

# Kronecker Product Rules

$${}_{n \times m} A \otimes B = \begin{pmatrix} a_{11}B & a_{12}B & \cdots & a_{1m}B \\ a_{21}B & a_{22}B & \cdots & a_{2m}B \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1}B & a_{n2}B & \cdots & a_{nm}B \end{pmatrix}$$

$$\text{vec } {}_{n \times m} A = \begin{pmatrix} a_{11} \\ a_{12} \\ \vdots \\ a_{1m} \\ a_{21} \\ \vdots \\ a_{nm} \end{pmatrix}$$

Properties:

$$(A \otimes B)(C \otimes D) = AC \otimes BD.$$

$$(A \otimes B)^{-1} = A^{-1} \otimes B^{-1}.$$

$$(A \otimes B)' = A' \otimes B'.$$

$$\left| \begin{matrix} A & B \\ n \times n & m \times m \end{matrix} \right| = |A|^m |B|^n.$$

$$\text{tr}(A \otimes B) = \text{tr} A \text{tr} B.$$

$$\text{vec}(BAC) = (B \otimes C') \text{vec} A.$$

$$\text{tr}(A'C) = (\text{vec} A)' (\text{vec} C).$$

$$\begin{aligned} \text{tr}(A'MAN) &= (\text{vec} A)' \text{vec}(MAN) \\ &= (\text{vec} A)' (M \otimes N) \text{vec} A. \end{aligned}$$

$$d\text{tr}(MAN) = \text{tr}(dMAN).$$

$$d\text{tr}(A'MAN) = \text{tr}(dA'MAN) + \text{tr}(NA'MdA).$$