

Vortex Panel Method for a 2-D Airfoil

Introduction

In this project we implement a two-dimensional version of the vortex panel method, a technique that may be used to find the ideal flow over a lifting surface. Here we will limit ourselves to vortex panels, but when used along with source or doublet panels, the technique introduced here may be used to find the ideal flow over a whole aircraft, including wings, fuselage, nacelles, etc. Calculations of this type are commonly used in industry for aircraft design; and despite being based on ideal flow, are considered useful for aircraft in clean configurations. Another more recently developed numerical scheme, the boundary element method (BEM), is closely related.

The surface of the 2-D airfoil will be represented by a closed polygon of "panels," each of which has a linear distribution of vortex strength. We will require that the flow be tangent to each panel at a particular point, presumably the middle, called the control point and that the flow come smoothly off the trailing edge of the airfoil in accordance with the Kutta condition. Much of the programming effort will be involved in computing the geometric relationships among the panels. This need is a consequence of the fact that each panel affects all others. The resulting system of linear equations for the unknown vortex strengths will be solved using a standard routine based on Gauss elimination, and the results will be used to find the lift distribution.

This project closely follows the description given in Kuethe and Chow (1986), who base their implementation on the work of Stevens et al. (1971). An excellent contemporary reference by the developer of the panel method is given in Hess (1990), while Katz and Plotkin (1991) discuss it in a more general context. That same volume includes an article discussing the use of the panel method in yacht design (Larsson, 1990).

Background

The airfoil outline approximated by a finite number of straight panels. A convenient scheme for dividing up an airfoil into panels, given an equation for its shape, will be discussed later in the implementation section. The panelization of a generic airfoil using that scheme is shown below. A total of 36 panels were used; the symbols are used to indicate the endpoints. Alternatively, station and ordinate data for tabulated shapes (see e.g., Abbott and Von Doenhoff, 1959) may be used as the ends (boundary points) of the panels.

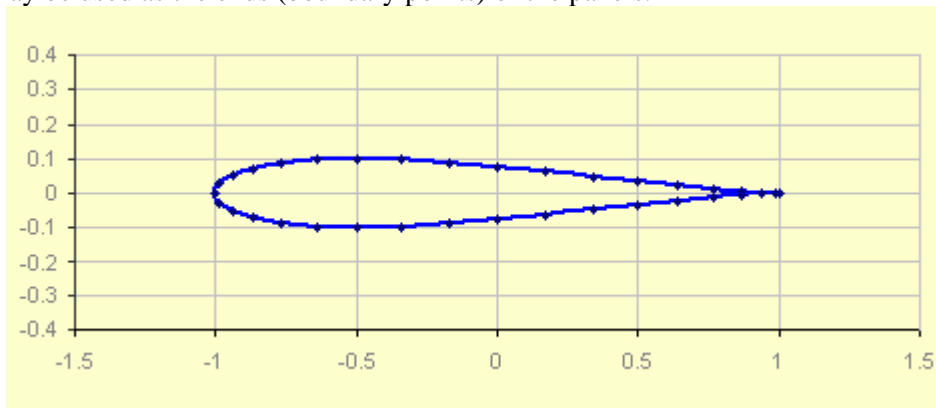


Figure 1. Panelization of a simple airfoil

Each panel is assumed straight and the ends of each are referred to as "boundary points". In the diagram below, which details the relationship between two representative panels, upper case refers to the boundary points. The midpoint of each segment, the point at which the condition that there be no normal velocity is applied, is referred to as the control point. In the same diagram lower case x and y refer to positions along the segment. While x_i, y_i refer to the control point on panel i, the position coordinates x_j, y_j refer to any point on panel j. This distinction is made because the no-normal-velocity requirement is made only at the control point, but each control point feels the effect of the vortex distribution along the whole length of the j^{th} panel.

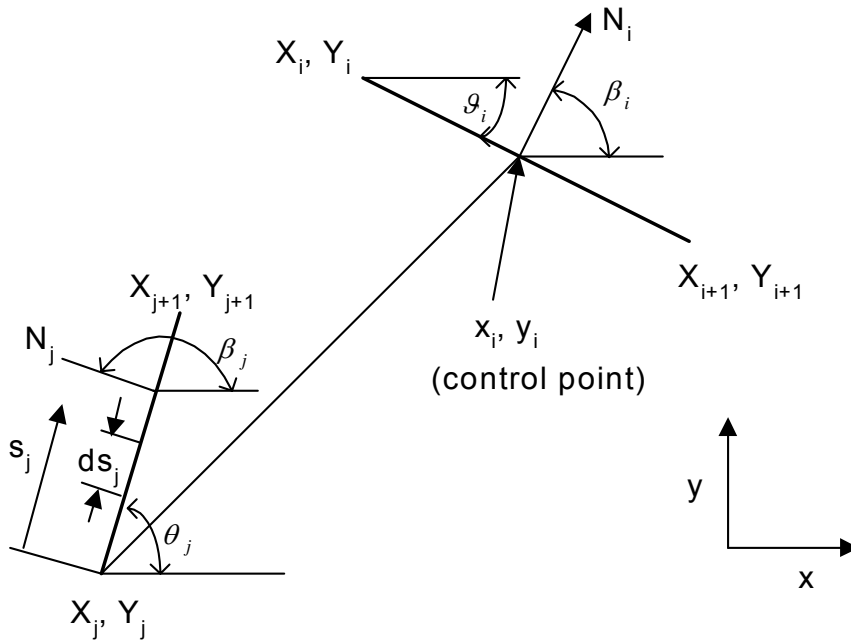


Figure 2. Panel geometry

The velocity potential at x_i, y_i , a representative control point, is the summation of that from the free stream velocity plus the effect of all vortex panels along the airfoil; i.e.,

$$\phi(x_i, y_i) = V_\infty (x_i \cos \alpha + y_i \sin \alpha) - \sum_{j=1}^m \int_j \frac{\gamma(s_j)}{2\pi} \tan^{-1} \left(\frac{y_i - y_j}{x_i - x_j} \right) ds_j. \quad (1)$$

Note that the integral is along the length of the j^{th} (contributing) panel and α is the angle that the free stream makes with the horizontal. Following Stevens et al. (1971) the vortex strength is taken to vary linearly along each panel; i.e.,

$$\gamma(s_j) = \gamma_j + (\gamma_{j+1} - \gamma_j) \frac{s_j}{S_j} \quad (2)$$

The denominator S_j is the total length of the j^{th} panel. While other formulations may take the vortex strength as uniform for each panel, the Stevens formulation has the advantage that the vortex strength is continuous at boundary points.

In terms of the velocity potential, the condition that there be no normal velocity at the control points is given by:

$$\frac{\partial}{\partial n_i} \phi(x_i, y_i) = 0, \quad (3)$$

where here i runs from 1 to m , the total number of panels. Carrying out the differentiation represented by Eqn. 3 on Eqn 1, Kuethe and Chow (1986) give the following results:

$$\sum_{j=1}^m (C_{n1_j} \gamma'_j + C_{n2_j} \gamma'_{j+1}) = \sin(\theta_i - \alpha) \quad (4)$$

This equation applies for all i from 1 to m , again that is, at each panel. Here $\gamma' = \frac{\gamma}{2\pi V_\infty}$ and θ_i is as indicated on the figure. The coefficients are given by

$$C_{n1_j} = 0.5DF + CG - C_{n2_j} \quad (5a)$$

and

$$C_{n2_j} = D + 0.5QF/S_j - (AC + DE)G/S_j \quad (5b)$$

Here, as also given by K&C:

$$A = -(x_i - X_j) \cos \theta_j - (y_i - Y_j) \sin \theta_j \quad (6a)$$

$$B = (x_i - X_j)^2 + (y_i - Y_j)^2 \quad (6b)$$

$$C = \sin(\theta_i - \theta_j) \quad (6c)$$

$$D = \cos(\theta_i - \theta_j) \quad (6d)$$

$$E = (x_i - X_j) \sin \theta_j - (y_i - Y_j) \cos \theta_j \quad (6e)$$

$$F = \ln \left(1 + \frac{S_j^2 + 2AS_j}{B} \right) \quad (6f)$$

$$G = \tan^{-1} \left(\frac{ES_j}{B + AS_j} \right) \quad (6g)$$

$$P = (x_i - X_j) \sin(\theta_i - 2\theta_j) + (y_i - Y_j) \cos(\theta_i - 2\theta_j) \quad (6h)$$

and

$$Q = (x_i - X_j) \cos(\theta_i - 2\theta_j) - (y_i - Y_j) \sin(\theta_i - 2\theta_j) \quad (6i)$$

Equations 5a and 5b for the normal coefficients are replaced by the simpler forms

$$C_{n1_i} = -1, C_{n2_i} = 1,$$

for the case when $i = j$. At this point we have m equations for the $m+1$ unknown vortex strengths. The last equation comes from the condition that the flow comes smoothly off the trailing edge. The "Kutta condition" requires that

$$\gamma'_1 + \gamma'_{m+1} = 0. \quad (7)$$

Once the vortex strengths have been found by solving Equations 4 and 7, you will want to determine the (tangential) velocity along the airfoil surface. Kuethe and Chow give:

$$V_i = \cos(\theta_i - \alpha) + \sum_{j=1}^m (C_{t1_j} \gamma'_j + C_{t2_j} \gamma'_{j+1}), \quad (8)$$

which is applicable again from $i = 1$ to m . Here the tangential coefficients are given by,

$$C_{t1_j} = 0.5 CF - DG - C_{t2_j} \quad (9a)$$

and

$$C_{t2_j} = C + 0.5PF/S_j + (AD - CE)G/S_j \quad (9b)$$

All quantities are as defined above. For the special case where $i = j$, Eqn. (9a) and (9b) reduce to $C_{t1_i} = C_{t2_i} = \pi/2$.

Implementation

1. The first step in this exercise is to either compute or input boundary point data for the chosen airfoil. Station and ordinate data for one particular airfoil, the NACA 747A315 (Abbott and von Doenhoff, 1959) are supplied in tabular form in the file NACA747.FOR (noting that that data is normalized by the chord length) and similar data for dozens of wing sections may be found in the same reference. As an alternative, Kuethe and Chow (1986) give a convenient way of "panelizing" an airfoil for which an equation is known. Briefly summarized, a circle centered at midchord and passing through the leading and trailing edge is drawn. The circumference of that circle is divided into as many arcs of equal length as the number of panels desired. The

intersections of vertical lines drawn from the endpoints of the arcs to the x-axis and the line outlining the airfoil become the boundary points. This scheme automatically distributes more (shorter) panels near the leading and trailing edges, where more are presumably needed. The K&C method is implemented in the program FOILDR.FOR where the coordinates of the boundary points are the variables XB and YB. (Note that in FOILDR the airfoil is already at a non-zero angle of attack. Either rotate it back to zero degrees or use a zero angle of attack in subsequent calculations.)

2. With the boundary points as determined in step 1, the next step (one DO (or For) LOOP) is to determine the "simple" geometric factors for each panel. These include the length of each panel S_j , the x and y coordinates of the control points (taken as the center of the panel), the angle θ (Hint, use the ATAN2 function), the sine and cosine of θ , and the right hand side of Equation 4.

3. The third step is to compute the geometric factors involving the interactions among panels. These include the quantities denoted A - G in Equations 6a - 6g, P and Q in Equations 6h and 6i and the normal coefficients C_{n1_j} and C_{n2_j} in Equations 5a and 5b. In the same nested DO LOOP you can also compute C_{t1_j} and C_{t2_j} as given above. Many of these quantities have already been coded for you in the program segment "PAN.FOR," which is provided.

4. After all the coefficients have been determined; they must be formed into a system of linear equations. For illustration purposes let us assume the airfoil has been divided into 6 panels. This gives $NPTS = 7$ unknown source strengths. The resulting system of equations is then of the form:

$$\begin{bmatrix} AN(1,1) & AN(1,2) & AN(1,3) & AN(1,4) & AN(1,5) & AN(1,6) & AN(1,7) \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 1.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 1.0 \end{bmatrix} \begin{bmatrix} Y'_1 \\ Y'_2 \\ Y'_3 \\ Y'_4 \\ Y'_5 \\ Y'_6 \\ Y'_7 \end{bmatrix} = \begin{bmatrix} RHS(1) \\ RHS(2) \\ RHS(3) \\ RHS(4) \\ RHS(5) \\ RHS(6) \\ RHS(7) \end{bmatrix}$$

Here we note that I increases downward from the top and J increases from left to right. Then for $I < NPTS$, i.e., for $I = 1$ to 6 here, the first column is given by $AN(I,1) = CN1(I,1)$. The middle, that is for $j = 2$ to $NPTS-1$, is given by $AN(I,J) = CN1(I,J) + CN2(I,J-1)$. The last column is given by $AN(I,NPTS) = CN2(I,NPTS-1)$; while the $RHS(I) = \sin(\theta_i - \alpha)$. The last row comes from the Kutta condition, which from Equation 7 above gives $AN(NPTS,1) = AN(NPTS,NPTS) = 1.0$. All other entries in this row, including the right hand side, are 0.0 and must be initialized as such.

5. Solve the system of equations. This is a full system, not sparse, because each panel interacts with all others. You might use the subroutines SGECO/SGESL from LINPACK. (LINPACK is obsolescent but the corresponding newer LAPACK subroutines are, because of their generality, very long.) The program LINEQN shows you how to call these subroutines and their internal documentation is very thorough.

6. Find the tangential velocity at each control point using Equation 8 and then use it to find the pressure coefficient applicable at each control point using $c_p = 1.0 - V^2$.
7. Run your program for angles of attack of 0° , 10° , and some higher value for which you know the assumptions used are invalid. Plot c_p versus chord for top and bottom surfaces for all three cases and comment.

Some Fun Extras

1. Set up a rectangular grid of points encompassing the airfoil, find the streamfunction at each of these points from the summed effect of all panels plus the freestream velocity and plot contours of streamfunction. Your flow should come smoothly off the trailing edge – even at very high angles of attack where you know there would be separation.
2. Integrate your computed values of local c_p numerically to get the section lift coefficient. Run your code for several values of angle of attack and compare with published values.
3. Add a simple flap (or aileron) to your calculation.
4. Implement a separation criterion that would warn the user when the ideal flow analysis used in your program is apt to break down.
5. According to Abbott and von Doenhoff, the significance of the NACA 747A315 designation is as follows: The first "7" indicates the NACA series number, the "4" indicates that at the design lift coefficient a favorable pressure gradient exists over the first 40% of the upper surface, while the second "7" indicates the same condition for the first 70% of the lower. The "3" indicates the design lift coefficient of 0.3 and the "15" indicates the maximum thickness in terms of the chord. Use your results from extra item 2 above to find the design angle of attack and verify the extent of the favorable pressure gradient.
6. Set up your program to analyze a two-dimensional car or truck. You will need to do some research on how to handle the rear of the vehicle and the effect of the ground. See, e.g., Hucho (1987).

Verification

At 0° angle of attack, the symmetric airfoil given by the equation above should have identical c_p 's on the top and bottom surfaces, resulting in no net lift.

Local c_p is given for many airfoils in Abbott and von Doenhoff (1959). Values of the section lift coefficient (c_l) for the NACA747A315 airfoil as a function of angle of attack (scaled from a plot in Abbott and von Doenhoff) are as follows: -8° , $c_l = -.75$; -4° , $c_l = -0.3$; 0° , $c_l = +.12$; 4° , $c_l = +.52$; 8° , $c_l = 0.9$.

References

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Software Provided

LINPACK - Includes SGECO and SGESL subroutines for solving linear equations.

LINEQN - Demonstrates how to call Linpack subroutines.

PAN.txt - A program segment that computes many of the geometric quantities

FOILDR.FOR, Foildraw.xls - A program that "panelizes" an airfoil using the Kuethe and Chow scheme and draws it on screen.

NACA747.FOR - Coordinate data for the NACA 747A315 airfoil.

panelmet.doc 2/19/08