Consider

$$\begin{aligned} & \Pr\left[u < V\right], \\ \underset{J\times 1}{u} \sim & N\left(0, \Omega\right). \end{aligned}$$

Write this as

$$\Pr\left[u < V\right] = \int_{-\infty}^{V} f\left(u\right) du$$

where $f(u)$ is the joint normal density function. To write this as an importance sampler, we want to write it as

$$\Pr\left[u < V\right] = \int_{-\infty}^{V} \frac{f\left(u\right)}{g\left(u\right)} g\left(u\right) du$$

where $g(u)$ is the importance sampling density function. In GHK, we simulate

$$\begin{aligned} u_1^r &\mid u_1^r < V_1, \\ u_2^r &\mid u_1^r, u_2^r < V_2, \\ &\vdots \\ u_{J-1}^r &\mid u_1^r, u_2^r, .., u_{J-2}^r, u_{J-1}^r < V_{J-1} \end{aligned}$$

and use it in

$$F_1\left(V_1\right) F_2\left(V_2 \mid u_1^r\right) \cdots F_J\left(V_J \mid u_1^r, u_2^r, .., u_{J-2}^r, u_{J-1}^r\right).$$

Thus, if we define

$$g\left(u\right) = \frac{f_1\left(u_1\right)}{F_1\left(V_1\right)} \frac{f_2\left(u_2 \mid u_1\right)}{F_2\left(V_2 \mid u_1\right)} \cdots \frac{f_{J-1}\left(u_{J-1} \mid u_1, u_2, .., u_{J-2}\right)}{F_{J-1}\left(V_{J-1} \mid u_1, u_2, .., u_{J-2}\right)},$$

then we can write

$$\begin{aligned} \Pr\left[u < V\right] &= \int_{-\infty}^{V} f\left(u\right) \left[\frac{F_1\left(V_1\right)}{f_1\left(u_1\right)} \frac{F_2\left(V_2 \mid u_1\right)}{f_2\left(u_2 \mid u_1\right)} \cdots \frac{F_{J-1}\left(V_{J-1} \mid u_1, u_2, .., u_{J-2}\right)}{f_{J-1}\left(u_{J-1} \mid u_1, u_2, .., u_{J-2}\right)}\right] \cdot \\ & \left[\frac{f_1\left(u_1\right)}{F_1\left(V_1\right)} \frac{f_2\left(u_2 \mid u_1\right)}{F_2\left(V_2 \mid u_1\right)} \cdots \frac{f_{J-1}\left(u_{J-1} \mid u_1, u_2, .., u_{J-2}\right)}{F_{J-1}\left(V_{J-1} \mid u_1, u_2, .., u_{J-2}\right)}\right] du \\ &= \int_{-\infty}^{V} F_1\left(V_1\right) F_2\left(V_2 \mid u_1\right) \cdots F_J\left(V_J \mid u_1, u_2, .., u_{J-1}\right) \cdot \qquad (1) \\ & \left[\frac{f_1\left(u_1\right)}{F_1\left(V_1\right)} \frac{f_2\left(u_2 \mid u_1\right)}{F_2\left(V_2 \mid u_1\right)} \cdots \frac{f_{J-1}\left(u_{J-1} \mid u_1, u_2, .., u_{J-2}\right)}{F_{J-1}\left(V_{J-1} \mid u_1, u_2, .., u_{J-2}\right)}\right] du \end{aligned}$$

because

$$f\left(u\right) = f_1\left(u_1\right) f_2\left(u_2 \mid u_1\right) \cdots f_J\left(u_J \mid u_1, u_2, .., u_{J-1}\right).$$

Equation (1) is the integral representation of the GHK algorithm.