

Quiz 12

Let X and Y be random variables with the following joint pdf.

$$f_{X,Y}(x,y) = \begin{cases} 1/2, & x \geq 0, y \geq 0, x+y \leq 1 \\ 3/2, & x \geq 0, y \geq 0, 1 \leq x+y \leq 2 \end{cases}$$

Find $P(Y \geq \frac{X}{2})$.

Solution

The region corresponding to the intersection of $Y \geq X/2$ and the support is the triangle $\{(0,0), (1,0), (1, 1/2)\}$. This triangle can be divided into two triangles A and B where the pdf over A equals $1/2$ and the pdf over B equals $3/2$. So

$$P(Y \geq X/2) = |A|(1/2) + |B|(3/2)$$

where $|A|$ is the area of A and $|B|$ is the area of B . The point shown by an arrow is obtained by solving the equations

$$\begin{cases} y = x/2, \\ x + y = 1, \end{cases}$$

hence the point is $(2/3, 1/3)$. Thus

$$\begin{aligned} P(Y \geq X/2) &= |A|(1/2) + |B|(3/2) \\ &= \frac{(1/3)(1)}{2}(1/2) + \frac{(1/3)(1/2)}{2}(3/2) \\ &= \frac{1}{12} + \frac{1}{8} = \frac{2+3}{24} = \frac{5}{24}. \end{aligned}$$

