Econ 771
Homework 5

1. Consider a vector of random variables $(X, Y, Z)$ with density $f(x, y, z) = a1(x^2 + y^2 + z^2 \leq c^2)$.
   - What is $P \left( X^2 + Y^2 < \left( \frac{c}{2} \right)^2 \mid Z = z \right)$?
   - What is $E(X + Y \mid X + Z = 0)$?

2. Let
   \[
   \begin{pmatrix} X \\ Y \end{pmatrix} \sim N \left( \begin{pmatrix} \mu_x \\ \mu_y \end{pmatrix}, \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{xy} & \sigma_{yy} \end{pmatrix} \right).
   \]
   - Prove that $X \mid Y$ is normal
   - Prove that $X$ is normal
   - Find $E(X \mid Y < 3)$

3. Let
   \[
   X_i \sim \text{Poisson}(\lambda_i),
   \log \lambda_i = \theta + u_i,
   u_i = e + \varepsilon_i,
   e \sim N(0, \sigma_e^2),
   \varepsilon_i \sim iidN(0, \sigma_e^2),
   \]
   - Find the joint density of $(X_1, X_2, \ldots, X_n)$.
   - Find the conditional joint density of $(X_1, X_2, \ldots, X_{n-1} \mid X_n)$
   - Find the conditional joint density of $(X_1, X_2, \ldots, X_n \mid e)$

4. Let $F(t_i) = 1 - \exp \{-\lambda_i t_i^\alpha\}$.
   - Define the hazard rate for $F(\cdot)$ as
     \[
     \frac{f(\cdot)}{1 - F(\cdot)}.
     \]
     Compute the hazard rate for this distribution, and show how its derivative depends on $\alpha$
   - Consider specifying the hazard rate as
     \[
     h(t_i \mid X_i) = \exp \{X_i \beta + \theta(\cdot)\}.
     \]
     Describe how to pick a functional form for $\theta(\cdot)$ and for $\lambda_i$ so that the distribution function above has the hazard rate above.

5. Show how to find the expected value of a Beta random variable.