

Quiz 13

Decide whether X and Y are independent for the following joint pdfs. If they are independent, identify the marginal pdf's f_X and f_Y . If they are not, give a reason. You may need $\int \ln t = t \ln t - t$.

- $f_{X,Y}(u, v) = -\frac{v \ln(u/2)}{12}$ for $0 \leq u \leq 2, 2 \leq v \leq 4$ and 0 elsewhere.

Solution

First, we check the support and we see that it is in fact a product set so the RVs *could* be independent. We find the marginals to verify. For $0 \leq u \leq 2$,

$$f_X(u) = -\frac{\ln(u/2)}{12} \int_2^4 v dv = -\frac{\ln(u/2)}{2}.$$

For $2 \leq v \leq 4$,

$$f_Y(v) = \frac{v}{6} \int_0^2 \frac{\ln(u/2)}{2} du = \frac{v}{6}.$$

The fact that $f_X(u) f_Y(v) = f_{X,Y}(u, v)$ over the support proves that X and Y are independent.