Midterm
Econ 772
March 6

Part I. Do 5 out of 6 questions. (50 points)

1. Let $y = X\beta + Z\gamma + u$ where $Euu' = \Omega$. Let $\beta^* = (\beta' \quad \gamma')'$ and $\hat{\beta}^*$ be the OLS estimator of β^*. Construct a test statistic and write its asymptotic distribution for $H_0 : \gamma = 0$ against $H_A : \gamma \neq 0$.

2. Let $y = X\beta + u$ where $Euu' = \sigma^2 I$. Construct an estimator for σ and show that it is consistent.

3. I am interested in testing whether the difference in the rate of return to education for black women and white women has changed over the last twenty years. Given reasonable panel data on wages and relevant individual characteristics, describe how to specify an equation and what the null and alternative hypothesis would be in terms of the parameters of your equation.

4. Let $y = X\beta + u$ where $Euu' = \Omega$. Let $\tilde{\beta}$ be the OLS estimate of β and $\hat{\beta}$ be the GLS estimate of β. Show that $D\left(\tilde{\beta}\right) \geq D\left(\hat{\beta}\right)$.

5. Let

$$u_{it} = e_i + v_{it}$$

where

$$\begin{aligned} e_i &\sim iid\left(0, \sigma_e^2\right), \\ v_{it} &= \rho v_{it-1} + \varepsilon_{it}, \\ \varepsilon_{it} &\sim iid\left(0, \sigma_\varepsilon^2\right). \end{aligned}$$

Let $u' = (u_{11}, u_{12}, .., u_{1T}, u_{21}, .., u_{2T}, u_{31}, .., u_{NT})$ be a vector of the errors. Construct $\Omega = Euu'$.

6. Let $y = X\beta + u$, and let $\hat{\beta}$ be the OLS estimate of β. Let $\hat{u} = y - X\hat{\beta}$. Show that $D(X'u) > D(X'\hat{u})$.

Part II. Do 1 out of 2 questions (20 points)

1. Let

$$y_i = A X_{1i}^{\beta} X_{2i}^{\gamma} u_i$$

where y_i is output at firm i, X_{1i} and X_{2i} are inputs at firm i, and $\log u_i \sim iidN\left(0, \sigma^2\right)$ is an error specific to firm i.

a) Assuming the inputs are exogenous, construct a consistent estimate of (A, β, γ).

b) Assume u_i is a measure of managerial ability. Why might it be unreasonable to assume that

$$\operatorname{plim} \frac{\sum_i X_{1i} u_i}{N} = \operatorname{plim} \frac{\sum_i X_{2i} u_i}{N} = 0?$$

2. Let $y_t = X_t \beta + u_t$ for $t = 1, 2, ..., T$ where $u_t = \rho u_{t-1} + \varepsilon_t$.

a) Assume $\varepsilon_t \sim iid\left(0, \sigma_{\varepsilon}^2\right)$. Consider the test $H_0 : \rho = 0$ against $H_A : \rho \neq 0$, and consider the test statistic

$$K = \left[\frac{1}{T-1} \sum_{t=2}^{T} \hat{u}_t \hat{u}_{t-1}\right] / \left[\frac{1}{T-1} \sum_{t=2}^{T} \hat{u}_{t-1}^2\right].$$

Find the plim K. Describe what else you would need to know to use K in testing the null hypothesis.

b) Instead assume $\varepsilon_t = a e_t + b e_{t-2}$ where $e_t \sim iid\left(0, \sigma_e^2\right)$. Find the autocovariance function for u_t.