Econ 772
Homework 5
Simultaneous Equations

1) Consider

\[ q_t = \beta_0 + \beta_1 p_t + \beta_2 y_t + u_t^d \]
\[ q_t = \alpha_0 + \alpha_1 p_t + \alpha_2 w_t + u_t^s \]

as a system of equations for supply and demand of bananas. Suggest how to test

\[ H_0 : \beta_2 = \alpha_2 = 0 \quad \text{vs.} \quad H_A : \beta_2 \neq \alpha_2 \neq 0. \]

2) Consider the model

\[ y = X \beta + u \]

where some of the \( X \) variables are potentially endogenous. Let \( Z \) be a valid set of instruments for \( X \).

a) Show that the OLS estimator of \( \beta \) is asymptotically biased if any of the \( X \) variables are endogenous and asymptotically unbiased otherwise.

b) Show that the IV estimator of \( \beta \) is asymptotically unbiased whether or not the \( X \) variables are endogenous.

c) Use this to construct a test statistic for

\[ H_0 : X \text{ is exogenous vs. } H_A : X \text{ is endogenous.} \]

3) Consider the model:

\[ y_i = A_i X_i \]

where \( y_i \) is output at firm \( i \), \( X_i \) is input at firm \( i \), and \( A_i \) is a firm-specific productivity factor.

a) Suggest how to estimate \( \alpha \) using OLS.

b) Suggest why, if firms are profit maximizing, \( X_i \) would be endogenous and show the precise relationship between \( X_i \) and the error.

c) Suggest a reasonable instrument for \( X_i \). Explain why it is a valid instrument.