1) Consider

\[ y = X\beta + u, \]
\[ Euu' = \Omega. \]

Show that GLS is BLUE.

2) Consider the model

\[ y_i = X_i\beta + u_i, \]
\[ u_i \sim iid(0, \sigma^2), \quad i = 1, 2, \ldots, N. \]

We know that the statistical properties of the OLS estimator of \( \beta \) improve as \( N \) increases. So consider doubling the sample size by just using every observation twice. Derive the statistical properties of an estimator that uses each observation twice.

3) Consider a population with a joint density of \((y, X)\):

\[ f(y, X). \]

Now consider a sample of this population, \( \{y_i, X_i\}_{i=1}^N \) where observation \( i \) is sampled with known probability \( p(X_i) \). Such a method is called stratified sampling, and it is used to oversample people with certain characteristics (e.g., race).

a) Given your sample, suggest an estimator of \( \mu_y = E_y \)

of the form

\[ \hat{\mu}_y = \sum_{i=1}^N \alpha_i y_i; \]

i.e., what are good choices of \( \{\alpha_i\}_{i=1}^N \)? Show that your estimator is unbiased and derive its variance.

b) Consider the true model

\[ y_i = X_i\beta + u_i, \]
\[ u_i \sim iid(0, \sigma^2). \]

How should you use the information about sampling probabilities in \( p(X_i) \) in a GLS framework to weight observations and get a more efficient estimator of \( \beta \) than the OLS estimator?
4) Consider the process

\[ u_t - \rho_1 u_{t-1} - \rho_2 u_{t-2} = a_0 \varepsilon_t + a_1 \varepsilon_{t-1}, \]
\[ \varepsilon_t \sim iid \left(0, \sigma^2 \right). \]

a) Find the autocovariance function for \( u_t \).
b) Let

\[ z_t - \theta z_{t-1} = u_t \]

where the process for \( u_t \) is the same as above. Write the process for \( z_t \) as an ARMA process.