Econ 772
Homework 1
Asymptotics

1) Let

$$H_0 : X \sim \chi_n^2 \text{ vs. } H_A : X \nsim \chi_n^2$$

with n large. Construct a test statistic using an asymptotic approximation for the distribution of X.

2) Let S_n be a statistic with $plim\ S_n = \theta$. Let $g(\cdot)$ be a differentiable function. Prove from first principals that

$$plim\ g(S_n) = g(\theta).$$

3) Let

$$X_T = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 2 & \log 2 \\ \vdots & \vdots & \vdots \\ 1 & T & \log T \end{pmatrix}.$$

Show that

$$\frac{X_T' X_T}{T}$$

does not converge to a finite matrix as $T \to \infty$. Find a matrix D_T such that

$$D_T X_T' X_T D_T^{'}$$

converges to a finite full-rank matrix as $T \to \infty$.