Econ 772
Homework 5
Simultaneous Equations

1) Consider

$$\begin{aligned} q_t &= \beta_0 + \beta_1 p_t + \beta_2 y_t + u_t^d \\ q_t &= \alpha_0 + \alpha_1 p_t + \alpha_2 w_t + u_t^s \end{aligned}$$

as a system of equations for supply and demand of bananas. Suggest how to test

$$H_0 : \beta_2 = \alpha_2 = 0 \text{ vs. } H_A : \beta_2 \neq \alpha_2 \neq 0.$$

2) Consider the model

$$y = X\beta + u$$

where some of the X variables are potentially endogenous. Let Z be a valid set of instruments for X.

a) Show that the OLS estimator of β is asymptotically biased if any of the X variables are endogenous and asymptotically unbiased otherwise.

b) Show that the IV estimator of β is asymptotically unbiased whether or not the X variables are endogenous.

c) Use this to construct a test statistic for

$$H_0 : X \text{ is exogenous vs. } H_A : X \text{ is endogenous.}$$

3) Consider the model:

$$y_i = A_i X_i^{\alpha}$$

where y_i is output at firm i, X_i is input at firm i, and A_i is a firm-specific productivity factor.

a) Suggest how to estimate α using OLS.

b) Suggest why, if firms are profit maximizing, X_i would be endogenous and show the precise relationship between X_i and the error.

c) Suggest a reasonable instrument for X_i. Explain why it is a valid instrument.