Online Appendix

Exchange Rates, Interest Rates, and Gradual Portfolio Adjustment

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This Online Appendix develops the algebra associated with the model with long terms bonds in Section 4 of the paper.

A Optimal Portfolios

Home agents maximize

$$E_t \frac{C_{t+1}^{1-\gamma}}{1-\gamma} - \frac{1}{4}\psi \sum_{i=1}^{4} \left(z_{it} - z_{i,t-1}\right)^2 \tag{A.1}$$

subject to

$$C_{t+1} = R_t + z_{1t}\left(\frac{Q_{t+1}}{Q_t} R_t^* e^{-\tau} - R_t\right) + z_{2t}\left(\frac{Q_{t+1}}{Q_t} R_{t+1}^{L,*} e^{-\tau_L} - R_t\right) + z_{3t}\left(R_{t+1}^L - R_t\right) + T_{t+1} \tag{A.2}$$

The aggregate of the cost of investing abroad is reimbursed through T_{t+1}, so that in the aggregate

$$C_{t+1} = R_t + z_{1t}\left(\frac{Q_{t+1}}{Q_t} R_t^* - R_t\right) + z_{2t}\left(\frac{Q_{t+1}}{Q_t} R_{t+1}^{L,*} - R_t\right) + z_{3t}\left(R_{t+1}^L - R_t\right) \tag{A.3}$$

Define $\hat{z}$ as a deviation from z_{4t}, so for example $\hat{z}_{1t} = z_{1t} - z_{4t}$. First-order conditions for optimal portfolio choice are then

$$E_t C_{t+1}^{-\gamma}\left(\frac{Q_{t+1}}{Q_t} R_t^* e^{-\tau} - R_t\right) = 0.5\psi(\hat{z}_{1t} - \hat{z}_{1,t-1}) \tag{A.4}$$

$$E_t C_{t+1}^{-\gamma}\left(\frac{Q_{t+1}}{Q_t} R_{t+1}^{L,*} e^{-\tau_L} - R_t\right) = 0.5\psi(\hat{z}_{2t} - \hat{z}_{2,t-1}) \tag{A.5}$$

$$E_t C_{t+1}^{-\gamma}\left(R_{t+1}^L - R_t\right) = 0.5\psi(\hat{z}_{3t} - \hat{z}_{3,t-1}) \tag{A.6}$$

Denoting logs with lower case letters, define the three excess returns as

$$er_{1,t+1} = q_{t+1} - q_t + r_t^* - r_t \tag{A.7}$$

$$er_{2,t+1} = q_{t+1} - q_t + r_{t+1}^{L,*} - r_t \tag{A.8}$$

$$er_{3,t+1} = r_{t+1}^L - r_t \tag{A.9}$$

We can then rewrite the first-order conditions as

$$E_t e^{-\gamma c_{t+1}+er_{1,t+1}-\tau} - E_t e^{-\gamma c_{t+1}} = 0.5\psi(\hat{z}_{1t} - \hat{z}_{1,t-1})e^{-r_t} \tag{A.10}$$

$$E_t e^{-\gamma c_{t+1}+er_{2,t+1}-\tau_L} - E_t e^{-\gamma c_{t+1}} = 0.5\psi(\hat{z}_{2t} - \hat{z}_{2,t-1})e^{-r_t} \tag{A.11}$$

$$E_t e^{-\gamma c_{t+1}+er_{3,t+1}} - E_t e^{-\gamma c_{t+1}} = 0.5\psi(\hat{z}_{3t} - \hat{z}_{3,t-1})e^{-r_t} \tag{A.12}$$

Using log normality of consumption and returns, and approximating $e^x = 1 + x$, we can write this as (also linearizing the right hand side)

$$E_t er_{1,t+1} - \tau + 0.5\sigma_1^2 - \gamma cov(er_{1,t+1}, c_{t+1}) = \frac{0.5\psi}{R}(\hat{z}_{1t} - \hat{z}_{1,t-1}) \tag{A.13}$$

$$E_t er_{2,t+1} - \tau_L + 0.5\sigma_2^2 - \gamma cov(er_{2,t+1}, c_{t+1}) = \frac{0.5\psi}{R}(\hat{z}_{2t} - \hat{z}_{2,t-1}) \tag{A.14}$$

$$E_t er_{3,t+1} + 0.5\sigma_3^2 - \gamma cov(er_{3,t+1}, c_{t+1}) = \frac{0.5\psi}{R}(\hat{z}_{3t} - \hat{z}_{3,t-1}) \tag{A.15}$$

Here $\sigma_i^2 = var(er_{i,t+1})$ and R is the steady state value of the returns.

Log-linearizing (A.3), we have

$$c_{t+1} = r_t + z_{1t}er_{1,t+1} + z_{2t}er_{2,t+1} + z_{3t}er_{3,t+1} \tag{A.16}$$

The first-order conditions then become

$$E_t er_{1,t+1} - \tau + 0.5\sigma_1^2 - \gamma z_{1t}\sigma_1^2 - \gamma z_{2t}\sigma_{12} - \gamma z_{3t}\sigma_{13} = \frac{0.5\psi}{R}(\hat{z}_{1t} - \hat{z}_{1,t-1}) \tag{A.17}$$

$$E_t er_{2,t+1} - \tau_L + 0.5\sigma_2^2 - \gamma z_{1t}\sigma_{12} + \gamma z_{2t}\sigma_2^2 - \gamma z_{3t}\sigma_{23} = \frac{0.5\psi}{R}(\hat{z}_{2t} - \hat{z}_{2,t-1}) \tag{A.18}$$

$$E_t er_{3,t+1} + 0.5\sigma_3^2 - \gamma z_{1t}\sigma_{13} - \gamma z_{2t}\sigma_{23} - \gamma z_{3t}\sigma_3^2 = \frac{0.5\psi}{R}(\hat{z}_{3t} - \hat{z}_{3,t-1}) \tag{A.19}$$

Here σ_{ij} is the covariance between $er_{i,t+1}$ and $er_{j,t+1}$.

Define $\mathbf{er}_{t+1} = (er_{1,t+1}, er_{2,t+1}, er_{3,t+1})'$ and $\mathbf{z}_t = (z_{1t}, z_{2t}, z_{3t})'$. $\hat{\mathbf{z}}_t$ subtracts z_{4t} from each element of $\mathbf{z}_t$. Then we can write the three first-order conditions for Home agents compactly as

$$E_t \mathbf{er}_{t+1} - \begin{pmatrix} \tau \\ \tau_L \\ 0 \end{pmatrix} + 0.5diag(\Sigma) - \gamma\Sigma\mathbf{z}_t = \frac{0.5\psi}{R}(\hat{\mathbf{z}}_t - \hat{\mathbf{z}}_{t-1}) \tag{A.20}$$

where Σ is the variance of $\mathbf{er}_{t+1}$.

Next consider the Foreign country. We have

$$C^*_{t+1} = R_t e^{-\tau}\frac{Q_t}{Q_{t+1}} + z^*_{1t}\left(R^*_t - R_t e^{-\tau}\frac{Q_t}{Q_{t+1}}\right) + z^*_{2t}\left(R^{L,*}_{t+1} - R_t e^{-\tau}\frac{Q_t}{Q_{t+1}}\right) +$$
$$z^*_{3t}\left(R^L_{t+1} e^{-\tau_L}\frac{Q_t}{Q_{t+1}} - R_t e^{-\tau}\frac{Q_t}{Q_{t+1}}\right) + T^*_{t+1} \tag{A.21}$$

The cost of investment abroad is reimbursed through T^*_{t+1}, so that aggregate Foreign consumption is

$$C^*_{t+1} = R_t\frac{Q_t}{Q_{t+1}} + z^*_{1t}\left(R^*_t - R_t\frac{Q_t}{Q_{t+1}}\right) + z^*_{2t}\left(R^{L,*}_{t+1} - R_t\frac{Q_t}{Q_{t+1}}\right) +$$
$$z^*_{3t}\left(R^L_{t+1}\frac{Q_t}{Q_{t+1}} - R_t\frac{Q_t}{Q_{t+1}}\right) \tag{A.22}$$

First-order conditions for optimal portfolio choice are

$$E_t(C^*_{t+1})^{-\gamma}\left(R^*_t - R_t e^{-\tau}\frac{Q_t}{Q_{t+1}}\right) = 0.5\psi(\hat{z}^*_{1t} - \hat{z}^*_{1,t-1}) \tag{A.23}$$

$$E_t(C^*_{t+1})^{-\gamma}\left(R^{L,*}_{t+1} - R_t e^{-\tau}\frac{Q_t}{Q_{t+1}}\right) = 0.5\psi(\hat{z}^*_{2t} - \hat{z}^*_{2,t-1}) \tag{A.24}$$

$$E_t(C^*_{t+1})^{-\gamma}\left(R^L_{t+1} e^{-\tau_L}\frac{Q_t}{Q_{t+1}} - R_t e^{-\tau}\frac{Q_t}{Q_{t+1}}\right) = 0.5\psi(\hat{z}^*_{3t} - \hat{z}^*_{3,t-1}) \tag{A.25}$$

where $\hat{z}^*_{it} = z^*_{it} - z^*_{4t}$. We can then rewrite the first-order conditions as

$$E_t e^{-\gamma c^*_{t+1}} - E_t e^{-\gamma c^*_{t+1} - er_{1,t+1} - \tau} = 0.5\psi(\hat{z}^*_{1t} - \hat{z}^*_{1,t-1})e^{-r^*_t} \tag{A.26}$$

$$E_t e^{-\gamma c^*_{t+1} + er_{2,t+1} - er_{1,t+1}} - E_t e^{-\gamma c^*_{t+1} - er_{1,t+1} - \tau} = 0.5\psi(\hat{z}^*_{2t} - \hat{z}^*_{2,t-1})e^{-r^*_t} \tag{A.27}$$

$$E_t e^{-\gamma c^*_{t+1} + er_{3,t+1} - er_{1,t+1} - \tau_L} - E_t e^{-\gamma c^*_{t+1} - er_{1,t+1} - \tau} = 0.5\psi(\hat{z}^*_{3t} - \hat{z}^*_{3,t-1})e^{-r^*_t} \tag{A.28}$$

Assuming again that consumption and returns are log-linear, taking expectations and then linearizing e^x as $1 + x$, we have

$$E_t er_{1,t+1} + \tau - 0.5\sigma_1^2 - \gamma cov(er_{1,t+1}, c^*_{t+1}) = \frac{0.5\psi}{R}(\hat{z}^*_{1t} - \hat{z}^*_{1,t-1}) \tag{A.29}$$

$$E_t er_{2,t+1} + \tau + 0.5\sigma_2^2 - \sigma_{12} - \gamma cov(er_{2,t+1}, c^*_{t+1}) = \frac{0.5\psi}{R}(\hat{z}^*_{2t} - \hat{z}^*_{2,t-1}) \tag{A.30}$$

$$E_t er_{3,t+1} + \tau - \tau_L + 0.5\sigma_3^2 - \sigma_{13} - \gamma cov(er_{3,t+1}, c^*_{t+1}) = \frac{0.5\psi}{R}(\hat{z}^*_{3t} - \hat{z}^*_{3,t-1}) \tag{A.31}$$

Log-linearizing (A.22), we have

$$c^*_{t+1} = r^*_t - er_{1,t+1} + z^*_{1t}er_{1,t+1} + z^*_{2t}er_{2,t+1} + z^*_{3t}er_{3,t+1} \tag{A.32}$$

The first-order conditions then become

$$E_t er_{1,t+1} + \tau + 0.5\sigma_1^2 - (1-\gamma)\sigma_1^2 - \gamma z_{1t}^* \sigma_1^2 - \gamma z_{2t}^* \sigma_{12} - \gamma z_{3t}^* \sigma_{13} = \frac{0.5\psi}{R}(\hat{z}_{1t}^* - \hat{z}_{1,t-1}^*)$$

$$E_t er_{2,t+1} + \tau + 0.5\sigma_2^2 - (1-\gamma)\sigma_{12} - \gamma z_{1t}^* \sigma_{12} - \gamma z_{2t}^* \sigma_2^2 - \gamma z_{3t}^* \sigma_{23} = \frac{0.5\psi}{R}(\hat{z}_{2t}^* - \hat{z}_{2,t-1}^*)$$

$$E_t er_{3,t+1} + \tau - \tau_L + 0.5\sigma_3^2 - (1-\gamma)\sigma_{13} - \gamma z_{1t}^* \sigma_{13} - \gamma z_{2t}^* \sigma_{23} - \gamma z_{3t}^* \sigma_3^2 = \frac{0.5\psi}{R}(\hat{z}_{3t}^* - \hat{z}_{3,t-1}^*)$$

We can write these first-order conditions compactly as

$$E_t \mathbf{er}_{t+1} + \begin{pmatrix} \tau \\ \tau \\ \tau - \tau_L \end{pmatrix} + 0.5 diag(\Sigma) - (1-\gamma)\Sigma_1 - \gamma \Sigma \mathbf{z}_t^* = \frac{0.5\psi}{R}(\hat{\mathbf{z}}_t^* - \hat{\mathbf{z}}_{t-1}^*) \quad \text{(A.33)}$$

where $\mathbf{z}_t^* = (z_{1t}^*, z_{2t}^*, z_{3t}^*)'$ is the vector of portfolio shares of Foreign agents and $\hat{\mathbf{z}}_t^*$ subtracts z_{4t}^* from each element of $\mathbf{z}_t^*$. Σ_1 is the first column of Σ.

Taking the average of (A.20) and (A.33), we have

$$E_t \mathbf{er}_{t+1} + \frac{1}{2} \begin{pmatrix} 0 \\ \tau - \tau_L \\ \tau - \tau_L \end{pmatrix} + \frac{1}{2} diag(\Sigma) - \frac{1}{2}(1-\gamma)\Sigma_1 - \gamma \Sigma \mathbf{z}_t^A = \frac{\psi}{2R}(\hat{\mathbf{z}}_t^A - \hat{\mathbf{z}}_{t-1}^A) \quad \text{(A.34)}$$

where $\mathbf{z}_t^A = 0.5(\mathbf{z}_t + \mathbf{z}_t^*)$ and $\hat{\mathbf{z}}_t^A = 0.5(\hat{\mathbf{z}}_t + \hat{\mathbf{z}}_t^*)$.

B Market Equilibrium

Next impose asset market equilibrium:

$$z_{1t} + Q_t z_{1t}^* = Q_t b^S \quad \text{(B.1)}$$

$$z_{2t} + Q_t z_{2t}^* = Q_t P_t^{L,*} b_t \quad \text{(B.2)}$$

$$z_{3t} + Q_t z_{3t}^* = P_t^L b_t \quad \text{(B.3)}$$

$$z_{4t} + Q_t z_{4t}^* = b^S \quad \text{(B.4)}$$

Here b^S is the constant supply of the short-term bond, while b_t is the quamtity of long-term bonds. Both are equal in the two countries. Adding up these market clearing conditions, we have

$$(1 + Q_t)(1 - b^S) = b_t \left(Q_t P_t^{L,*} + P_t^L \right) \quad \text{(B.5)}$$

The steady state value of b_t must then be $\bar{b} = (1-b^S)/\bar{P}^L$, where $\bar{P}^L = \kappa/(R-1+\delta)$ is the steady state long term bond price. It follows that $\bar{b}\bar{P}^L = 1 - b^S$. We refer to $\bar{b}\bar{P}^L$ as b^L, the value (in terms of purchasing power) of long term bonds in both countries. Therefore $b^S + b^L = 1$. Furthermore, linearizing (B.5) gives

$$b_t = -p_t^{L,A} \tag{B.6}$$

where $p_t^{L,A} = 0.5(p_t^L + p_t^{L,*})$ is the average log bond price.

In log-linear form the first three market clearing conditions are then

$$\mathbf{z}_t^A = 0.5 \begin{pmatrix} b^S \\ b^L \\ b^L \end{pmatrix} + 0.5 \begin{pmatrix} b^S \\ b^L \\ 0 \end{pmatrix} q_t - 0.5\bar{\mathbf{z}}^* q_t + 0.25 b^L \begin{pmatrix} 0 \\ -p_t^{L,D} \\ p_t^{L,D} \end{pmatrix} \tag{B.7}$$

where $\bar{\mathbf{z}}^*$ is the steady state of $\mathbf{z}_t^*$ and $p_t^{L,D} = p_t^L - p_t^{L,*}$ is the relative log long term bond price.

Since the steady state portfolio shares $\bar{\mathbf{z}}^*$ enter in (B.7), we need to say something about them. We will relate then to portfolio home bias. Let $\bar{z}_i$ and $\bar{z}_i^*$ be the steady state portfolio shares of Home and Foreign agents. By symmetry

$$\bar{z}_1 + \bar{z}_4 = \bar{z}_1^* + \bar{z}_4^* = b^S \tag{B.8}$$

$$\bar{z}_2 + \bar{z}_3 = \bar{z}_2^* + \bar{z}_3^* = b^L \tag{B.9}$$

So both Home and Foreign investors invest a fraction b^S in short term bonds and a fraction b^L in long term bonds. Within short-term bonds and within long-term bonds, the extent of home bias is determined by τ and τ_L, which we can use to set home bias at any value. Denoting home bias as h for both short-term and long-term bonds, we have

$$h = 1 - \frac{\bar{z}_1/b^S}{0.5} = 1 - \frac{\bar{z}_4^*/b^S}{0.5} \tag{B.10}$$

$$h = 1 - \frac{\bar{z}_2/b^L}{0.5} = 1 - \frac{\bar{z}_3^*/b^L}{0.5} \tag{B.11}$$

Therefore

$$\bar{z}_1 = \bar{z}_4^* = 0.5(1-h)b^S \tag{B.12}$$

$$\bar{z}_2 = \bar{z}_3^* = 0.5(1-h)b^L \tag{B.13}$$

These equations, together with (B.8) and (B.9) map the home bias parameter h into all steady state portfolio shares in both countries. We have

$$\bar{z}_4 = \bar{z}_1^* = 0.5(1+h)b^S \tag{B.14}$$

$$\bar{z}_3 = \bar{z}_2^* = 0.5(1+h)b^L \tag{B.15}$$

Define

$$\mathbf{v} = 0.25(1-h)\begin{pmatrix} b^S \\ b^L \\ -b^L \end{pmatrix} \tag{B.16}$$

Then (B.7) becomes

$$\mathbf{z}_t^A = 0.5\begin{pmatrix} b^S \\ b^L \\ b^L \end{pmatrix} + \mathbf{v}q_t + 0.25b^L\begin{pmatrix} 0 \\ -p_t^{L,D} \\ p_t^{L,D} \end{pmatrix} \tag{B.17}$$

Combining these market equilibrium conditions with (A.34), and focusing on the deviation from the steady state, we have

$$E_t\mathbf{er}_{t+1} - \gamma\Sigma\mathbf{v}q_t - 0.25\gamma b^L p_t^{L,D}\Sigma\begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} = \tag{B.18}$$

$$\frac{0.5\psi}{R}\mathbf{v}(q_t - q_{t-1}) + \frac{\psi b^S}{8R}(1-h)\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}(q_t - q_{t-1}) + \frac{\psi b_L}{8R}(p_t^{L,D} - p_{t-1}^{L,D})\begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$$

C Solution

In order to solve the model, we will write (B.18) as a second-order difference equation in the variables $(q_t, p_t^{L,D}, p_t^{L,A})'$. We first need to write $\mathbf{er}_{t+1}$ in terms of these variables. Log-linearizing the long term bond returns, we have

$$r_{t+1}^L = \lambda p_{t+1}^L - p_t^L \tag{C.1}$$

$$r_{t+1}^{L,*} = \lambda p_{t+1}^{L,*} - p_t^{L,*} \tag{C.2}$$

where $\lambda = (1-\delta)/R$. We then have

$$er_{1,t+1} = q_{t+1} - q_t + r_t^D \tag{C.3}$$

$$er_{2,t+1} = q_{t+1} - q_t + \lambda p_{t+1}^{L,*} - p_t^{L,*} - r_t \tag{C.4}$$

$$er_{3,t+1} = \lambda p_{t+1}^L - p_t^L - r_t \tag{C.5}$$

We can also write

$$er_{2,t+1} = q_{t+1} - q_t - 0.5\lambda p_{t+1}^{L,D} + \lambda p_{t+1}^{L,A} + 0.5p_t^{L,D} - p_t^{L,A} + 0.5r_t^D - r_t^A \quad (C.6)$$
$$er_{3,t+1} = 0.5\lambda p_{t+1}^{L,D} + \lambda p_{t+1}^{L,A} - 0.5p_t^{L,D} - p_t^{L,A} + 0.5r_t^D - r_t^A \quad (C.7)$$

where $r_t^D = r_t^* - r_t$.

Next a couple of comments on the matrix Σ. Let $\sigma_q^2 = var_t(q_{t+1})$, $\sigma_L^2 = var_t(\lambda p_{t+1}^L)$, $\sigma_{LL} = cov(\lambda p_{t+1}^L, \lambda p_{t+1}^{L,*})$ and $\sigma_{qL} = cov_t(q_{t+1}, \lambda p_{t+1}^L)$. Then we have

$$\Sigma = \begin{pmatrix} \sigma_q^2 & \sigma_q^2 - \sigma_{qL} & \sigma_{qL} \\ \sigma_q^2 - \sigma_{qL} & \sigma_q^2 + \sigma_L^2 - 2\sigma_{qL} & \sigma_{qL} + \sigma_{LL} \\ \sigma_{qL} & \sigma_{qL} + \sigma_{LL} & \sigma_L^2 \end{pmatrix} \quad (C.8)$$

Let σ_{ij} be element (i,j) of the matrix Σ. Denote $\sigma_i^2 = \sigma_{ii}$. In the data we compute σ_1^2, σ_3^2, σ_{13} and σ_{23}. Then

$$\Sigma = \begin{pmatrix} \sigma_1^2 & \sigma_1^2 - \sigma_{13} & \sigma_{13} \\ \sigma_1^2 - \sigma_{13} & \sigma_1^2 + \sigma_3^2 - 2\sigma_{13} & \sigma_{23} \\ \sigma_{13} & \sigma_{23} & \sigma_3^2 \end{pmatrix} \quad (C.9)$$

Consider the system (B.18). First take the third equation, plus the second equation, minus the first equation. This gives

$$E_t(\lambda p_{t+1}^{L,A} - p_t^{L,A} - r_t^A) = 0 \quad (C.10)$$

Assuming that r_t^A follows an AR process with AR coefficient ρ, the solution is

$$p_t^{L,A} = -\frac{1}{1-\lambda\rho} r_t^A \quad (C.11)$$

Next consider the first equation of (B.18), together with the third minus second plus first equation. This gives

$$E_t q_{t+1} - q_t + r_t^D + a_1 q_t + 0.25\gamma(\sigma_1^2 - 2\sigma_{13})b^L p_t^{L,D} = \frac{\psi}{4R} b^S (1-h)(q_t - q_{t-1}) \quad (C.12)$$
$$\lambda E_t p_{t+1}^{L,D} - p_t^{L,D} + r_t^D + 2a_2 q_t + 0.5\gamma(\sigma_{23} - \sigma_3^2)b^L p_t^{L,D} = \frac{\psi}{4R}(1-h)(b^S - b^L)(q_t - q_{t-1}) + \frac{\psi}{4R} b^L (p_t^{L,D} - p_{t-1}^{L,D}) \quad (C.13)$$

where

$$a_1 = -0.25\gamma(1-h)\left(\sigma_1^2 b^S + (\sigma_1^2 - 2\sigma_{13})b^L\right) \tag{C.14}$$

$$a_2 = -0.25\gamma(1-h)\left(\sigma_{13} b^S + (\sigma_{23} - \sigma_3^2)b^L\right) \tag{C.15}$$

This system can also be written as

$$A_1 E_t \begin{pmatrix} q_{t+1} \\ p_{t+1}^{L,D} \end{pmatrix} + A_2 \begin{pmatrix} q_t \\ p_t^{L,D} \end{pmatrix} + A_3 \begin{pmatrix} q_{t-1} \\ p_{t-1}^{L,D} \end{pmatrix} + A_4 r_t^D = 0 \tag{C.16}$$

The matrices are defined as follows. We have

$$A_1 = \begin{pmatrix} 1 & 0 \\ 0 & \lambda \end{pmatrix} \tag{C.17}$$

$$A_2 = \begin{pmatrix} -1 + a_1 - \frac{\psi}{4R}(1-h)b^S & 0.25\gamma(\sigma_1^2 - 2\sigma_{13})b^L \\ 2a_2 - \frac{\psi}{4R}(1-h)(b^S - b^L) & -1 + 0.5\gamma(\sigma_{23} - \sigma_3^2)b^L - \frac{\psi}{4R}b^L \end{pmatrix} \tag{C.18}$$

$$A_3 = \frac{\psi}{4R} \begin{pmatrix} b^S(1-h) & 0 \\ (1-h)(b^S - b^L) & b^L \end{pmatrix} \tag{C.19}$$

$$A_4 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{C.20}$$

The system is driven by exogenous AR processes for r_t^D:

$$r_t^D = \rho r_{t-1}^D + \varepsilon_t \tag{C.21}$$

Let σ^2 be the variance of ε_t.

One can write the system as a first-order difference equation of the form $AE_t x_{t+1} + Bx_t = 0$, where $x_t = (q_t, p_t^{L,D}, q_{t-1}, p_{t-1}^{L,D}, r_t^D)'$. This allows us to solve for the control variables $(q_t, p_t^{L,D})$ as a function of the state variables $(q_{t-1}, p_{t-1}^{L,D}, r_t^D)'$. Define

$$v_t = \begin{pmatrix} q_t \\ p_t^{L,D} \end{pmatrix} \tag{C.22}$$

Then the solution takes the form

$$v_t = M_1 v_{t-1} + M_2 r_t^D \tag{C.23}$$

We can also integrate this and write

$$v_t = \sum_{k=0}^{\infty} M_1^k M_2 r_{t-k}^D \tag{C.24}$$

with M_1^0 being the identity matrix.

D Model Moments

Consider the regression of excess returns on r_t^D. First consider the excess return

$$er_{4,t+1} = -\lambda p_{t+1}^{L,D} + p_t^{L,D} + q_{t+1} - q_t \tag{D.1}$$

This is equal to $er_{2,r+1} - er_{3,t+1}$, which is the excess return of the Foreign long term bond over the Home long term bond. The coefficient of a regression of $er_{4,t+1}$ on r_t^D is equal to

$$\beta_1 = \frac{cov(er_{4,t+1}, r_t^D)}{var(r_t^D)} \tag{D.2}$$

Define the vectors $e_1 = (1, -\lambda)$ and $e_2 = (-1, 1)$. Then

$$er_{4,t+1} = e_1 M_2 r_{t+1}^D + \sum_{k=0}^{\infty} \left(e_1 M_1^{k+1} + e_2 M_1^k\right) M_2 r_{t-k}^D \tag{D.3}$$

We then have

$$\beta_1 = \rho e_1 M_2 + (e_1 M_1 + e_2)(I - \rho M_1)^{-1} M_2 \tag{D.4}$$

Next consider $er_{1,t+1}$, the excess return of the Foreign short term bond over the Home short term bond. Defining $e_1 = (1, 0)$ and $e_2 = (-1, 0)$, the regression coefficient of $er_{1,t+1}$ on r_t^D is

$$\beta_2 = \rho e_1 M_2 + (e_1 M_1 + e_2)(I - \rho M_1)^{-1} M_2 + 1 \tag{D.5}$$

Finally consider the difference between the Foreign and the Home local excess returns of long term over short term bonds. This is equal to $-\lambda p_{t+1}^{b,D} + p_t^{b,D} - r_t^D$. Defining $e_1 = (0, -\lambda)$ and $e_2 = (0, 1)$, this coefficient of a regression on r_t^D is

$$\beta_3 = \rho e_1 M_2 + (e_1 M_1 + e_2)(I - \rho M_1)^{-1} M_2 - 1 \tag{D.6}$$

We can also consider predictability reversal in this model for the FX excess return. Defining again $e_1 = (1, 0)$ and $e_2 = (-1, 0)$, the regression coefficient of $er_{1,t+k}$ on r_t^D is

$$\beta_k = \rho^k e_1 M_2 + \sum_{i=0}^{k-2} (e_1 M_1^{i+1} + e_2 M_1^i) M_2 \rho^{k-i-1} + (e_1 M_1 + e_2) M_1^{k-1} (I - \rho M_1)^{-1} M_2 + \rho^{k-1} \tag{D.7}$$