1) Consider the model

\[ y^*_ij = X_i \beta_j + u_{ij}, \quad j = 1, 2, ..., J; i = 1, 2, ..., I \]

with

\[ F(u_{ij}) = \exp \{-e^{-u_{ij}}\}. \]

The econometrician does not observe \( y^*_ij \); instead she observes

\[ y_{ij} = \max_k y^*_{ik}; \]

she observes which choice \( j \) is the best.

a) Find \( \Pr[y_{ij} = 1 \mid X_i] \).
b) Find a MLE for \( \beta \).
c) Find a MOM estimator for \( \beta \).

2) Consider a model where there is a distribution of prices \( F(\cdot) \) for bananas. Assume consumer \( i \) purchases a banana if he encounters a price \( p < r_i \) where \( r_i \) satisfies

\[ G(r_i, X_i) = 0. \]

Given a random sample of accepted banana prices and personal characteristics \( \{p_i, X_i\}_{i=1}^n \), show how you can estimate parameters implicit in \( F(\cdot) \) and \( G(\cdot, \cdot) \). What reasonable identifying assumptions might you have to make?

3) Let

\[ y = X\beta + Z\gamma + u, \quad u \sim (0, \sigma^2 I). \]

Note that it was not assumed that the errors were normal. Consider

\[ H_0: \gamma = 0 \quad \text{vs} \quad H_A: \gamma \neq 0. \]

Suggest a LM-like test, i.e. one that requires estimation of only the restricted model to test the null hypothesis.