Kronecker Product Rules

\[ A_{n \times m} \otimes B = \begin{pmatrix}
    a_{11}B & a_{12}B & \cdots & a_{1m}B \\
    a_{21}B & a_{22}B & \cdots & a_{2m}B \\
    \vdots & \vdots & \ddots & \vdots \\
    a_{n1}B & a_{n2}B & \cdots & a_{nm}B
\end{pmatrix} \]

\[ \text{vec } A_{n \times m} = \begin{pmatrix}
    a_{11} \\
    a_{12} \\
    \vdots \\
    a_{1m} \\
    a_{21} \\
    \vdots \\
    a_{nm}
\end{pmatrix} \]

Properties:

\((A \otimes B)(C \otimes D) = AC \otimes BD.\)

\((A \otimes B)^{-1} = A^{-1} \otimes B^{-1}.\)

\((A \otimes B)' = A' \otimes B'.\)

\[ |A_{n \times n} \otimes B_{m \times m}| = |A|^m |B|^n. \]

\(tr (A \otimes B) = tr A tr B.\)

\(\text{vec}(BAC) = (B \otimes C') \text{vec}A.\)

\(\text{tr} (A'C) = (\text{vec}A)' (\text{vec}C).\)

\(\text{tr} (A'MAN) = (\text{vec}A)' \text{vec}(MAN) = (\text{vec}A)' (M \otimes N) \text{vec}A.\)

\(\text{dtr} (MAN) = tr (dMAN).\)

\(\text{dtr} (A'MAN) = tr (dA'MAN) + tr (NA'MdA).\)