Econ 771
Homework 7

1. Let $Z_i \sim ind \chi^2_{k_i}$. Note that $M_{Z_i}(t) = \left(\frac{1}{1-2it}\right)^{k_i/2}$.

- Find moment generating function of $Y = \sum_i Z_i$
- Use it to derive the density of $Y$.

2. Let $X_i \sim iid (\mu, \sigma^2)$, $i = 1, 2, \ldots, n$. Define $\bar{X} = \frac{1}{n} \sum_i X_i$. Consider a function $g(\cdot)$. Find the asymptotic distribution of $g(\bar{X})$.

3. Let $X_i \sim ind (\mu, \sigma_i^2)$, $\sigma_i^2 = e^{i\theta}$, $i = 1, 2, \ldots, n$, with $\theta > 0$.

- Find the distribution of $\bar{X} = \frac{1}{n} \sum_i X_i$.
- Find the distribution of $\sqrt{n}(\bar{X} - \mu)$.

4. Let $X_i \sim ind (\mu_i, \sigma^2)$, $i = 1, 2, \ldots, n$.

- Let $\mu_i = ai + b$, and find the distribution of $\sqrt{n}\bar{X}$.
- Let $\mu_i = \exp\{ai + b\}$ with $a < 0$, and find the distribution of $\sqrt{n}\bar{X}$.