sigma}{\sqrt{\frac{(n-1)s^2}{(n-1)\sigma^2}}} = \frac{\sqrt{n}\left(\overline{X} - \mu\right)}{s} \sim t_{n-1}. \end{aligned}$$

4.

$$\left.\begin{array}{l} U \sim \chi^2_m \\ V \sim \chi^2_n \end{array}\right\} \text{independent}$$
$$\Rightarrow \quad \frac{U}{m}/\frac{V}{n} \sim F_{m,n}$$

5. Relationships using the Law of Large Numbers (LLN):

(a)

$$mF_{m,n} \to \chi^2_m \text{ as } n \to \infty.$$

To see this, write

$$mF_{m,n} = m\frac{U}{m}/\frac{V}{n} = U/\frac{V}{n}.$$

Write

$$V = \sum_i X_i$$

where $X_i \sim iid\chi^2_1$. The LLN $\Rightarrow$

$$V/n \to 1$$

because

$$EX_i = 1 \Rightarrow EV/n = 1.$$

Therefore,

$$U/\frac{V}{n} \to U.$$

(b)

$$t_n \to N(0,1) \text{ as } n \to \infty.$$

To see this, write

$$t_n = \frac{U}{\sqrt{V/n}}$$

where $U \sim N(0,1)$ and $V \sim \chi^2_n$. Then

$$V/n \to 1 \Rightarrow t_n \to U.$$

(c)

$$t_n^2 \sim F_{1,n}.$$

To see this write

$$t_n^2 = \frac{U^2/1}{V/n} \sim F_{1,n}.$$

6. Central Limit Theorem:

$$\sqrt{n}\left(\overline{X} - \mu\right) \sim N\left(0, \sigma^2\right).$$

7. Hypothesis Tests: Give two examples

2 Matrix Algebra

Topics to Mention:

1. Addition
2. Multiplication
3. Dimension
4. Rank
5. Inverse
6. Definiteness and Semidefiniteness
7. Eigenvalues
8. Trace
9. Partitioned Matrices
10. Projection
11. Spaces and Subspaces