Midterm
Econ 772
March 2011

Part I. (Do 5 out of 6 questions) 25 points

1) Construct a test statistic to see if women are as price sensitive as men to the price of bananas when making decisions about purchasing bananas. You can ignore endogeneity and corner solution issues.

2) Let

$$\begin{aligned} y_i &= X_i\beta + u_i, \\ u_i &= \log z_i, \\ z_i &\sim iidGamma(\lambda, \alpha). \end{aligned}$$

Note that the support of z_i is $(0, \infty)$ and that the moments of u_i exist. Show that the OLS estimate of β is consistent.

3) Let

$$\begin{aligned} u_t &= a_0\varepsilon_t + a_1\varepsilon_{t-1} + a_2\varepsilon_{t-2}, \\ \varepsilon_t &\sim iid\left(0, \sigma_\varepsilon^2\right), \\ z_t &= \rho z_{t-1} + \varepsilon_t. \end{aligned}$$

Construct the covariance matrix of u.

4)Let

$$\begin{aligned} y &= X\beta + u, \\ u &\sim (0, \Omega). \end{aligned}$$

Let $\widehat{\beta}$ be the GLS estimator of β and

$$\widehat{u} = y - X\widehat{\beta}.$$

Compute

a) $E\widehat{u}'\widehat{u}$;
b) $X'\widehat{u}$.

5) Let

$$\begin{aligned} y_{it} &= X_{it}\beta + u_i + \varepsilon_{it}, \\ u_i &\sim iid\left(0, \sigma_u^2\right), \\ \varepsilon_{it} &\sim iid\left(0, \sigma_\varepsilon^2\right). \end{aligned}$$

Suggest a good estimate of the covariance matrix needed to do GLS.

6) Consider the model,

$$\begin{aligned} y &= X\beta + Z\gamma + u, \\ u &\sim (0, \Omega). \end{aligned}$$

Construct a test for $H_0 : \gamma_3 = \gamma_4 = \gamma_5$ against the general alternative.

Part II. (Do 2 out of 3 questions) 50 points

1) Consider the model,

$$\begin{aligned} y &= X\beta + u, \\ u &\sim N\left(0, \sigma^2 I\right). \end{aligned}$$

Show, using geometry, that the OLS estimator of β is unbiased. Hint: the normal density is symmetric around zero.

2) Consider the model,

$$\begin{aligned} y_i &= X_i\beta + \alpha z_i + u_i, \\ u_i &\sim iidN\left(0, \sigma^2\right). \end{aligned}$$

Assume, instead of observing z_i, we observe

$$\begin{aligned} w_{1i} &= z_i + e_{1i}, \\ w_{2i} &= z_i + e_{2i} \end{aligned}$$

where

$$\begin{pmatrix} e_{1i} \\ e_{2i} \end{pmatrix} \sim iid\left(0, \sigma_e^2 I\right).$$

Let

$$\begin{aligned} W_1 &= (w_{11}, w_{12}, .., w_{1n})', \\ W_2 &= (w_{21}, w_{22}, .., w_{2n})', \end{aligned}$$

and consider the properties of an estimator of (β', α) that solves the orthogonality condition,

$$\begin{pmatrix} X & W_2 \end{pmatrix}' \left[y - \begin{pmatrix} X & W_1 \end{pmatrix} \begin{pmatrix} \beta \\ \alpha \end{pmatrix} \right] = 0.$$

Show that such an estimator is consistent.

3a) Consider the model,

$$\begin{aligned} y_i &= \sum_{j=0}^{J} \beta_j x_{ij} + u_i, \\ u_i &\sim iid\left(0, \sigma^2\right), \end{aligned}$$

and the restriction that $\beta_3 = \beta_4$. Suggest how to estimate the parameters of the model subject to the restriction *without* using Lagrange Multiplier methods.

b) Generalize your approach (still no Lagrange Multiplier methods) to the model

$$\begin{aligned} y &= X\beta + u, \\ u &\sim \left(0, \sigma^2 I\right), \\ A\beta &= c. \end{aligned}$$