ECE313		**Summary 2012**
	Problem Set 11	
Reading: Gaussian RVs, Scaling of pdfs		Quiz Date: Fri, July 17

Note: It is very important that you solve the problems first and check the solutions afterwards.

Problem 1

Examples 3.6.1, 3.6.2, 3.6.3, 3.6.4, 3.6.5, 3.6.6

Problem 2

Let μ and σ respectively denote the statistical mean and standard deviation of scores in an exam. The grading policy suggests the following assignment of grades:

- A: scores at least $\mu + \sigma$
- B: scores in the range $[\mu, \mu + \sigma)$
- C: scores in the range $[\mu - \sigma, \mu)$
- D: scores in the range $[\mu - 2\sigma, \mu - \sigma)$
- F: scores less than $\mu - 2\sigma$

Assume that the scores can be approximated by a Normal density function with mean μ and standard deviation σ. For each grade, find the percentage of the students receiving that grade. In a class of size 30, approximately, how many students receive A? Use the table on page 191.

Solution

Let $X \sim N\left(\mu, \sigma^2\right)$.
Percentage who receive A $= P\left(\mu + \sigma \leq X\right) = P\left(\frac{X-\mu}{\sigma} \geq 1\right) = 1 - \Phi\left(1\right) = 16\%$.
Percentage who receive B $= P\left(\mu \leq X < \mu + \sigma\right) = P\left(0 \leq \frac{X-\mu}{\sigma} < 1\right) = \Phi\left(1\right) - \Phi\left(0\right) = 34\%$.
Percentage who receive C $= P\left(\mu - \sigma \leq X < \mu\right) = P\left(-1 \leq \frac{X-\mu}{\sigma} < 0\right) = \Phi\left(0\right) - \Phi\left(-1\right) = 34\%$.
Percentage who receive D $= P\left(\mu - 2\sigma \leq X < \mu - \sigma\right) = P\left(-2 \leq \frac{X-\mu}{\sigma} < -1\right) = \Phi\left(-1\right) - \Phi\left(-2\right) = 14\%$.
Percentage who receive F $= P\left(X < \mu - 2\sigma\right) = P\left(\frac{X-\mu}{\sigma} < -2\right) = \Phi\left(-2\right) = 2\%$.
In a class of size 30, approximately $30 \cdot 0.16 = 4.8 \simeq 5$ students receive A.

Problem 3

Let X be an exponential random variable with parameter λ and let $Y = aX, a > 0$. Show that Y is an exponential random variable.

Solution

For pdf of Y, we have

$$f_Y\left(v\right) = \frac{1}{a} f_X\left(\frac{v}{a}\right) = \frac{\lambda}{a} e^{-\lambda\left(\frac{v}{a}\right)} = \frac{\lambda}{a} e^{-\frac{\lambda}{a}v}, \qquad v \geq 0.$$

So Y is an exponential random variable with parameter $\frac{\lambda}{a}$.