1 Do 9 out 10 questions. (90 points)

1. Let \( X_i \sim iid \text{Bernoulli}(p), i = 1, 2, \ldots, n \). Using a Lagrange Multiplier test, test
\[
H_0 : p = 0.3 \text{ vs } H_A : p \not= 0.3.
\]

2. Let \( X = (X_1, X_2)' \) and assume that \( X \sim N [0, I] \). Define \( Y = X_1^2 + X_2^2 \). What is the distribution (or density) of \( Y \mid X_1 = 3 ? \)

3. Derive the moment generating function for \( X \sim U (a, b) \), and use it to find \( E [X - EX]^2 \).

4. Define \( X' = (X_1, X_2, \ldots, X_K) \), and assume that \( X \sim N (\mu, \Omega) \). Find
\[
\Pr [X_2 + X_3 < X_4 \mid X_5].
\]

5. Construct the moment generating function corresponding to the Heckman-Singer unobserved heterogeneity density with 3 mass points.

6. Let \( X_i \sim iid U (0, 1), i = 1, 2, \ldots, n \), and let \( G (\cdot) \) be some arbitrary decreasing function of \( X_i \) over \( (0, 1) \). Find the distribution of \( Y_i = G (X_i). \)

7. Let \( X_i \sim iid f (\cdot), i = 1, 2, \ldots, n \) with
\[
 f (x) = \frac{\exp \{-x/\gamma\}}{\gamma}.
\]
Find the MOM estimator of \( \gamma \), and show that it is consistent.

8. Let \( T_i \) be a random variable with hazard rate,
\[
h (t \mid \varepsilon) = \exp \{\lambda (t) + \varepsilon\}
\]
where \( \lambda (\cdot) \) is the baseline hazard function and the unobserved heterogeneity is \( \varepsilon \sim g (\cdot) \). Let
\[
S (t \mid \varepsilon) = \Pr [T > t \mid \varepsilon] = \exp \left\{ - \int_0^t h (s \mid \varepsilon) \, ds \right\}.
\]
Show how to write
\[
S (t) = \Pr [T > t]
\]
in terms of the moment generating function corresponding to \( g (\cdot) \).
9. Let
\[
\begin{align*}
\Pr [X_i = 0] &= p, \\
\Pr [X_i = Y_i] &= 1 - p,
\end{align*}
\]
where \(Y_i \sim \text{iidPoisson} (\lambda), i = 1, 2, \ldots, n\). Given data, \(\{x_i\}_{i=1}^n\), derive the MLE of \(\theta = (p, \lambda)\) and show that it is consistent.

10. Let \(X_i \sim \text{iidN} (0, \sigma^2_x), i = 1, 2, \ldots, m\), and let \(Z_i \sim \text{iidN} (0, \sigma^2_z), i = 1, 2, \ldots, n\). Suggest a test statistic for \(H_0 : \sigma^2_x = \sigma^2_z\) vs \(H_A : \sigma^2_x \neq \sigma^2_z\) and compute its power function for some specified size.

2 Do 3 out of 4 question. (90 points)

1. a) Let \(X_i \sim \text{iidF} (\cdot), i = 1, 2, \ldots, n\). Let \(Y_i = \Phi^{-1} [F (X_i)]\) where \(\Phi [\cdot]\) is the standard normal distribution function. What is the distribution of \(Y_i\)?

Let \(\Omega_{n \times n}\) be a covariance matrix with all diagonal elements equal to 1 and \(R_{n \times n}\) be a matrix such that \(RR' = \Omega\), and let \(Y = (Y_1, Y_2, \ldots, Y_n)\). What is the distribution of \(Z = RY\)?

Define \(W_i = F^{-1} (\Phi (Z_i)), i = 1, 2, \ldots, n\). What can you say about the joint distribution of \(W = (W_1, W_2, \ldots, W_n)'\)?

2. Explain as carefully as possible the sources of bias associated with kernel estimation given a model
\[
Y_i = g (X_{1i}, X_{2i}) + u_i.
\]

3. For US presidential elections, each person eligible to vote gets one vote. Prior to the actual election people are asked whether they plan to vote and, if so, who they plan to vote for. Construct a model of an individual answering a polling question, deciding whether to vote, and who to vote for that has the property that polling results and voting results can deviate. Hint: the critical problem here is the existence of the law of large numbers.

4. Construct a model of a husband and wife making decisions about the timing of children. Hints: You might think of this in terms of repeated survival events. Alternatively, you might construct a model in discrete time where the husband and wife decide each period whether to get pregnant (or whether to try to get pregnant).