

# Gravity in International Finance<sup>1</sup>

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## **Abstract**

The past decade has witnessed an explosion of papers estimating gravity equations for cross-border financial holdings without much of a theoretical foundation. In this paper we develop a theory for bilateral asset holdings that takes a gravity form. We discuss how to estimate international financial frictions and conduct comparative statics analysis within the context of the theory. We also find though that reasonable extensions of the model no longer generate a gravity form. While this does not significantly complicate estimation and comparative statics analysis, it raises questions about the empirical validity of gravity specifications for cross-border financial holdings that need to be addressed in future work.

# 1 Introduction

The past decade has witnessed an explosion of papers estimating gravity equations for cross-border financial holdings. This used to be the territory of the international trade literature, in which there is a long tradition of estimating gravity equations that relate trade flows to country size and various proxies for trade barriers. At least three factors are driving this interest in estimating gravity equations applied to international finance. One is the discovery that gravity equations for international asset trade fit the data at least as well as for goods trade. The contribution by Portes and Rey (2005) is central in this regard. Second, the release of the Coordinated Portfolio Investment Survey by the International Monetary fund, which contains bilateral portfolio holdings for 67 countries since 2001, has been a key driver as well and most of the recent contributions use this data set.<sup>1</sup> Finally, there is a wealth of potential policy questions that can be addressed through the estimation of gravity equations, such as the impact on globalization of harmonization of financial regulations or the formation of monetary or trade unions.

However, this explosion of empirical work on gravity for cross-border financial holdings has taken place without a solid theoretical foundation. As has been well established in the trade literature (e.g. Anderson and van Wincoop (2003)), estimating gravity equations that are not founded in economic theory can lead to biased estimation results due to omitted variables. It also leads to incorrect comparative statics analysis that does not take into account the general equilibrium effects of changes in cross-border barriers.

This paper is a response to this need for a theoretical foundation of a gravity equation for cross-border asset holdings. We will show that under a certain set

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<sup>1</sup>A substantial number of papers also use data on external claims by banks from the BIS. Some recent papers that have estimated empirical gravity equations for equity, bond and bank holdings include Ahearne, Grier and Warnock (2004), Aviat and Coeurdacier (2007), Balli (2008), Balli et.al. (2008), Balta and Delgado (2008), Berkel (2007), Bertaut and Kole (2004), Buch (2000,2002), Chan et.al. (2005), Coeurdacier and Martin (2009), Coeurdacier and Guibaud (2005), Daude and Fratzscher (2008), de Santis and Gerard (2009), Eichengreen and Luengnaruemitchai (2006), Faruquee, Li and Yan (2004), Forbes (2008), Gande et.al. (2009), Garcia-Herrero et.al. (2009), Gelos and Wei (2005), Ghosh and Wolf (2000), Hahm and Shin (2009), Jeanneau and Micu (2002), Kim, Sung and Wei (2007), Kim, Lee and Shin (2006), Lane and Milesi-Ferretti (2005a,b), Lane (2005), Lee (2008), Martin and Rey (2004), Pendle (2007), Portes and Rey (2005), Portes, Rey and Oh (2001), Rose and Spiegel (2004), Salins and Benassy-Quere (2006), Vlachos (2004) and Yu (2009).

of assumptions it is possible to derive a gravity equation for asset trade. We discuss how to estimate cross-border financial frictions in this context and how to conduct proper comparative statics analysis. The empirical work to date is often inconsistent with the theory in that either proper source and destination country fixed effects are not included or variables are included in the gravity equation that have no theoretical justification for being there (e.g. asset return correlations).<sup>2</sup>

However, we also show that when relaxing the assumptions of the model in many reasonable directions it is no longer possible to write bilateral asset holdings in a gravity form. It is still possible to estimate international financial frictions in this case and to conduct comparative statics analysis. But this is based on more complex non-linear equations that relate bilateral asset holdings to all bilateral financial frictions, measures of country size and asset return risk.

The paper has several parallels to the contribution by Anderson and van Wincoop (2003) in the trade literature. Just like in this paper, their work was motivated by a large empirical gravity literature without any theoretical foundation. They showed how to derive a simple and intuitive gravity equation from theory and developed the implications for empirical estimation and comparative statics. The gravity equation that we derive for cross-border asset trade is closely analogous to that derived by Anderson and van Wincoop (2003) for goods trade. Bilateral financial positions depend on *relative* barriers: bilateral financial barriers relative to average barriers (multilateral resistance) faced by both source and destination countries.

As discussed in Anderson and van Wincoop (2004), two key assumptions are needed to generate a gravity specification for trade in goods where bilateral trade is a product of measures of economic size, a bilateral barrier and multilateral resistance indices. The first is trade separability, which says that total production and expenditure are separable from the bilateral allocation of trade across countries. The second condition is that demand depends on a relative price, such as the price of goods from a particular country relative to an overall price index. These

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<sup>2</sup>There are a couple of exceptions though, including Coeurdacier and Martin (2009), Lane (2005) and Vlachos (2004), where estimation is done in a way that is consistent with the theory that we will develop here. It should also be said that while presently there is no justification for many of the existing empirical gravity specifications, we cannot prove that they have no theoretical foundation. All we can say is that currently there is no theory justifying such specifications and it is best for empirical work to be consistent with existing theory.

conditions are satisfied in a large class of models, including models with product differentiation by country of origin, models with monopolistic competition, the Heckscher-Ohlin model with specialization and even the Ricardian model of Eaton and Kortum (2002).

Such conditions also need to be satisfied to derive a gravity specification for asset trade. A condition analogous to trade separability is that decisions about the overall demand for assets (affected by saving) are separable from the portfolio allocation across assets. This condition is the least problematic and holds in many models. The second condition, that asset demand depends on a relative price, is far less trivial than for goods trade. Asset demand naturally takes a very different form than demand for goods. Optimal portfolio choice leads to asset demand that depends on the inverse of a covariance matrix of all returns times a vector of expected returns of all assets. In that context it is not trivial to relate demand for individual assets to a relative price. Not surprising therefore, we find that a gravity specification for asset trade is much less robust to changes in model assumptions than in the trade literature.

In order to derive our theoretical gravity equation, we start from a simple static portfolio choice framework. Investors can hold claims on risky assets from a large number of countries. Asset returns are affected both by a country-specific and a global component. In addition we allow for trade in a riskfree asset and in an asset whose return is only related to global risk; both are in zero net supply. We introduce international financial frictions in the form of information asymmetries about the country-specific return components.<sup>3</sup> After imposing asset market equilibrium in all markets we show that this leads to a gravity equation where bilateral financial holdings depend on the product of economic size variables (stock market capitalization in the destination country and total investment in stock in the source country) divided by a relative financial friction. The relative friction is equal to the bilateral financial friction divided by the product of multilateral resistance terms from the perspective of source and destination countries.

We consider a variety of generalizations of this benchmark model in which the gravity result falls apart. In particular, we consider the case where there do not exist separate assets that allow agents to hedge factors contributing to cross-border

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<sup>3</sup>A substantial literature has documented the relevance of such information asymmetries across countries. See for example Bae, Stulz and Tan (2008), Ahearne et.al. (2004), Portes and Rey (2005), Kang and Stulz (1997) any many references in those papers.

return correlations. We also consider different financial frictions that take the form of a tax on foreign returns. And finally, we consider the case of only trade in risky assets, which captures an extreme case of borrowing constraints associated with the riskfree asset. In all these cases it is no longer possible to write bilateral asset holdings in a gravity form as the product of country-specific variables (economic size, multilateral resistance or any other country-specific variable) and a bilateral friction.

There are two other theories in the literature that generate a gravity specification for asset trade. One approach is that by Martin and Rey (2004), who derive a gravity equation for financial holdings when countries trade claims on Arrow Debreu securities. An extension by Coeurdacier and Martin (2009) shows that this can lead to a gravity equation that is similar to that for goods trade, with bilateral holdings depending both on bilateral frictions and multilateral resistance indices of source and destination countries. The reason for this is that demand for Arrow Debreu securities takes a similar form as the demand for goods under CES preferences. The differentiation of goods by type in the trade literature is now replaced by an analogous differentiation of assets by states in which they have a payoff. Standard constant relative risk-aversion expected utility can then be written as a function of Arrow Debreu asset holdings in a way that is analogous to CES utility as a function of consumption of differentiated goods.

The main limitation of this approach though is that it is not applicable to the types of financial holdings for which we have cross-border data: bilateral equity, bond and bank holdings. The reason is that these assets, on which the empirical gravity literature is based, have non-zero payoffs in multiple states. More precisely, if the asset from one country has a non-zero payoff, assets from other countries generally have a non-zero payoff as well. In the AD framework, if the asset of a country has a positive payoff, the assets of all other countries have a zero payoff.<sup>4</sup> Turning the argument around, it is sometimes argued that any risky asset can be written as a combination of AD securities. But the problem is that these will then

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<sup>4</sup>This also implies that correlations between the returns on Arrow Debreu securities are actually negative. To see this, let  $r_1$  and  $r_2$  be the return on assets that only have a payoff in respectively state 1 and 2 (e.g.  $r_1(1) > 0$ ,  $r_1(s) = 0$  for  $s \neq 1$ ). Assuming that states 1 and 2 have non-zero probabilities  $\pi(1)$  and  $\pi(2)$ , we have  $cov(r_1, r_2) = Er_1r_2 - Er_1Er_2 = -\pi(1)\pi(2)r_1(1)r_2(2) < 0$ . This stands in contrast to the generally positive correlation between asset returns across countries when applied to stocks, bonds or bank earnings.

be a combination of AD securities from different countries, so that the risky asset is not specific to a particular country.

A second alternative way to derive a theoretical gravity equation, suggested by Lane and Milesi-Ferretti (2005a), is a multi-country extension of the model in Obstfeld and Rogoff (2000) that relates barriers in goods trade to portfolio home bias. While theoretically possible, this approach has drawbacks as well. The main problem is that the real exchange rate hedge channel, through which barriers in goods trade affect asset trade in Obstfeld and Rogoff (2000), does not appear to be operative in practice. Using data on equity returns and real exchange rates, van Wincoop and Warnock (2010) show that hedging real exchange rate risk cannot account for portfolio home bias. Consistent with these findings, Coeurdacier (2009) develops an extension of Obstfeld and Rogoff (2000) to show that for realistic model parameters trade barriers cannot generate a portfolio home bias.

The remainder of the paper is organized as follows. Section 2 derives a gravity theory for financial holdings from a static multi country portfolio choice framework. It discusses what assumptions are needed to derive such a gravity specification. We also discuss some extensions that preserve the gravity result. Section 3 considers several extensions of the benchmark model where we no longer obtain a gravity specification. Section 4 discusses how to estimate bilateral financial frictions and conduct comparative statics analysis, both when the theory leads to gravity and when it does not. Section 5 concludes.

## **2 A Gravity Theory of Financial Holdings**

In this section we develop a gravity model for bilateral asset holdings in a one-good, two-period,  $N$  country framework.

### **2.1 The model**

#### **The Assets**

There are  $N + 2$  assets. The first  $N$  assets are country-specific risky assets. The gravity equation that we will derive applies to these  $N$  assets. We will refer to them as equity, although they could also be other risky assets such as corporate bonds, long-term bonds or bank holdings. The supply of the asset in country  $i$  is

$K_i$ . One can think of this as the capital stock. The equity claim of country  $i$  has a real payoff of  $D_i$  in period 2, where

$$D_i = 1 + \epsilon_i + \theta_i \epsilon_g \quad (1)$$

Here  $\epsilon_i$  is a country-specific payoff innovation and  $\epsilon_g$  is a global payoff innovation. The constant term is 1, which is simply a normalization. The country-specific payoff innovations are uncorrelated across countries and with the global innovation. We allow the response to global innovations to be country-specific. We assume that  $\epsilon_g$  has a mean of 0 and variance  $\sigma_g^2$ . The distribution of the country-specific innovation  $\epsilon_i$  is discussed below. The price of a country  $i$  equity claim in period 1 is  $Q_i$ .

The second asset is a riskfree bond that is in zero net supply. The bond pays one unit of the good in period 2 and has a period 1 price of  $Q_f$ . Finally, there is an asset whose return is perfectly correlated with the global shock. This asset is also in zero net supply. It has a period 1 price of  $Q_g$  and a period 2 payoff of  $D_g = 1 + \theta_g \epsilon_g$ . This asset allows agents to hedge global risk.

We will write the returns on the  $N + 2$  assets as

$$R_i = \frac{D_i}{Q_i} \quad i = 1 \dots N \quad (2)$$

$$R_f = \frac{1}{Q_f} \quad (3)$$

$$R_g = \frac{D_g}{Q_g} \quad (4)$$

These assumptions about the asset market structure are obviously restrictive and we will discuss below how results change when we relax them. At this point we only briefly comment on the global asset. It allows agents to hedge the global risk factor, so that the only risk that matters for portfolio allocation across the  $N$  equity is the country-specific risk. This significantly simplifies the portfolio allocation problem and we will see that it is critical to derive a gravity equation for bilateral asset holdings.

One way to interpret the global asset is as a global equity futures contract, allowing one to buy or sell a claim on the global equity payoff at a futures price of  $f^g$ . The payoff on such a contract is

$$1 + \theta_g \epsilon_g + \sum_{i=1}^N (K_i/K) \epsilon_i - f^g \quad (5)$$



where  $K$  is the global capital stock and  $\theta_g = \sum_{i=1}^N (K_i/K)\theta_i$ . The payoff depends on the global shock through the term  $\theta_g\epsilon_g$  in exactly the same way as the assumed global asset. Note though that it is not exactly the same as our global asset when the third term that depends on the idiosyncratic shocks is not zero. As a result of the law of large numbers this term will be close to zero when there are many small countries. But with some big countries like the United States and Japan, this is not necessarily the case.

A second, and closely related, possibility is to interpret the global asset as an equity futures contract on a set of multinational firms. For such firms country-specific shocks naturally play less of a role as a result of their global operations. A third possibility is to interpret the global asset as a derivative whose payoff is specifically connected to shocks that affect the entire world economy, such as an oil price futures contract. Admittedly though, each of these interpretations of the global asset clearly has their limitations. We will therefore discuss below how results change when we do not allow for such an asset.

### Consumption and Portfolio Choice

Agents in country  $j$  are born with an endowment of  $Y_j$  in period 1 plus a claim on all country  $j$  equity. The wealth of country  $j$  agents in period 1 after consumption is therefore

$$W_j = Y_j + Q_j K_j - C_j^1$$

where  $C_j^1$  is period 1 consumption.

In period 1 agents decide how much to consume and how to allocate the remainder of the wealth across the  $N + 2$  assets. The budget constraint is

$$C_j^2 = W_j R_j^p = (Y_j + Q_j K_j - C_j^1) R_j^p \quad (6)$$

where the portfolio return is

$$R_j^p = \sum_{i=1}^N \alpha_{ij} R_i + \alpha_{gj} R_g + \alpha_{fj} R_f \quad (7)$$

Here  $\alpha_{ij}$  is the fraction invested in country  $i$  equity,  $\alpha_{gj}$  the fraction invested in the global asset and  $\alpha_{fj}$  the fraction invested in the riskfree asset. These portfolio shares sum to 1.

Agents maximize

$$\frac{(C_j^1)^{1-\gamma}}{1-\gamma} + \beta \frac{E(C_j^2)^{1-\gamma}}{1-\gamma} \quad (8)$$

The first-order conditions for consumption and portfolio choice are

$$(C_j^1)^{-\gamma} = \beta E (C_j^2)^{-\gamma} R_j^p \quad (9)$$

$$E (C_j^2)^{-\gamma} (R_i - R_f) = 0 \quad i = 1 \dots N \quad (10)$$

$$E (C_j^2)^{-\gamma} (R_g - R_f) = 0 \quad (11)$$

(9) is the standard consumption Euler equation that represents the tradeoff between consumption in periods 1 and 2. (10) is a portfolio Euler equation that represents the tradeoff between investment in the equity claim of country  $i$  and the riskfree asset. Finally, (11) is a portfolio Euler equation that represents the tradeoff between investment in the global and riskfree assets.

The market clearing conditions for country  $i$  equity, the global asset and the riskfree asset are

$$\sum_{j=1}^N \alpha_{ij} W_j = Q_i K_i \quad (12)$$

$$\sum_{j=1}^N \alpha_{gj} W_j = 0 \quad (13)$$

$$\sum_{j=1}^N \alpha_{fj} W_j = 0 \quad (14)$$

The period 1 and 2 goods market clearing conditions are

$$\sum_{j=1}^N C_j^1 = \sum_{j=1}^N Y_j \quad (15)$$

$$\sum_{j=1}^N C_j^2 = \sum_{j=1}^N D_j \quad (16)$$

## Information Asymmetry

We assume that due to differences in language and regulatory systems, and easier access to local information, domestic agents are more informed than foreigners about the idiosyncratic payoff innovations on domestic equity claims. From the perspective of agents in country  $j$ ,  $\epsilon_i$  has a mean of 0 and variance

$$\tau_{ij} \sigma_i^2 \quad (17)$$

Information asymmetry is therefore captured by  $\tau_{ij} > \tau_{ii}$  when  $j \neq i$ .<sup>5</sup>

Since this assumption is critical to the derivation of the gravity equation for asset trade, it deserves further discussion. What makes the derivation of a gravity equation for asset trade different from goods trade is that asset trade necessarily involves risk. Without risk there would just be a single riskfree asset that is the same for each country. We know from covered interest rate arbitrage that riskfree returns are indeed equalized across industrialized countries. When introducing financial frictions it is therefore natural to relate them to risk.

There is a substantial body of evidence showing that information asymmetries exist and are relevant in explaining portfolio home bias. Without conducting an extensive survey, we mention just a couple of relevant papers. Bae, Stulz and Tan (2008) find that the absolute forecast error of annual earnings per share is 7.8% higher for foreign analysts than local analysts. Ahearne et.al. (2004) find that home bias of U.S. investors relative to other countries is significantly reduced when the stock of foreign countries is traded on centralized exchanges. This reduces information barriers as a result of the regulatory and accounting burden imposed on such foreign firms. Portes and Rey (2005) find that “the geography of information is the main determinant of the pattern of international (financial) transactions”, documenting the effect of a variety of information frictions on cross-border equity flows. Kang and Stulz (1997) document that investors tend to invest in foreign firms for which information barriers are lower (large firms with good accounting performance, low unsystematic risk and low leverage).

Information is not exogenous. Investors may acquire more information about countries that they are less informed about. However, this will not necessarily eliminate information asymmetries. van Nieuwerburgh and Veldkamp (2009) show that information asymmetries will in fact be amplified when allowing agents to acquire information about different asset payoffs. The reason for this is that it is optimal to acquire more information about assets that have a large weight in the portfolio, which happen to be assets that agents are already relatively well informed about.

Modeling the financial friction  $\tau_{ij}$  as an information friction differs from the

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<sup>5</sup>While we assume that agents in different countries have different quality signals about  $\epsilon_i$ , we assume that the expectation of  $\epsilon_i$  is the same across countries. This can be justified in models with a continuum of agents in each country. See for example van Nieuwerburgh and Veldkamp (2009).

approach in a number of papers that introduce a financial friction simply as a tax or transaction cost that reduces the return on foreign investment. Examples are Tille and van Wincoop (2010a,b), Coeurdacier (2009), Coeurdacier and Guibaud (2005) and Martin and Rey (2004). Many types of capital controls can be thought of as a tax. Danthine et.al. (2000) show that transaction costs are larger for cross-border than domestic transactions. We will discuss in Section 4 how results change if instead we model the friction as a tax or transaction cost.<sup>6</sup>

## 2.2 Derivation of Gravity Equation

In solving the model we apply the local approximation solution method developed by Tille and van Wincoop (2010a) and Devereux and Sutherland (2011). We focus on what in a more dynamic model would be called the “deterministic steady state” of asset allocation. In more technical terms, this is the zero-order component. Leaving the algebraic derivations to the Appendix, and omitting the technical order component notation used in the Appendix, we obtain the following intuitive expression for equity portfolio shares:

$$\alpha_{ij} = \frac{1}{\gamma R \sigma_i^2 \tau_{ij}} \left[ E(R_i - R_f) - \frac{\theta_i}{\theta_g} E(R_g - R_f) \right] \quad (18)$$

where  $R$  is the zero-order component of asset returns that is the same for all assets. As is quite standard, portfolio shares depend on the ratio of the expected excess return (second-order component) and the variance of the excess return. As global risk can be separately hedged, both the expected excess return and its variance remove the global components. The expected excess return therefore subtracts the part that is a compensation for global risk. Analogously, the variance of the excess return only refers to country-specific risk.

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<sup>6</sup>Two other explanations for portfolio home bias that have received extensive attention in the literature are associated with a hedge against uncertainty about the return on non-traded assets (e.g. labor income) and a hedge against real exchange rate risk (e.g. non-traded goods or any other source of deviations from PPP). However, empirically these explanations have not fared very well. van Wincoop and Warnock (2010) show that the second explanation can explain virtually no home bias at all. Bottazzi et.al. (1994) and Julliard and Rosa (2009) find that the non-traded asset explanation also does not generate much home bias. It should be said though that there remains an ongoing debate about the role of non-financial wealth (non-traded assets). See Coeurdacier and Gourinchas (2009) for a recent contribution.

Now define

$$\frac{1}{p_i} = \frac{1}{\gamma R \sigma_i^2} \mathbb{E} \left[ R_i - R_f - \frac{\theta_i}{\theta_g} (R_g - R_i) \right] \quad (19)$$

The variable  $p_i$  is proportional to a risk-return ratio: the amount of country-specific risk of asset  $i$  as captured by the variance  $\sigma_i^2$ , divided by the expected excess return. The higher  $p_i$ , the lower the demand for the asset. The variable  $p_i$  is endogenous as it depends on the second-order component of the expected excess return that in equilibrium adjusts to clear equity markets through second-order changes in asset prices. Given the definition of  $p_i$ , portfolio allocation (18) becomes

$$\alpha_{ij} = \frac{1}{\tau_{ij} p_i} \quad (20)$$

We can think of  $\tau_{ij} p_i$  as the “price” (risk-return ratio) faced by agents from country  $j$  investing in country  $i$ .

Write total equity holdings by agents from country  $j$  as

$$E_j = \sum_{i=1}^N \alpha_{ij} W_j \quad (21)$$

Substituting (20) yields

$$W_j = E_j P_j \quad (22)$$

where

$$\frac{1}{P_j} = \sum_{i=1}^N \frac{1}{\tau_{ij} p_i} \quad (23)$$

Using this, we can write the total equity claim  $X_{ij} = \alpha_{ij} W_j$  by country  $j$  on country  $i$  as

$$X_{ij} = \frac{P_j}{\tau_{ij} p_i} E_j \quad (24)$$

This equation is critical to what follows. Bilateral asset demand depends on a relative price: the “price” (risk-return ratio) of country  $i$  equity relative to an overall price index.

Similar to goods trade, we can now derive a gravity specification by combining this demand equation with a set of market clearing equations. The asset market clearing condition for country  $i$  equity is

$$\sum_{j=1}^N X_{ij} = S_i \quad (25)$$

where  $S_i = Q_i K_i$  is the country  $i$  equity supply. Also define  $E = S = \sum_{j=1}^N E_j = \sum_{i=1}^N S_i$  as the world demand and supply of equity. Then the market clearing condition (25) gives the following solution for  $p_i$ :

$$p_i = \frac{S}{S_i} \frac{1}{\Pi_i} \quad (26)$$

where

$$\frac{1}{\Pi_i} = \sum_{j=1}^N \frac{P_j}{\tau_{ij}} \frac{E_j}{E} \quad (27)$$

Substituting this solution for  $p_i$  back into (23) and (24), we get the following gravity specification for bilateral asset holdings:

$$X_{ij} = \frac{S_i E_j}{E} \frac{\Pi_i P_j}{\tau_{ij}} \quad (28)$$

$$\frac{1}{P_j} = \sum_{i=1}^N \frac{\Pi_i}{\tau_{ij}} \frac{S_i}{S} \quad (29)$$

$$\frac{1}{\Pi_i} = \sum_{j=1}^N \frac{P_j}{\tau_{ij}} \frac{E_j}{E} \quad (30)$$

$$P_j E_j = W_j \quad (31)$$

For given asset supplies  $S_i$ , (zero-order components of) wealth  $W_j$  and bilateral frictions  $\tau_{ij}$ , equations (29), (30) and (31) can be used to jointly solve for  $P_j$ ,  $E_j$  and  $\Pi_i$  for  $i = 1, \dots, N$  and  $j = 1, \dots, N$ . Together with (28) this determines bilateral asset holdings  $X_{ij}$ .

The gravity equation (28) implies that bilateral asset holdings  $X_{ij}$  are driven by two factors. The first is a size factor: the product of total equity holdings  $E_j$  of country  $j$  and the supply of equity  $S_i$  of country  $i$ , divided by the world demand or supply. The second factor is a relative friction. Just as is the case for trade flows, bilateral asset holdings are driven not simply by the bilateral friction  $\tau_{ij}$ , but rather by the relative friction

$$\frac{\tau_{ij}}{\Pi_i P_j} \quad (32)$$

Here  $\Pi_i$  and  $P_j$  are so-called multilateral resistance variables that measure the average financial frictions for respectively country  $i$  as a destination country and country  $j$  as a source country. Given the size factor  $S_i E_j / E$ , it is this *relative*

*financial friction* that drives the bilateral asset holding  $X_{ij}$ .<sup>7</sup>

In order to understand why bilateral asset holdings are driven by this relative financial friction, as opposed to just  $\tau_{ij}$ , first consider the source country  $j$ . Investors from  $j$  invest a total of  $E_j$  in equity. They will allocate more of this to destination countries for which the bilateral financial friction  $\tau_{ij}$  is low in comparison to the average financial friction  $P_j$  that it faces relative to all destination countries. The relative financial friction (32) is also affected by the multilateral resistance  $\Pi_i$  of the destination country. When  $\Pi_i$  is high, country  $i$  faces high financial frictions with many source countries. In order to generate equilibrium in the market for country  $i$  equity, it will have to offer a low “price”  $p_i$  through a high expected return. For a given bilateral barrier  $\tau_{ij}$  this will raise  $X_{ij}$ .

There is one difference relative to the goods trade gravity literature that is worth pointing out. Since the zero-order component of  $W_j$  does not depend on financial frictions (see Appendix), (31) implies that the total equity investment  $E_j$  by country  $j$  goes down when its multilateral resistance rises. The reason is that higher financial frictions lead to a shift away from risky assets and towards the riskfree asset. This is not usually the case in gravity models for goods trade where  $E_j$  represents the total demand for differentiated goods in country  $j$ . However, when introducing a homogeneous good as well as differentiated goods, one can derive an analogous gravity specification for goods trade.<sup>8</sup> As we will see in Section 4, this relationship between total expenditure on risky assets and multilateral resistance has implications for estimation and comparative statics.

## 2.3 Extensions that Retain Gravity

A key question that we need to address is how robust the gravity specification is to the various assumptions that we have made in the benchmark model. We start by discussing some extensions under which the gravity form is retained. In the next section we discuss a variety of extensions under which gravity no longer applies.

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<sup>7</sup>In the goods trade literature the friction is an ad valorem tariff, which has a non-unitary elasticity in the gravity specification that depends on the elasticity of substitution between the goods. Here instead we have an asymmetric information friction. A 1% increase in a bilateral friction raises the country-specific variance by 1%, which gives rise to a 1% drop in the portfolio share invested in that country (holding all else constant) and therefore a unitary elasticity.

<sup>8</sup>For further discussion of this comparison to the goods trade gravity literature, see the 2010 working paper version of this paper.

We leave most of the algebra related to these extensions to a separate Technical Appendix, only discussing the main results here.

One generalization of the model that leaves the gravity system (28)-(31) intact is to allow for a more general asset payoff structure, while at the same time assuming that there are separate assets that can hedge uncertainty associated with factors responsible for return co-movements. More precisely, assume that the payoff structure is

$$D_i = 1 + \epsilon_i + \sum_{l=1}^L \theta_{il} u_l \quad (33)$$

Where all the innovations  $\epsilon_i$  ( $i = 1, \dots, N$ ) and  $u_l$  ( $l = 1, \dots, L$ ) are uncorrelated. The innovations  $u_l$  are common across countries and lead to return co-movement. The benchmark model is a special case of this where  $L = 1$  and  $u_1 = \epsilon_g$ . The extension allows for additional factors generating co-movement, such as for example regional factors. At the same time we assume that there are  $L$  assets whose respective payoffs only depend on the common factors  $u_l$ . An example is a European equity futures contract when  $u_l$  is a European regional factor.

Under this extension it remains the case that any common asset return risk can be separately hedged, so that it is really only the country-specific risk that matters for portfolio allocation among the  $N$  equity. While this extension has the advantage that the gravity result can hold under a very general covariance structure of asset returns, obviously the assumption that all common components of returns can be separately hedged is a strong one.<sup>9</sup>

Another extension is to allow for fixed costs associated with investment abroad. If this fixed cost is such that investors only hold claims on a subset of foreign countries, so that some of the  $X_{ij}$  are zero, the gravity system (28)-(31) changes very little. All that needs to be changed is the summation over  $i$  in the definition of  $P_j$  and the summation over  $j$  in the definition of  $\Pi_i$ . For  $P_j$  the summation should only be over countries on which country  $j$  investors hold positive claims. For  $\Pi_i$  summation should be over countries for which country  $i$  has positive liabilities.

A third extension, also related to fixed costs, is perhaps more interesting. It separates agents into two groups. For one group the fixed cost of investing abroad is

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<sup>9</sup>By far the most important common component is the global component. In the 2010 working paper version of this paper we find that the average absolute value of the covariance between quarterly stock returns among 24 industrialized countries (2000-2007) is reduced by 88% after controlling for the first principal component.



so large that agents only invest in domestic stocks and bonds. For the other group fixed costs are not large enough to provide a barrier to investment abroad. They behave just like the investors in the benchmark model. This setup is consistent with extensive evidence that many investors only invest in the domestic stock market, as documented by Christelis and Georgarakos (2011), Kyrychenko and Shumb (2009) and many others. The latter paper finds that only about 10% of U.S. investors with directly held stock hold any foreign stock.

This fixed cost is also consistent with a relatively large share of domestic equity (usually well above 50%) held by even very small countries. Without the fixed cost the benchmark model implies that the share of domestic equity should approach zero when the size of the country becomes small. For example, with  $N$  countries of equal size and  $\tau_{ij} = \tau > 1$  for  $i \neq j$  and  $\tau_{ii} = 1$ , the equilibrium share held domestically is  $\tau/(N + \tau - 1)$ , which goes to zero when  $N$  becomes big.

In the Technical Appendix we show that this extension again leaves the gravity system (28)-(31) unchanged. The only difference is that the information friction  $\tau_{ij}$  is now multiplied by what may be called a fixed cost friction  $\delta_{ij}$  where

$$\begin{aligned}\delta_{ij} &= 1 & i \neq j \\ \delta_{ii} &= W_i^A/W_i\end{aligned}$$

Here  $W_i^A/W_i$  represents the share of wealth held by diversified agents of country  $i$  ( $A$  stands for access to foreign markets). For any source country  $j$  this equally raises all the cross-border frictions relative to the domestic friction by a magnitude  $W_j/W_j^A$ . While gravity is retained, this extension does have some implications for estimation and comparative statics that we discuss in Section 4.

A final extension addresses in a slightly different way the large domestic holdings for even very small countries. In the previous extension, when agents do not have access to foreign markets, they optimally diversify their wealth across domestic stocks and bonds. But some holdings of domestic stock may not be the result of a diversification motive at all, not even between domestic stocks and bonds. One example is insider trading. Kho et.al. (2009) report that as much as 50% of stock is held by insiders in industrialized countries. As a result of agency problems, it is often optimal for an executive to invest in the firm at which the executive is employed. This has nothing to do with diversification motives. The absence of diversification may apply to less wealthy investors as well. First, fixed costs may prevent them from being globally diversified. Second, low collateral may prevent

them from borrowing. In that case all wealth may be allocated to domestic stock (or domestic risky assets in general).

Assume that a fraction  $\mu_i$  of the wealth of country  $i$  is invested exclusively in the domestic stock market for reasons entirely unrelated to diversification. In that case the gravity system (28)-(31) remains unchanged. All we need to do is to subtract  $\mu_i W_i$  from the asset supply  $S_i$ , the wealth  $W_i$  and domestic holdings  $X_{ii}$ . Essentially, we need to take the  $\mu_i W_i$  “out of the market”.

### 3 Limitations to Gravity

As already emphasized in the introduction, the gravity result derived in the previous section is far from a general one. In this section we will discuss three quite reasonable extensions of the benchmark model under which the gravity result no longer holds. Most algebraic detail is again left to the Technical Appendix.

#### 3.1 General Covariance Structure of Returns

The first extension is to allow for a general covariance structure of asset returns, while assuming that factors generating return co-movement cannot be separately hedged (e.g. global risk cannot be separately hedged through a global asset). Consider the payoff structure

$$D_i = 1 + \epsilon_i + v_i \tag{34}$$

Here  $\epsilon_i$  is the same country-specific shock as before, with the same variance  $\tau_{ij}\sigma_i^2$  from the perspective of agents from country  $j$ . But payoffs are now also affected by a shock  $v_i$  (uncorrelated with  $\epsilon_i$ ) that is correlated across countries with  $\text{var}(v_1, \dots, v_N)' = \mathbf{\Omega}$ . Note that in the benchmark specification in the previous section  $v_i = \theta_i \epsilon_g$  only captures global shocks. In that case  $\mathbf{\Omega} = \boldsymbol{\theta} \boldsymbol{\theta}' \sigma_g^2$ , where  $\boldsymbol{\theta} = (\theta_1, \dots, \theta_N)'$ . But while we have now further generalized the covariance matrix, the more important assumption is that we no longer allow for assets that hedge the risk associated with the  $v_i$ .

In this case portfolio demand becomes quite complex. Defining the vector of portfolio shares for country  $j$  investors as  $\boldsymbol{\alpha}_j = (\alpha_{1j}, \dots, \alpha_{Nj})'$ , we have

$$\boldsymbol{\alpha}_j = \frac{1}{\gamma} \boldsymbol{\Phi}_j^{-1} \mathbf{E} \mathbf{R} \tag{35}$$

where  $\Phi_j = \Omega + \mathbf{L}_j$ ,  $\mathbf{L}_j$  is a diagonal matrix with  $\tau_{ij}\sigma_i^2$  as the  $i$ 'th diagonal element, and  $\mathbf{ER}$  is a vector of expected excess returns defined as

$$\mathbf{ER} = \frac{1}{R} \begin{pmatrix} E(R_1 - R_f) \\ \dots \\ E(R_N - R_f) \end{pmatrix}$$

These portfolio shares, together with  $X_{ij} = \alpha_{ij}W_j$ , imply

$$X_{.,j} = \frac{1}{\gamma} W_j \Phi_j^{-1} \mathbf{ER} \quad (36)$$

where  $X_{.,j} = (X_{1j}, \dots, X_{Nj})'$ . Imposing the market clearing conditions  $\sum_{j=1}^N X_{ij} = S_i$  implies that the vector of expected excess returns is

$$\mathbf{ER} = \gamma \left( \sum_{k=1}^N \Phi_k^{-1} W_k \right)^{-1} \mathbf{S} \quad (37)$$

where  $\mathbf{S} = (S_1, \dots, S_N)'$  is the vector of equity supplies. Substituting this solution for  $\mathbf{ER}$  back into (36) gives

$$X_{.,j} = W_j \Phi_j^{-1} \left( \sum_{k=1}^N \Phi_k^{-1} W_k \right)^{-1} \mathbf{S} \quad (38)$$

This is a complicated non-linear expression. It relates  $X_{ij}$  to the entire vectors  $(S_1, \dots, S_N)$  and  $(W_1, \dots, W_N)$  of country size variables, the entire covariance matrix  $\Omega$ , all the country-specific payoff variances  $\sigma_i^2$  as well as all the financial frictions  $\tau_{ij}$ .

In the Technical Appendix we show that we can no longer relate  $X_{ij}$  to a relative price as in (24), no matter how we define the price  $p_i$  and price index  $P_j$ . This implies that we can no longer derive the system of gravity equations (28). Even more generally, we cannot write  $X_{ij}$  in any gravity-form, perhaps a different one than derived in the previous section.

In order to see this last point, consider the following very broad definition of a ‘‘gravity’’ specification:

$$X_{ij} = \frac{z}{d_{ij}} Z_i H_j \quad (39)$$

Here  $z$  is a constant,  $d_{ij}$  is a bilateral friction and  $Z_i$  and  $H_j$  are country specific variables. The term gravity originates from physics, where  $X_{ij}$  is the gravitational

force between two objects  $i$  and  $j$ ,  $z$  is the gravitational constant,  $d_{ij}$  is the square of the distance between the objects and  $Z_i$  and  $H_j$  are their masses.

In economics  $d_{ij}$  is often interpreted as distance as well, but more generally as a barrier between  $i$  and  $j$  (trade barrier for goods trade or financial friction for asset trade). Of course for any specification of bilateral asset trade there are always  $d_{ij}$  such that (39) holds. In order for (39) to have meaning as a gravity equation,  $d_{ij}$  must be exclusively related to (financial) frictions between  $i$  and  $j$ . It should not be related to variables unrelated to such frictions, such as moments of asset returns and country size variables.

In theory-based gravity specifications (such as in the previous section)  $Z_i$  and  $H_j$  are products of multilateral resistance and size. However, (38) is inconsistent with (39) for any specification of  $Z_i$  and  $H_j$ , no matter the interpretation. In order to illustrate this we focus on the simple case where  $N = 2$ , where it is possible to analytically invert the various matrices in (38).

Start by defining for  $i, j = 1, 2$

$$\begin{aligned} a_{ij} &= \tau_{ij}\sigma_i^2 + \mathbf{\Omega}_{ii} & b_j &= a_{1j}a_{2j} - \mathbf{\Omega}_{12}^2 \\ e_1 &= \sum_{k=1}^2 W_k a_{2k}/b_k & e_2 &= -\sum_{k=1}^2 W_k \mathbf{\Omega}_{12}/b_k & e_3 &= \sum_{k=1}^2 W_k a_{1k}/b_k \\ h_1 &= e_3 S_1 - e_2 S_2 & h_2 &= e_1 S_2 - e_2 S_1 \end{aligned}$$

We then have

$$\begin{pmatrix} X_{1j} \\ X_{2j} \end{pmatrix} = \frac{1}{e_1 e_3 - e_2^2} \frac{W_j}{b_j} \begin{pmatrix} a_{2j} h_1 - \mathbf{\Omega}_{12} h_2 \\ a_{1j} h_2 - \mathbf{\Omega}_{12} h_1 \end{pmatrix} \quad (40)$$

The question is whether this takes the general form (39), which implies

$$\frac{X_{12} X_{21}}{X_{11} X_{22}} = \frac{d_{11} d_{22}}{d_{12} d_{21}} \quad (41)$$

It is important to emphasize that  $d_{ij}$  is nothing other than a barrier between  $i$  and  $j$ , which in our application must be either equal to  $\tau_{ij}$  or some function of that. Importantly, it should not be a function of other variables like variances, covariances and country size variables.

(40) implies that

$$\frac{X_{12} X_{21}}{X_{11} X_{22}} = \frac{(a_{22} h_1 - \mathbf{\Omega}_{12} h_2)(a_{11} h_2 - \mathbf{\Omega}_{12} h_1)}{(a_{21} h_1 - \mathbf{\Omega}_{12} h_2)(a_{12} h_2 - \mathbf{\Omega}_{12} h_1)} \quad (42)$$

This expression is clearly not just a function of the bilateral barriers  $\tau_{ij}$ . Even when  $\mathbf{\Omega}_{12} = 0$ , so that the expression boils down to  $a_{22} a_{11}/(a_{21} a_{12})$ , it still depends on

the variances  $\sigma_i^2$  and  $\Omega_{ii}$ . Only when we set the entire matrix  $\Omega$  equal to zero does this become  $\tau_{11}\tau_{22}/(\tau_{12}\tau_{21})$ , consistent with (41). This confirms that it is simply not possible to express bilateral asset holdings as a gravity form in a general setup.

### 3.2 Financial Friction as Tax or Transaction Cost

As discussed in Section 2, international financial frictions are often modeled in the literature as a tax or transaction cost. One can introduce this in different ways. Consider agents from country  $j$  who invest in the assets from country  $i$ . In the absence of a tax the return is  $R_i$ . One can introduce an additive tax, making the return  $R_i - \tau_{ij}$ . Alternatively one can introduce a multiplicative tax, making the return  $(1 - \tau_{ij})R_i$ . One can also tax the price of the asset, making the price  $(1 + \tau_{ij})Q_i$  for investors from country  $j$ , or tax the dividend. All of these alternative ways of introducing a tax (or transaction cost) lead to fundamentally the same expression once we take a second-order approximation of the first-order conditions.

Leaving all algebra to the Technical Appendix, introducing a second-order multiplicative tax  $\tau_{ij}$ , such that the return becomes  $(1 - \tau_{ij})R_i$ , gives

$$\alpha_{ij} = \frac{1}{p_i} - \frac{\tau_{ij}}{\gamma\sigma_i^2} \quad (43)$$

with  $p_i$  as defined in Section 2. Note that the financial friction now enters in the form of a separate additive term in  $\alpha_{ij}$  rather than multiplicative in the first term. The reason is that it subtracts a second-order component from the expected excess return of all assets.

Imposing market equilibrium, we have

$$X_{ij} = \frac{W_j S_i}{W} + \frac{W_j}{\gamma\sigma_i^2} (\hat{\tau}_i - \tau_{ij}) \quad (44)$$

where  $W = \sum_{j=1}^N W_j$  is world financial wealth and  $\hat{\tau}_i = \sum_{j=1}^N (W_j/W)\tau_{ij}$  is a weighted average financial friction that destination country  $i$  faces with all source countries. As we illustrate in the Technical Appendix, it is impossible to write this in the general gravity form (39). The reason for this is the additive term on the right hand side of (44). As was the case with a general covariance structure, bilateral asset holdings are now a complex non-linear function of country size variables, second moments of asset returns and financial frictions.

### 3.3 Only Trade in Equity

Finally we consider the case in which there is only trade in equity. In the benchmark model all equity positions are positive while bond holdings are both positive and negative (they aggregate to zero). However, there are no restrictions on borrowing (negative bond holdings). In reality such restrictions can be quite severe and lenders demand collateral from the borrowers. This reduces the extent of the holdings of the riskfree asset, both positive and negative. Rather than explicitly introducing such borrowing constraints based on collateral, here we will only briefly discuss the extreme case that rules out borrowing altogether. In that case there is only trade in equity. Less severe borrowing constraints, based on collateral, lead to the same qualitative point: gravity falls apart.<sup>10</sup>

As shown in the Technical Appendix, equilibrium bilateral holdings in this case (after imposing market equilibrium) are highly complex. To be precise, we get

$$X_{.,j} = b_j W_j + W_j \mathbf{M}_j \left( \sum_{j=1}^N W_j \tilde{\mathbf{M}}_j \right)^{-1} \left( \mathbf{S} - \sum_{j=1}^N W_j \tilde{b}_j \right) \quad (45)$$

where  $b_j$  is a vector of size  $N$  with element  $i$  equal to  $\frac{1}{\sigma_i^2 \tau_{ij} h_j}$ ,  $h_j = \sum_{i=1}^N 1/[\sigma_i^2 \tau_{ij}]$ ,  $\mathbf{S} = (S_2, \dots, S_N)'$  and  $\mathbf{M}_j$  a  $N$  by  $N - 1$  matrix with

$$\mathbf{M}_j[i, k - 1] = -\frac{1}{\gamma R \sigma_i^2 \sigma_k^2 \tau_{ij} \tau_{kj} h_j} \quad k \neq i \quad (46)$$

$$\mathbf{M}_j[i, i - 1] = -\frac{1}{\gamma R \sigma_i^4 \tau_{ij}^2 h_j} + \frac{1}{\gamma R \sigma_i^2 \tau_{ij}} \quad (47)$$

$\tilde{b}_j$  and  $\tilde{\mathbf{M}}_j$  refer to the last  $N - 1$  rows of respectively  $b_j$  and  $\mathbf{M}_j$ .

As was the case with the other two extensions, this is a complex expression that relates bilateral asset holdings to measures of country size, second moments of asset returns and financial frictions. As shown in the Technical Appendix, it cannot be written in the general gravity form (39).

We should finally emphasize that of course the extensions that we have discussed in this section are by no means exhaustive. Others, such as non-financial wealth, may need to be considered as well. But the overall message is that most extensions will not deliver a gravity form.

<sup>10</sup>This case is also of interest in analogy to the gravity theory for goods trade, where agents usually can buy only differentiated goods.

## 4 Estimation and Comparative Statics

In this section we will describe how to estimate the size of financial frictions and conduct comparative statics analysis with respect to changes in financial frictions. We will discuss how to do so both for gravity system (28) as well as various extensions of it.

### 4.1 Estimation

We first discuss three estimation methods for the bilateral financial frictions based on the gravity system (28)-(31). The first method is analogous to that commonly used in the trade gravity literature today. We first relate the unobservable bilateral financial frictions to various observables. Specifically, assume that

$$\ln(\tau_{ij}) = \sum_{m=1}^M \phi_m z_{ij}^m \quad (48)$$

The variables  $z_{ij}^m$  need to be such that they can be thought of as affecting financial frictions and particularly information frictions. Examples are language, legal and regulatory similarities. They cannot be things like asset returns or correlations of returns.

Substituting (48) into the logarithm of the gravity equation (28), and replacing  $\ln(S_i) + \ln(\Pi_i/E)$  and  $\ln(E_j) + \ln(P_j)$  with respectively destination and source dummies  $\eta_i$  and  $\xi_j$ , we have

$$\ln(X_{ij}) = - \sum_{m=1}^M \phi_m z_{ij}^m + \eta_i + \xi_j + \epsilon_{ij} \quad (49)$$

An error term is added that can be interpreted for example as data measurement error of bilateral financial holdings. Regressing the log of bilateral holdings on the  $z_{ij}^m$ , as well as source and destination country dummies, provides us with estimates of  $\phi_m$  and therefore the relationship between financial frictions and various observables.<sup>11</sup> Note that when one of the  $z_{ij}^m$  variables is a border dummy  $Home_{ij}$  that

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<sup>11</sup>This method is easily extended to panel data by adding time subscripts to the  $z_{ij}^m$  and the source and destination dummies. Note that time-varying financial frictions lead to time-varying multilateral resistance, so that for each period there need to be separate source and destination country dummies.

is 1 when  $i = j$  and 0 otherwise, it allows us to also estimate the average of all residual cross-border frictions that are not captured by any of the other variables  $z_{ij}^m$ .

The second estimation method exploits the fact that when using  $W_j = E_j P_j$  we can also write the gravity equation (28) as

$$X_{ij} = \frac{W_j S_i \Pi_i}{E \tau_{ij}} \quad (50)$$

Taking logs, defining  $\theta_i = \ln(S_i) + \ln(\Pi_i/E)$  as a destination country dummy, and adding an error term, we have

$$\ln(X_{ij}/W_j) = - \sum_{m=1}^M \phi_m z_{ij}^m + \theta_i + \epsilon_{ij} \quad (51)$$

The difference in comparison to (49) is that there is no source country dummy in this regression. This implies that source country specific frictions can now be identified as well: some of the  $z_{ij}^m$  may only depend on  $j$ . Examples are regulatory quality and financial market sophistication of the source country.

The reason that such source country specific frictions can be identified is as follows. An increase in source country specific frictions does not change relative financial frictions for that source country because its multilateral resistance rises proportionally. However, the higher multilateral resistance lowers  $E_j$ . It causes a general shift out of equity and into bonds by country  $j$ . It is this general shift out of equity by a source country that allows us to identify such frictions.

Finally, a third method estimates bilateral frictions directly by using

$$\left( \frac{X_{ij} X_{ji}}{X_{ii} X_{jj}} \right)^{-0.5} = \left( \frac{\tau_{ij} \tau_{ji}}{\tau_{ii} \tau_{jj}} \right)^{0.5} \quad (52)$$

or

$$\left( \frac{X_{ij}/W_j}{X_{ii}/W_i} \right)^{-1} = \frac{\tau_{ij}}{\tau_{ii}} \quad (53)$$

A drawback of these measures is that they are very sensitive to measurement error of bilateral equity holdings for individual pairs. Such measurement error can be significantly reduced by computing the following weighted harmonic mean of frictions of country  $i$  as a destination, which follows from (53):

$$\frac{\sum_{j \neq i} W_j}{\sum_{j \neq i} \frac{1}{\tau_{ij}} W_j} = \frac{X_{ii}}{\sum_{j \neq i} X_{ij}} \frac{\sum_{j \neq i} W_j}{W_i} \quad (54)$$



where  $X_{ii} = S_i - \sum_{j \neq i} X_{ij}$ . All that is needed to compute this is the aggregate external equity liabilities of country  $i$ , measures of wealth and aggregate stock market capitalization.

So far we have only discussed estimation of (28) based on the benchmark model. We now turn to extensions. First consider the fixed cost extensions, which have the advantage that the overall gravity form is retained. If the fixed cost is such that agents invest only in a subset of the destination countries (some of the  $X_{ij}$  are zero), all of the estimation methods described above continue to hold when we remove the country pairs for which  $X_{ij} = 0$ .

Next consider the case where as a result of fixed costs only a fraction of the agents is globally diversified and the other agents invest only in domestic equity and bonds. Defining  $W_i^A$  as the wealth of agents that are globally diversified, we have seen that this extension implies that the overall financial friction becomes  $\tau_{ij}\delta_{ij}$  with  $\delta_{ii} = W_i^A/W_i$  and  $\delta_{ij} = 1$  when  $i \neq j$ . This means that for all  $i \neq j$  the friction is still  $\tau_{ij}$ . One approach is therefore to adopt the first estimation method described above, applied to only cross border holdings ( $i \neq j$ ).<sup>12</sup> Gravity estimation based on cross-border holdings alone (ignoring the  $X_{ii}$  observations) is in fact most common in the existing empirical gravity literature.

This has the drawback though that it is impossible to measure the overall magnitude of cross-border information frictions. In particular, we could not identify the coefficient on the residual border dummy  $Home_{ij}$ , which is zero for all  $i \neq j$ . We could use the third method described above, based on any of the equations (52) through (54), to measure overall financial frictions  $\tau_{ij}\delta_{ij}$ . But it does not allow us to distinguish between information frictions  $\tau_{ij}$  and the fixed cost friction  $\delta_{ij}$ .

Another approach is to relate the unobservable  $W_i^A/W_i$  to a set of country-specific variables. These would be related to individual-specific variables that have been identified in the literature as affecting whether agents hold any foreign assets. Examples are financial sophistication, resources, education and age, for which it is easy to develop corresponding country-wide measures. So assume

$$\ln(\delta_{ii}) = \sum_{l=1}^L \mu_l h_i^l \quad (55)$$

Let the first variable,  $h_i^1$  be a constant set at 1. Also, let  $z_{ij}^1 = Home_{ij}$ .

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<sup>12</sup>This method can also be applied to the case discussed at the very end of section 2 where some of domestic equity holdings are entirely unrelated to a diversification motive.

The gravity specification then becomes

$$\ln(X_{ij}) = (\psi_1 + \mu_1)Home_{ij} + \sum_{m=2}^M \psi_m z_{ij}^m + \sum_{l=2}^L \mu_l h_j^l Home_{ij} + \eta_i + \xi_j \quad (56)$$

Using data on both cross-border and domestic asset holdings we can estimate the coefficients  $\psi_1 + \mu_1$ ,  $\psi_m$  ( $m = 2, \dots, M$ ) and  $\mu_l$  ( $l = 2, \dots, L$ ). We are unable to distinguish the information and fixed cost frictions only to the extent that the former cannot be attributed to variables  $z_{ij}^m$  ( $m > 1$ ) and the latter cannot be attributed to the source country variables  $h_i^l$  ( $l > 1$ ).

Finally, consider extensions such as those discussed in Section 3, where we do not get a gravity specification at all. For concreteness, consider the first generalization of Section 3, where we introduced a general covariance structure. Substituting (48) into (38), taking logs and adding an error term, we get

$$\ln(X_{ij}) = f(\phi_1, \dots, \phi_M; \mathbf{\Omega}, \sigma_k^2, W_k, S_k, z_{kl}^m \quad k, l = 1, \dots, N, m = 1, \dots, M) + \epsilon_{ij} \quad (57)$$

This relates bilateral holdings to the unknown parameters  $\phi_1, \dots, \phi_M$  that need to be estimated and a set of data that includes variances and covariances of asset returns, country size variables and the variables impacting the bilateral frictions. This system can then be estimated for example with non-linear least squares.

The same applies to the other extensions discussed in Section 3. While deviations from gravity therefore do not pose any particularly difficult new problems in estimation of international financial frictions, the method obviously stands in stark contrast to the existing empirical gravity literature. An important direction for future empirical work will be to understand whether, and to what extent, such generalizations fit the bilateral asset data better than the gravity specification (28).

## 4.2 Comparative Statics

First consider comparative statics analysis in the context of the gravity system (28). Consider the impact of a change in  $\tau_{ij}$  of any magnitude on bilateral asset holdings  $X_{kl}$  for any country pair  $(k, l)$ .<sup>13</sup> Using  $E_l = W_l/P_l$ , the gravity equation becomes

$$X_{kl} = \frac{S_k W_l \Pi_k}{E \tau_{kl}} \quad (58)$$

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<sup>13</sup>Of course we could simultaneously change many bilateral frictions, but this simply involves repeating the steps for different  $i$  and  $j$ , with a multiplicative impact on  $X_{kl}$ .

The bilateral financial claim  $X_{kl}$  is only affected through a change in  $\Pi_k/\tau_{kl}$ . All we therefore need to know is the change in  $\Pi_k$ . Substituting  $E_l = W_l/P_l$  into (30), we have

$$\frac{1}{\Pi_k} = \sum_{s=1}^N \frac{1}{\tau_{ks}} \frac{W_s}{E} \quad (59)$$

A change in  $\tau_{ij}$  only affects  $X_{kl}$  when  $k = i$ . Using (58) and (59), a change from  $\tau_{ij}$  to  $\tau'_{ij}$  implies

$$X'_{il} = X_{il} \frac{1}{1 + \frac{X_{ij}}{S_i} \left( \frac{\tau_{ij}}{\tau'_{ij}} - 1 \right)} \frac{\tau_{il}}{\tau'_{il}} \quad (60)$$

where the last ratio is 1 when  $l \neq j$ .

Introducing fixed costs does not change this formula at all, whether it leads to zero cross-border holdings for some country pairs or to a group of agents that does not hold any foreign equity. Note that in the latter case  $\tau_{ij}$  needs to be replaced by  $\tau_{ij}\delta_{ij}$ , but when considering only the impact of changes in information frictions  $\delta_{ij} = \delta'_{ij}$  and therefore (60) still applies.

While a simple analytic comparative statics result such as (60) no longer applies under the generalizations considered in Section 3, it is still straightforward to compute the impact of changes in financial frictions even there. Consider the first generalization, a more general covariance structure. For given values of  $\mathbf{\Omega}$ ,  $\sigma_k^2$ ,  $W_k$  and  $S_k$  ( $k = 1, \dots, N$ ), which do not depend on bilateral frictions, we can use (38) to compute the changes in all bilateral asset holdings resulting from changes in bilateral barriers. The same can be done for the other generalizations.

## 5 Conclusion

The rapidly growing empirical gravity literature on cross-border asset holdings clearly calls out for a theory. We have developed a theory for bilateral asset holdings that takes a gravity form and we discussed how to estimate international financial frictions and conduct comparative statics analysis within the context of the theory. Nonetheless some strong assumptions needed to be made to derive at such a theory. In contrast to goods trade, where many different types of models generate a gravity structure, reasonable changes in assumptions of our model do not deliver a gravity form for bilateral asset holdings.

This paper has been entirely theoretical, but it has laid a clear foundation for future empirical work. Even if one accepts the assumptions of our model that lead to a gravity form, existing empirical work often suffers from omitted variables (fixed effects) or the inclusion of variables that do not belong (e.g. return correlations). But perhaps more importantly, future empirical work needs to evaluate the empirical relevance of various extensions such as those we discussed. This is important both to understand what type of model better describes the data and ultimately to estimate the magnitude of cross-border financial frictions.

## Appendix

In this Appendix we apply the local approximation solution method developed by Tille and van Wincoop (2010a) and Devereux and Sutherland (2011) to derive portfolio demand equation (20). We decompose the model variables across components of different orders. Any variable  $x$  can be written as the sum of its zero, first and higher-order components:  $x = x(0) + x(1) + x(2) + \dots$ . The zero-order component,  $x(0)$ , is the value of  $x$  when all standard deviations of model innovations approach zero. The first-order component is proportional to model innovations. The second-order component is proportional to the variance, covariance or product of model innovations, and so on.

There are a total of  $N^2 + 5N + 4$  variables in the model:  $N^2 + N$  portfolio shares  $\alpha_{ij}$ ,  $\alpha_{gj}$ ;  $N + 2$  asset prices  $Q_i$ ,  $Q_g$  and  $Q_f$ ;  $N + 2$  corresponding asset returns;  $N$  period 1 consumption variables  $C_{i,1}$ ; and  $N$  period 2 consumption variables  $C_{i,2}$ . There are  $N^2 + 5N + 6$  equations:  $N^2 + N$  portfolio Euler equations;  $N$  consumption Euler equations;  $N + 2$  asset market clearing conditions; 2 goods market clearing conditions;  $N + 2$  definitions of asset returns; and  $N$  budget constraints. As there are two periods, we can drop two equations due to Walras' Law. We will drop the market clearing conditions for the riskfree and global assets.

We first need to impose the zero-order components of all equations. This gives:

$$R_i(0) = R_g(0) = R_f(0) \equiv R(0) = \frac{1}{\beta} \left( \frac{Y_w}{D_w} \right)^{-1/\gamma} \quad (61)$$

$$Q_i(0) = Q_g(0) = Q_f(0) = \frac{1}{R(0)} \quad (62)$$

$$C_{i,1}(0) = \frac{\beta^{-1/\gamma} R(0)^{1-1/\gamma}}{1 + \beta^{-1/\gamma} R(0)^{1-1/\gamma}} (Y_i + Q_i(0)K_i) \quad (63)$$

$$C_{i,2}(0) = W_i(0)R(0) \quad (64)$$

$$\sum_{j=1}^N \alpha_{ij}(0)W_j(0) = K_i Q_i(0) \quad (65)$$

where  $Y_w = \sum_{i=1}^N Y_i$ ,  $D_w = \sum_{i=1}^N D_i$  and  $W_j(0) = Y_j + Q_j(0)K_j - C_{j1}(0)$ .

The next step of the solution method involves jointly imposing the second-order component of the difference in portfolio Euler equations across countries together with the first-order component of all equations. This yields a solution to the zero-order component of the difference across countries in portfolio shares together with

the first-order component of all other variables. We will follow this method, with one small difference. Rather than just imposing the second-order component of the difference in portfolio Euler equations across countries, we impose the second-order component of all portfolio Euler equations without taking the difference across countries. This will in addition give us a solution to the second-order component of the  $N$  equilibrium expected excess returns (which enter in the  $p_i$  that are solved from the zero-order component of the market clearing conditions—see the text).

First impose the first-order components of all equations. This gives

$$E(R_i(1)) = E(R_g(1)) = E(R_f(1)) \quad (66)$$

$$R_i(1) = R(0)(\epsilon_i + \theta_i \epsilon_g) \quad (67)$$

$$R_g(1) = R(0)\theta_g \epsilon_g \quad (68)$$

$$R_f(1) = Q_f(1) = Q_i(1) = Q_g(1) = 0 \quad (69)$$

$$C_{j1}(1) = 0 \quad (70)$$

$$C_{j2}(1) = W_j(0)R_j^p(1) = W_j(0) \left( \sum_{i=1}^N \alpha_{ij}(0)R_i(1) + \alpha_{gj}(0)R_g(1) \right) \quad (71)$$

Next we impose the second-order component of the portfolio Euler equations. This gives

$$C_{j2}(0)E(R_i(2) - R_f(2)) = \gamma EC_{j2}(1)(R_i(1) - R_f(1)) \quad (72)$$

$$C_{j2}(0)E(R_g(2) - R_f(2)) = \gamma EC_{j2}(1)(R_g(1) - R_f(1)) \quad (73)$$

Using our result in (69) that  $R_f(1) = 0$  and the expression for  $C_{j2}(1)$  in (71), these equations can be rewritten as

$$\begin{aligned} \frac{1}{R(0)}E(R_i(2) - R_f(2)) &= \gamma \sigma_g^2 \theta_i \left( \sum_{k=1}^N \alpha_{kj}(0)\theta_k + \alpha_{gj}(0)\theta_g \right) \\ &\quad + \gamma \alpha_{ij}(0)\sigma_i^2 \tau_{ij} \end{aligned} \quad (74)$$

$$\frac{1}{R(0)}E(R_g(2) - R_f(2)) = \gamma \sigma_g^2 \theta_g \left( \sum_{k=1}^N \alpha_{kj}(0)\theta_k + \alpha_{gj}(0)\theta_g \right) \quad (75)$$

Substituting (75) into (74) yields

$$\alpha_{ij}(0) = \frac{1}{\gamma R(0)\sigma_i^2 \tau_{ij}} \left[ E(R_i(2) - R_f(2)) - \frac{\theta_i}{\theta_g} E(R_g(2) - R_f(2)) \right] \quad (76)$$

which is (18) in the text.

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