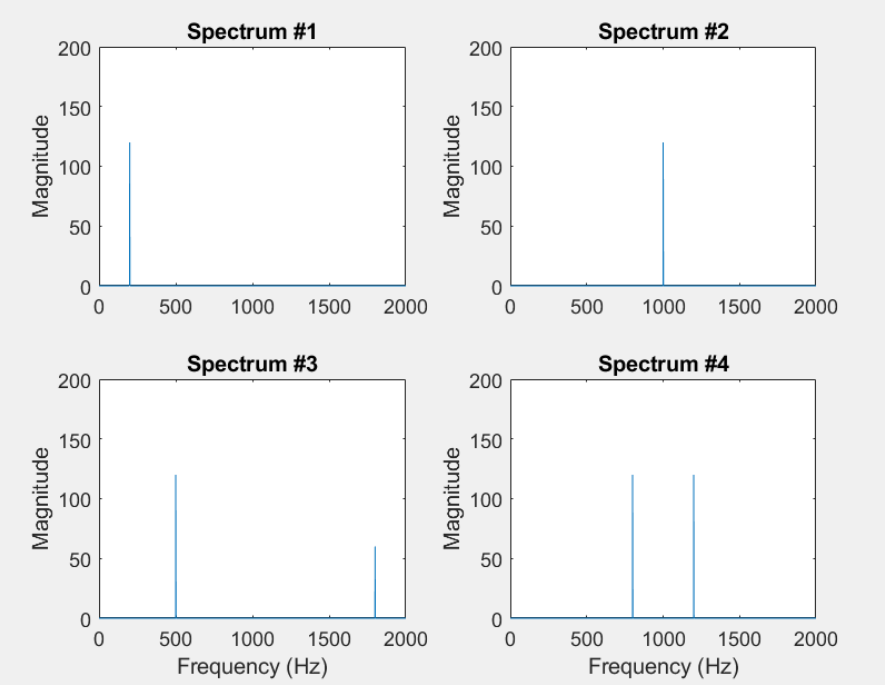
ECE 2066: Science of Information

Fall 2018

Homework 3

Instructions: Put both your name and computing ID (e.g., ffh8x) on the homework. You must show your work to receive full credit for a problem. Please turn in legible and complete answers. While you are encouraged to discuss your approach with your classmates, all your answers must be your own.

1. We're making a guitar tuner app for the iPhone. We first want to match pure tones (sinusoids) with their spectra. The magnitude spectra corresponding to 4 signals (a) – (d) are listed below. For each signal, list the plot that best matches that signal's magnitude spectrum.
    a. $120\cos(2\pi 500t) - 60\cos(2\pi 1800t)$
    b. $120\cos(2\pi 1000t)$
    c. $-120\cos(1256t)$
    d. $120\sin(2\pi 800t) + 120\cos(2\pi 1200t)$



2. Apple wants to make the new iPhone compatible with more service providers, so they are adding more signals to their coverage. However, they don't know what the spectra of these new signals would look like. Apple's top engineers couldn't figure it out, so they asked engineers from NASA, Google, and AT&T, but they couldn't figure it out either. Help out the Apple engineers by sketching the positive spectra of the signals
    a. $x(t) = \cos(628t)$
    b. $x(t) = \sin(1257t) + \cos(942t)$
    c. $x(t) = \cos(1885t) * \cos(1257t)$

i. Consider using the identity $\cos(a)\cos(b) = \frac{1}{2}(\cos(a+b) + \cos(a-b))$

3. Have you ever heard that odd hum when you are strolling outside by overhead power lines? Well, we took a frequency spectrum of that hum:

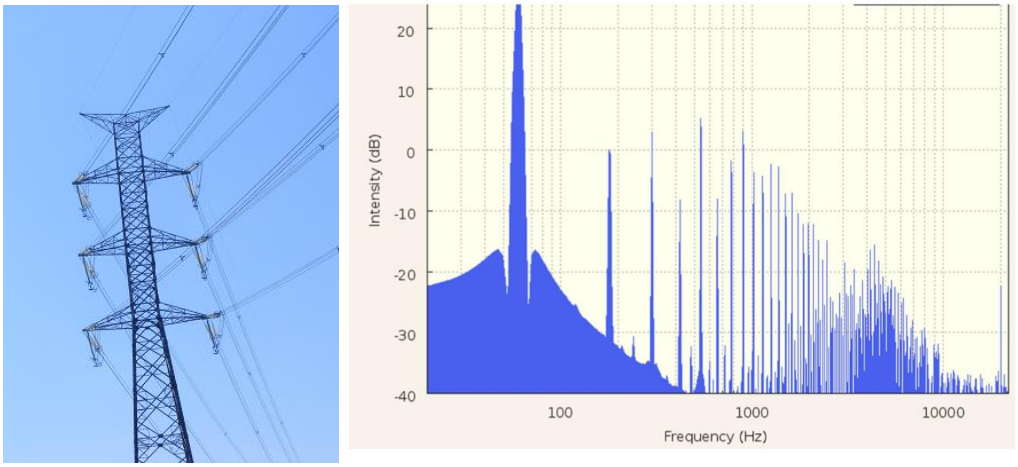


Image of overhead power lines and its spectrum

a. What is the fundamental frequency of this wave?
b. At what frequencies are the harmonics appearing?
c. Why is it at this frequency?

4. When is run-length coding useful?

Consider the following run-length coding algorithm that maps binary sequences (e.g. a file) to sequences over the alphabet {0,1,*}:

- Replace each run (same symbol repeating in a row) by its length in binary and separate the runs by *
- If the first run is a run of 1s, indicate that by adding a * to the beginning of the encoded sequence

For example: $001110 \rightarrow$ 10*11*1 and $10 \rightarrow$*1*1

1. Encode the following two sequences:
   a. 000000000000010000000000011
   b. 011001011101001110110011
   c. In which case is run-length coding more efficient? Why?
2. Among all binary sequences of length 1024 (1kbits)[1]:
   a. Which sequence has the shortest encoded form? What is this length?
   b. Which sequence has the longest encoded form? What is this length?

[1] 1k can either mean $2^{10} = 1024$ or $10^3 = 1000$.

3. Bonus (0.1% of course score): Here we encoded a binary sequence as a sequence over the alphabet {0,1,*} because we needed to separate the runs? Can you suggest a way to do run-length coding with binary alphabet?