

# Superposition of Elementary Flows

## Introduction

In this assignment we will create some fairly complicated, but still ideal, flows using superposition of more elementary flows that individually satisfy Laplace's equation. In addition to reviewing several aspects of fluid kinematics, another goal of this project is to gain some experience with whatever language and graphics package you intend to use for the rest of this course. Here we will use such a package to plot contours of streamfunction.

## Background

The expressions for streamfunction (and for reference the velocity potentials) as well as schematics for four elementary flows are given below (Sabersky et al., 1989).

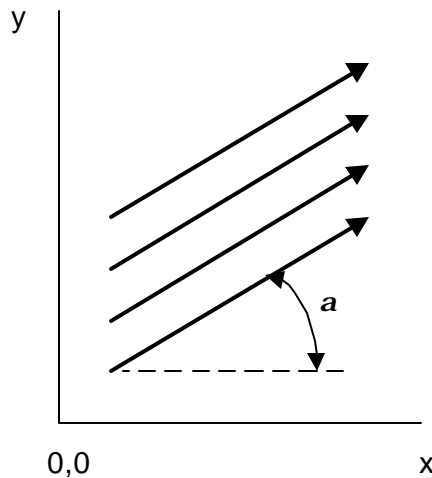
### *Uniform Flow:*

For uniform flow at an angle  $\mathbf{a}$  to the horizontal, the streamfunction is given by:

$$\Psi = U (y \cos \mathbf{a} - x \sin \mathbf{a}) \quad (1)$$

A schematic of streamfunction for uniform flow is given below. Similarly, the velocity potential is:

$$\Phi = U (x \cos \mathbf{a} + y \sin \mathbf{a}) \quad (2)$$



### *Line Source:*

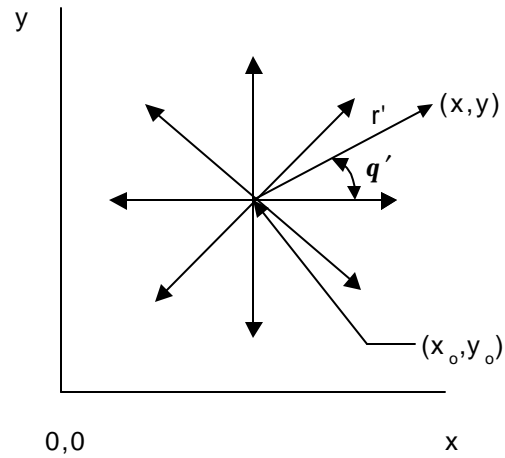
For a line source of strength  $\Lambda$ , the streamfunction is given by:

$$\Psi = \frac{\Lambda}{2\mathbf{p}} \tan^{-1} \frac{y - y_o}{x - x_o} = \frac{\Lambda}{2\mathbf{p}} \mathbf{q}' \quad (3)$$

and the velocity potential is:

$$\Phi = \frac{\Lambda}{2\rho} \ln r' \quad (4)$$

A schematic of the corresponding streamfunction is as follows:



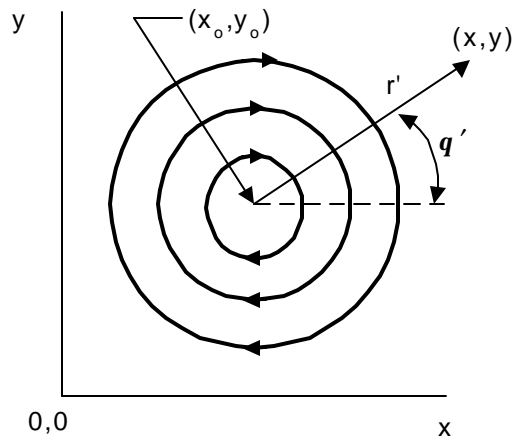
*Line Vortex:*

For a line vortex the streamfunction is given by:

$$\Psi = \frac{\Gamma}{2\rho} \ln \left[ (x - x_0)^2 + (y - y_0)^2 \right]^{\frac{1}{2}} = \frac{\Gamma}{2\rho} \ln r' \quad (5)$$

A schematic of streamfunction is given below. Similarly the velocity potential is:

$$\Phi = \frac{\Gamma}{2\rho} q' \quad (6)$$



## Doublet

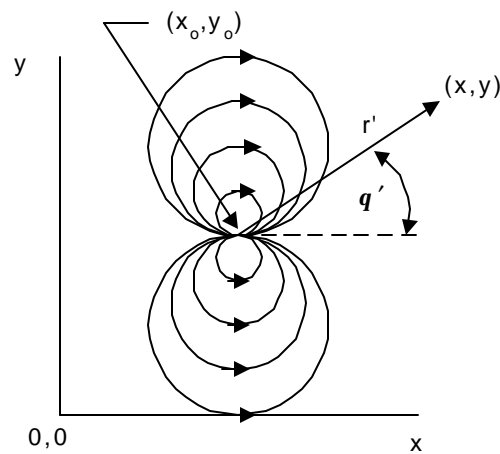
If a source and sink are brought together such that the product  $2\Delta x \Lambda$  remains a constant (here  $\Delta x$  is the separation), one gets a doublet. The streamfunction is given by:

$$\Psi = -\frac{K \sin \mathbf{q}'}{2\mathbf{p} r'} \quad (7)$$

while the velocity potential is:

$$\Phi = \frac{K \cos \mathbf{q}'}{2\mathbf{p} r'} \quad (8)$$

A schematic of streamfunction is seen below.



## Implementation

Set up a uniform mesh with the origin at the center. If you use an even number of grid points, e.g., 26 horizontally and 20 vertically, then the origin of your coordinate system will fall between grid points. Also making your minimum y value equal to -0.5, maximum y value equal to +0.5, minimum x value equal to -0.66 and maximum x value equal to +0.66 will give a well-proportioned figure. Then if you happen to place a singularity at the origin, you will not have to worry about dividing by zero. For each of the three cases described below, first evaluate the streamfunction at each point on your mesh due to the prescribed elements and then plot contours of these values (streamlines). You will need to have separate function subprograms for each of the four elementary flows given above. To get you started, Fortran code for two of the four are provided in the file *ELEM.FOR*. When you are writing the function for the line source, you will note that the usual *ATAN* function returns values only in the first and fourth quadrants. Consider using the Fortran function *ATAN2(a,b)*, which returns the arctangent (in radians) of a/b instead. The range for the output of *ATAN2* is  $-\mathbf{p}$  to  $\mathbf{p}$ , so you may want your function to change the output as necessary to run from 0 to  $2\mathbf{p}$ . Similar considerations hold if you are programming in other languages, e.g., VBA.

The three cases to be studied are:

- Uniform flow from left to right ( $U = 1.0$ ,  $\mathbf{a} = 0$ ) plus unit source ( $\Lambda = 1.0$ ) at  $(-0.20, 0.0)$  and unit sink ( $\Lambda = -1.0$ ) at  $(+0.20, 0.0)$ . This will give the flow over a Rankine oval.

b. Uniform flow from left to right ( $U = 1.0$ ,  $\mathbf{a} = 0.0$ ) plus doublet of strength  $K = 0.50$  at  $(0.0, 0.0)$ . This combination will give you ideal flow over a cylinder.

c. Uniform flow from left to right ( $U = 1.0$ ,  $\mathbf{a} = 0.0$ ), doublet of strength  $K = 0.25$  at  $(0.0, 0.0)$  and vortex of strength  $\Gamma = 1.0$  at  $(0.0, 0.0)$ . For this case you are to evaluate numerically the net force (the Magnus force) on the spinning cylinder you have created.

The strengths and locations given are based on your region running from  $-0.66 < x < 0.66$ ,  $-0.5 < y < 0.5$ . One difficulty you will encounter in this project is choosing the right vector of contour values to be plotted, so that one of them is the dividing streamline and the interesting part of the flow is evenly covered by contours. If you merely scan over the whole field for the maximum and minimum values and divide this range by, e.g., 10, you may find the plot unsatisfactory. Streamfunction values computed close to a doublet (and completely inside the cylinder anyway) will have such great magnitude that you will find the entire region outside the cylinder covered by only one or a few contour increments. You will also want one of your increments to fall exactly on the dividing streamline. To do this you will want one of the contour values to have that desired value. This will be easy for the first two cases; you'll have to consult a fluids book to find the value corresponding to the dividing streamline for the case of the spinning cylinder.

## Verification

Plots of flows resulting from the superposition of the elementary flows above are given in most undergraduate fluids books, e.g., John and Haberman (1988).

## References

John, J.E.A. and Haberman, W.L., *Introduction to Fluid Mechanics*, 3rd Ed., Prentice Hall, Englewood Cliffs, N.J. (1988).

Sabersky, R.H., Acosta, A.J., and Hauptmann, E.G., *Fluid Flow - A First Course in Fluid Mechanics*, 3rd Ed., Macmillan, New York (1989).

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