Econ 772
Homework 4
Generalized Least Squares

1) Consider

$$\begin{aligned} y &= X\beta + u, \\ Euu' &= \Omega. \end{aligned}$$

Show that GLS is BLUE.

2) Consider the model

$$\begin{aligned} y_i &= X_i\beta + u_i, \\ u_i &\sim iid\left(0, \sigma^2\right), \quad i = 1, 2, .., N. \end{aligned}$$

We know that the statistical properties of the OLS estimator of β improve as N increases. So consider doubling the sample size by just using every observation twice. Derive the statistical properties of an estimator that uses each observation twice.

3) Consider a population with a joint density of (y, X):

$$f(y, X).$$

Now consider a sample of this population, $\{y_i, X_i\}_{i=1}^N$ where observation i is sampled with known probability $p(X_i)$. Such a method is called stratified sampling, and it is used to oversample people with certain characteristics (e.g., race).

a) Given your sample, suggest an estimator of

$$\mu_y = Ey$$

of the form

$$\widehat{\mu}_y = \sum_{i=1}^N \alpha_i y_i;$$

i.e., what are good choices of $\{\alpha_i\}_{i=1}^N$? Show that your estimator is unbiased and derive its variance.

b) Consider the true model

$$\begin{aligned} y_i &= X_i\beta + u_i, \\ u_i &\sim iid(0, \sigma^2). \end{aligned}$$

How should you use the information about sampling probabilities in $p(X_i)$ in a GLS framework to weight observations and get a more efficient estimator of β than the OLS estimator?

4) Consider the process

$$
\begin{aligned}
u_t - \rho_1 u_{t-1} - \rho_2 u_{t-2} &= a_0 \varepsilon_t + a_1 \varepsilon_{t-1}, \\
\varepsilon_t &\sim iid\left(0, \sigma_\varepsilon^2\right).
\end{aligned}
$$

a) Find the autocovariance function for u_t.
b) Let

$$z_t - \theta z_{t-1} = u_t$$

where the process for u_t is the same as above. Write the process for z_t as an ARMA process.