Econ 772
Midterm
5/02

OLS,GLS,SimEq,MLE,MOM,Sim,SemiPar

Part I: Do 4 out 5 questions. (60 points)

1) Let

$$y = X\beta + Z\gamma + u.$$

a) Consider using OLS to estimate

$$y = Xb + e.$$

What is the asymptotic distribution of $\widehat{b}$?

b) Consider using OLS to estimate

$$y = Xb + Zc + Qd + v.$$

Let $a' = (b', c', d')$. What is the asymptotic distribution of $\widehat{a}$?

2) Let

$$y = X\beta + u$$

with

$$\begin{aligned} \beta &\sim N(\delta, \Omega), \\ u &\sim N(0, \Sigma). \end{aligned}$$

Construct the GLS estimator of δ and find its asymptotic covariance matrix.

3) Explain carefully why the order condition is a necessary but not sufficient condition for identification.

4) Show that the plim of the score statistic is zero.

5) Consider the model

$$\begin{aligned} y_{it}^* &= X_{it}\beta + u_i + \varepsilon_{it}, \\ u_i &\sim iidN\left(0, \sigma_u^2\right), \\ \varepsilon_{it} &\sim iidN\left(0, \sigma_\varepsilon^2\right), \\ y_{it} &= 1\left(y_{it}^* > 0\right). \end{aligned}$$

a) Derive

$$\Pr\left[y_{it} > 0 \mid u_i\right].$$

b) Use the answer to part (a) to write a likelihood function for the model.

c) Suggest how to simulate the likelihood function.

6) Construct the first three orthonormal polynomials, $i = 0, 1, 2$, such that

$$\int_{-\infty}^{\infty} p_i(x)\, p_j(x)\, \phi(x)\, dx = 1\,(i = j)$$

where $\phi(\cdot)$ is the standard normal density function.

Part II. Do 2 out of 3 questions. (60 points)
1) Consider the model

$$y = X\beta + u$$

and the test

$$H_0 : A\beta = c \;\text{ vs. }\; H_A : A\beta > c$$

Note the inequality in the alternative. Explain why a Wald test statistic will not have the appropriate χ^2 distribution and construct an alternative way to test H_0.
2) Consider the model

$$y_i = m(X_i\beta) + u_i$$

for some unspecified $m(\cdot)$ function where some of the variables in X_i are endogenous. Assume there is a valid set of instruments Z_i such that

$$\text{plim}\frac{1}{n}\sum_{i=1}^{n} Z_i' X_i = 0.$$

Suggest a nonparametric estimator of β.

Part III. Do 2 out of 3 question. (60 points)
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