

Quiz 4

Quiz Date: Friday, June 22

Problem 1

Five cards are randomly drawn from a standard deck of 52 cards. What is the conditional probability of TWO PAIRS given that the first two cards have different numbers. TWO PAIRS is the event that there are two of the cards have one number, another two have a different number, and the fifth card has a number different from the either of the other numbers. Example:

$$2\heartsuit 2\spadesuit K\heartsuit K\spadesuit 9\spadesuit$$

Solution

As in PS4, we may assume that the first two cards are 1H and 2H. We have the following cases. Suppose x is a number other than either 1 or 2.

a) Hands of the form 1122x: there are $\binom{3}{1}\binom{3}{1}11\binom{4}{1}$ such hands.

b) Hands of the form 11xx2: there are $\binom{3}{1} \cdot 11 \cdot \binom{4}{2}$ such hands.

c) Hands of the form 22xx1: there are $\binom{3}{1} \cdot 11 \cdot \binom{4}{2}$ such hands.

The size of the *reduced* sample space is $\binom{50}{3}$. So the conditional probability is

$$\frac{\binom{3}{1}\binom{3}{1}11\binom{4}{1} + 2\binom{3}{1} \cdot 11 \cdot \binom{4}{2}}{\binom{50}{3}} = \frac{99}{2450} = 0.0404082.$$

Just for fun!

Let β denote the conditional probability obtained above.

The (unconditional) probability of TWO PAIRS is

$$\alpha = \frac{\binom{13}{2}\binom{4}{2}\binom{4}{2}11\binom{4}{1}}{\binom{52}{5}} = 0.047539.$$

The conditional probability of TWO PAIRS given that the first two cards are of the same number is

$$\gamma = \frac{12\binom{4}{2}11\binom{4}{1}}{\binom{50}{3}} = 0.161633.$$

The probability that two randomly chosen cards have the same number is

$$\frac{13\binom{4}{2}}{\binom{52}{2}} = \frac{1}{17}$$

and the probability that they have different numbers is

$$\frac{\binom{13}{2}4^2}{\binom{52}{2}} = \frac{16}{17}.$$

Verify that

$$\alpha = \frac{16}{17}\beta + \frac{1}{17}\gamma.$$

Why is this true?