# Solving DSGE Portfolio Choice Models with Dispersed Private Information

Cédric Tille*

*Graduate Institute of International and Development Studies, and CEPR*

Eric vanWincoop**

*University of Virginia, and NBER*

**Abstract**

Noisy rational expectations models, in which agents have dispersed private information and extract information from an endogenous asset price, are widely used in finance. However these linear partial equilibrium models do not fit well in modern macroeconomics that is based on non-linear dynamic general equilibrium models. We develop a method for solving a DSGE model with portfolio choice and dispersed private information. We combine and extend existing local approximation methods applied to public information DSGE settings with methods for solving noisy rational expectations models in finance with dispersed private information.

*Key words:* local approximation method, dispersed information, private information, noisy rational expectations model, dynamic general equilibrium model
*JEL:* C60, F30, F41, G11

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*Department of Economics, PO Box 136, 1211 Geneva, Switzerland, cedric.tille@graduateinstitute.ch

**Corresponding author, Department of Economics, 2015 Ivy Road Charlottesville, VA 22904, USA, phone +1-434-924-3997, fax +1-434-982-2904, vanwincoop@virginia.edu

## 1. Introduction

There is a long tradition in finance of noisy rational expectations (NRE) models in which asset prices aggregate dispersed private information.[1] Despite the known relevance of dispersed information for asset prices, this feature is absent from the workhorse DSGE macro models with portfolio choice. This reflects the simplifying assumptions of linear, partial equilibrium NRE models (with a riskfree asset that is in infinite supply), which stand in contrast to the non-linear general equilibrium DSGE models.

In this paper we combine the two literatures and solve a DSGE portfolio choice model that contains the standard features of NRE models, namely dispersed private information and noise that prevents asset prices from completely revealing the aggregate of the private information (e.g. so-called noise or liquidity traders). This paper focuses on the solution method. We consider the implications of the model for capital flows in a companion paper (Tille and van Wincoop 2012).

While there is by now a large literature in macroeconomics that emphasizes the role of dispersed private information,[2] it does not consider the issue of portfolio choice. In addition, most contributions abstract from a feature of central interest under dispersed information. When agents act on their private information, it is reflected in macroeconomic aggregates such as consumption, the capital stock, asset and goods prices. By assuming that agents cannot extract information from such macroeconomic aggregates, most of the literature shuts down a central element of NRE models, where agents extract information from asset prices that they use alongside their own private information.

Outside of the finance NRE literature there are some models where agents extract information from endogenous variables (usually goods prices or a price index). These models are however either linear, or can be solved using a standard linearization.[3] The standard simple linearization method does not work in portfolio choice models for several reasons. First, non-linear

[1] See Brunnermeier (2001) for a review.

[2] See footnote 4 in Angeletos and La'O (2009) for a long list of such papers.

[3] Examples of the former are Angeletos, Lorenzoni and Pavan (2010), Townsend (1983) and Vives (1993, 1997). Examples of the latter are Amador and Weil (2010), Lorenzoni (2009) and Rondina (2008).

equations are approximated around a point where the standard deviation $\sigma$ of shocks is zero (deterministic steady state). Portfolio choice is however not well defined in a deterministic world. Second, we show that when $\sigma \rightarrow 0$ the cross-sectional variance of portfolio shares across investors approaches infinity and prices become fully revealing, unless we make an assumption on agents' private signals that requires taking higher order expansions of equations to solve the model.

Our solution method combines and extends methods used to solve linear NRE models and local approximation methods that are widely used to solve DSGE models in macroeconomics. We present a general characterization of local approximations in section 2, and show how it applies in models without private information. Section 3 discusses the challenges that arise from private information and how they are handled. In Section 4 we consider a simple linear NRE model that can be solved in closed form. This allows us to illustrate the approximation method and compare it to the closed form solution. In Section 5 we apply the method to a more complex two-country DSGE model in which agents make consumption, portfolio and investment decisions.[4] Section 6 discusses aggregation issues and the accuracy of the method. Section 7 concludes.

## 2. Local Approximation in Models with Public Information

In this section we describe how local approximation methods work in models with public information. This provides a good starting point for understanding the issues that arise when considering local approximation methods in models with private information that we take up in the next section.

### *2.1. Some Notation*

It is useful to start with some basic notation. Let $x_t$ be a vector of variables of the model, which we can write as $x_t = (c_t, k_t, a_t)'$, where $c_t$ is a

[4]Papers that apply NRE models to open economy settings include Albuquerque, Bauer and Schneider (2007,2009), Bacchetta and van Wincoop (2006), Brennan and Cao (1997), Gehrig (1993) and Veldkamp and van Nieuwerburgh (2009). But like all models in the NRE literature, these are linear partial equilibrium models, in contrast to the DSGE open economy model with dispersed information that we will consider in Section 4 as an illustration of the method.

vector of control variables, $k_t$ a vector of predetermined state variables (i.e. $E_t k_{t+1} = k_{t+1}$) and $a_t$ a vector of state variables that follow an exogenous forcing process. The model consists of equations of the form $E_t \varphi(x_t, x_{t+1}) = 0$ as well as the exogenous forcing process of the form:

$$a_{t+1} = N a_t + \sigma \nu_{t+1} \tag{1}$$

$\nu_{t+1}$ is an exogenous innovation with a distribution with mean zero that does not depend on $\sigma$. The standard deviation of all model innovations $\varepsilon_{t+1} = \sigma \nu_{t+1}$ is therefore proportional to $\sigma$.

*2.2. Local Approximation*

Local approximation methods provide a solution around the point $\sigma = 0$. We can then think of the solution of $x_t$ as a function of $\sigma$. A local approximation includes the first couple of terms of the Taylor expansion of this function around $\sigma = 0$. Notice that approximation methods are usually presented differently, specifically in the form of control variables as an approximate linear or quadratic function of state variables rather than as a linear or quadratic function of $\sigma$. We show below that our characterization and the usual one in fact identical. Thinking of the solution of $x_t$ explicitly as a function of $\sigma$ has conceptual advantages that facilitate the understanding of local approximations in more complicated settings with portfolio choice and private information.

The solution of $x_t$ as a function of $\sigma$ is conditional on innovations $\nu$ at time $t$ and earlier. Assuming that $x_t$ is a fully differentiable function of $\sigma$, we can write it as a Taylor expansion around $\sigma = 0$:

$$x_t = g_0 + \sum_{s=1}^{\infty} g_{s,t} \sigma^s \tag{2}$$

where the $g_0$ and $g_{s,t}$ in general depend on innovations $\nu$ at time $t$ and earlier. We refer to $g_{s,t}\sigma^s$ as the order $s$ component of $x_t$, which is also denoted as $x_t(s)$. The order 0 component is simply $x(0) = g_0$. As an example, an order 2 component could be $\sigma^2 + (\sigma \nu_t)^2$. The approximate solution of order $s$ contains all components of $x_t$ through order $s$.

In order to solve for $x_t$ as a polynomial function of $\sigma$ as in (2), for each equation $E_t \varphi(x_t, x_{t+1}) = 0$ we need to write $E_t \varphi(x_t, x_{t+1})$ as a polynomial function of $\sigma$:

$$E_t \varphi(x_t, x_{t+1}) = \sum_{s=0}^{\infty} \varphi_s^E \sigma^s \tag{3}$$

The term $\varphi_s^E \sigma^s$ is the order $s$ component of $E_t\varphi(x_t, x_{t+1})$.

Solving for the $\varphi_s^E$ takes three steps. The first is to derive a polynomial expression for $\varphi(x_t, x_{t+1})$ as a function of $\sigma$, conditional on known and unknown variables that do not depend on $\sigma$. This is discussed in Section 2.3 below. The second step involves taking the expectation of the resulting expression and the final step uses this result to compute the order components $\varphi_s^E \sigma^s$ of $E_t\varphi(x_t, x_{t+1})$. Steps 2 and 3 are straightforward in the absence of private information and are discussed in Section 2.4. The presence of private information makes steps 2 and 3 substantially more complex and is discussed in Section 3.

### 2.3. Expansion around $\sigma = 0$

We start by computing a polynomial expression for $\varphi(x_t, x_{t+1})$ as a function of $\sigma$, and show that it is equivalent to the usual approximation approach in terms of control and state variables.

Our approach first substitutes $x_t = g_0 + \sum_{s=1}^{\infty} g_{s,t}\sigma^s$ and $x_{t+1} = g_0 + \sum_{s=1}^{\infty} g_{s,t+1}\sigma^s$ into $\varphi(x_t, x_{t+1})$. This makes $\varphi(x_t, x_{t+1})$ an explicit function of $\sigma$, conditional on $g_0$, $g_{s,t+1}$ and $g_{s,t}$ for all $s \geq 1$. A Taylor expansion around $\sigma = 0$ gives

$$\varphi(x_t, x_{t+1}) = \omega_0 + \sum_{s=1}^{\infty} \omega_{s,t+1}\sigma^s \tag{4}$$

where the $\omega_0$ and $\omega_{s,t+1}$ depend on $g_0$, $g_{j,t+1}$ and $g_{j,t}$ for all $j \leq s$. Specifically, we have for $s = 0, 1, 2$

$$\omega_0 = \varphi(g_0, g_0) \tag{5}$$

$$\omega_{1,t+1} = \varphi_1' g_{1,t} + \varphi_2' g_{1,t+1} \tag{6}$$

$$\omega_{2,t+1} = \varphi_1' g_{2,t} + \varphi_2' g_{2,t+1} + \frac{1}{2} g_{1,t}' \varphi_{11} g_{1,t} + \frac{1}{2} g_{1,t+1}' \varphi_{22} g_{1,t+1} + g_{1,t}' \varphi_{12} g_{1,t+1} \tag{7}$$

The subscripts of $\varphi$ refer to derivatives with respect to $x_t$ and $x_{t+1}$, evaluated at $g_0$.

As pointed out above, thinking of equations and variables as functions of $\sigma$ differs from the usual representation. The two approaches are however identical. Usually, we start by writing a Taylor expansion of the expression

$\varphi(x_t, x_{t+1})$ around $x_t = x_{t+1} = x(0) = g_0$:

$$\varphi(x_t, x_{t+1}) = \varphi(x(0), x(0)) + \varphi_1'\hat{x}_t + \varphi_2'\hat{x}_{t+1} \\ +\frac{1}{2}\hat{x}_t'\varphi_{11}\hat{x}_t + \frac{1}{2}\hat{x}_{t+1}'\varphi_{22}\hat{x}_{t+1} + \hat{x}_t'\varphi_{12}\hat{x}_{t+1} + ... \tag{8}$$

where $\hat{x}_t = x_t - x(0)$ and where we have only explicitly written out up to quadratic terms for brevity. We now substitute (2) and focus on the terms that are constant, linear and quadratic in $\sigma$:

$$\varphi(x_t, x_{t+1}) = \varphi(g_0, g_0) + [\varphi_1' g_{1,t} + \varphi_2' g_{1,t+1}]\,\sigma + \Big[\varphi_1' g_{2,t} + \varphi_2' g_{2,t+1} \\ +\frac{1}{2}g_{1,t}'\varphi_{11}g_{1,t} + \frac{1}{2}g_{1,t+1}'\varphi_{22}g_{1,t+1} + g_{1,t}'\varphi_{12}g_{1,t+1}\Big]\,\sigma^2 + ... \tag{9}$$

This corresponds exactly to (4), combined with (5)-(7), showing that our representation matches exactly with the standard one.

An alternative way of writing (9), obtained by substituting $\hat{x}_t = \sum_{s=1}^{\infty} x_t(s)$ into (8), is

$$\varphi(x_t, x_{t+1}) = \varphi(g_0, g_0) + [\varphi_1' x_t(1) + \varphi_2' x_{t+1}(1)] + \Big[\varphi_1' x_t(2) + \varphi_2' x_{t+1}(2) + \\ \frac{1}{2}x_t(1)'\varphi_{11}x_t(1) + \frac{1}{2}x_{t+1}(1)'\varphi_{22}x_{t+1}(1) + x_t(1)'\varphi_{12}x_{t+1}(1)\Big] + ... \tag{10}$$

The first term in brackets consists of first-order terms and is equal to $\omega_{1,t+1}\sigma$. The second term in brackets contains second-order terms and is equal to $\omega_{2,t+1}\sigma^2$. These correspond exactly to the same terms in (9).

A noteworthy point from comparing (9) and (10) is that one should not confuse linear terms in (10) with first-order terms. For instance the term $\varphi_1' x_t(2)$ in (10) is linear in $x_t(2)$, but is second-order as it corresponds to $\varphi_1' g_{2,t}\sigma^2$ in (9). In general, a linear term $x_t$ has components of all orders.

### *2.4. Solution with Public Information*

We now move to the second step by taking the expectation of (4). For now we assume that there is only public information known to all agents, including the unconditional distribution of $\nu_{t+1}$, which is independent of $\sigma$. The expectation of (4) is:

$$E_t\varphi(x_t, x_{t+1}) = \omega_0 + \sum_{s=1}^{\infty} [E_t\omega_{s,t+1}]\,\sigma^s \tag{11}$$

A key point is that with public information $E_t\omega_{s,t+1}$ is zero-order for any $s \geq 1$. To see this, recall that $\omega_{s,t+1}$ depends on $g_0$, $g_{j,t}$ and $g_{j,t+1}$ for all $j \leq s$. In turn, $g_{j,t}$ depends on values of $\nu_t$ at time $t$ and earlier, while $g_{j,t+1}$ depends on values of $\nu_t$ at time $t+1$ and earlier. The values of $\nu$ at time $t$ and earlier are known at time $t$ by definition, and they are not a function of $\sigma$ by construction (i.e. they are zero-order). The expectation of $\nu_{t+1}$ is computed on the available information, namely the unconditional distribution of $\nu_{t+1}$ that does not depend on $\sigma$ either. It follows that $E_t\omega_{s,t+1}$ does not depend on $\sigma$, and is thus zero-order.

As $E_t\omega_{s,t+1}$ is zero-order, the order $s$ of the expectation $E_t\varphi(x_t, x_{t+1})$ corresponds to the expectation of the order $s$ of $\varphi(x_t, x_{t+1})$, that is $\varphi_s^E\sigma^s = [E_t\omega_{s,t+1}]\,\sigma^s$. This correspondence between the orders of expectations and expectations of orders greatly facilitates the computation of the solution. As shown below however, it only holds under public information.

We can now solve for the order components of $x_t$ by imposing the order components of the model equations. Imposing the order $s$ component of $E_t\varphi(x_t, x_{t+1}) = 0$ implies setting $\omega_0 = 0$ when $s = 0$ and $E_t\omega_{s,t+1} = 0$ for $s \geq 1$. We can write this in a more familiar way in terms of the order-components of $x_t$. For the zero, first and second-order we have

$$0 = \varphi(x(0), x(0)) \tag{12}$$

$$0 = \varphi_1' x_t(1) + \varphi_2' E_t x_{t+1}(1) \tag{13}$$

$$\begin{aligned} 0 = {} & \varphi_1' x_t(2) + \varphi_2' E_t x_{t+1}(2) + 0.5 x_t(1)'\varphi_{11} x_t(1) \\ & + 0.5 E_t x_{t+1}(1)'\varphi_{22} x_{t+1}(1) + E_t x_t(1)'\varphi_{12} x_{t+1}(1) \end{aligned} \tag{14}$$

These correspond respectively to the zero, first and second-order components of $E_t\varphi(x_t, x_{t+1}) = 0$. They also follow from (10) by setting the expectation of its corresponding order components equal to zero.

Two observations are noteworthy at this point. First, the solution for various orders can proceed sequentially, with the solution for order $s$ building on the solution for order $s-1$. Consider for instance the solution of the zero, first and second-order components of $x_t$. We sequentially impose the zero, first and second-order components of the equations. Imposing the zero-order component (12) of the equations allows us to solve for the zero-order component of the variables $x(0) = g_0$. Using this, we impose the first-order component (13) of the equations to solve for the first-order component of the variables $x_t(1)$. Using these results, we then impose the second-order component (14) of the equations to solve for the second-order component of the variables $x_t(2)$.

The second observation is that our characterization in terms of functions of $\sigma$ maps into the usual characterization in terms of a mapping from current state variables to controls variables and future state variables. Consider for instance the first-order solution. We usually think of it as a linear approximate solution of $c_t$ and $k_{t+1}$ as a function of the current state variables $a_t$ and $k_t$, together with the exogenous forcing process for $a_t$. One may write this system as (in terms of deviations from $x(0)$)

$$
\begin{aligned}
c_t(1) &= A_1 a_t(1) + A_2 k_t(1) && (15)\\
k_{t+1}(1) &= B_1 a_t(1) + B_2 k_t(1) && (16)\\
a_{t+1}(1) &= N a_t(1) + \sigma \nu_{t+1} && (17)
\end{aligned}
$$

with $A_i$ and $B_i$ matrices with zero-order constants. We included the first-order notation "(1)", although this is often omitted. (15)-(16) follow from imposing the first-order component (13) of the model equations, using standard techniques to solve such first-order difference equations. Each equation in the system (15)-(17) can be written as a zero-order term times $\sigma$:

$$
\begin{aligned}
c_t(1) &= \left[A_1(I - NL)^{-1}\nu_t + A_2(I - B_2L)^{-1}B_1L(I - NL)^{-1}\nu_t\right]\sigma && (18)\\
k_t(1) &= \left[(I - B_2L)^{-1}B_1L(I - NL)^{-1}\nu_t\right]\sigma && (19)\\
a_t(1) &= \left[(I - NL)^{-1}\nu_t\right]\sigma && (20)
\end{aligned}
$$

where $L$ is the lag operator. The terms in brackets can jointly be written as $g_{1,t}$ and are linear in innovations of $\nu$ at time $t$ and earlier. This shows that the familiar solution (15)-(17) can also be written in the form $x_t(1) = g_{1,t}\sigma$.

*2.5. Portfolio Choice Models*

The method needs to be modified a bit if we allow for portfolio choice, maintaining for now the assumption that there is only public information. Imposing the zero, first and second-order components of equations continues to take the form (12)-(14) in models with portfolio choice. But it is no longer possible to solve the order components of the variables by sequentially imposing the order components of the model equations.

Consider for example imposing the zero-order component $\varphi(x(0), x(0)) = 0$ of model equations. In models without portfolio choice we can usually solve for the entire vector $x(0)$ from this set of equations, sometimes referred to as the deterministic steady state. But with portfolio choice this is no longer the case as long as there are agents with different portfolios, as in open-economy settings. Intuitively, optimal portfolio shares (including their zero-order component) depend on risk, which is at least second-order and requires us to impose second and higher-order components of portfolio Euler equations.[5] Specifically, in a two-country model the part of $x(0)$ that consists of the zero-order component of the difference across countries in portfolio shares can only be solved by imposing the second-order component of the difference across countries in portfolio Euler equations.

Devereux and Sutherland (2010) and Tille and van Wincoop (2010) show how in this case the various order components of equations can be used to solve various order components of the variables.[6] Consider a two-country model with Home (H) and Foreign (F) agents that choose a different portfolio. We need to distinguish between the difference in portfolio shares between H and F and all "other variables". Similarly, we need to distinguish between the difference between H and F portfolio Euler equations and all "other equations".[7]

First the zero-order component of all "other variables" is computed from the zero-order component of all "other equations". Then the second-order component of the difference in portfolio Euler equations and the first-order component of all other equations are used to jointly solve for the zero-order component of the difference in portfolio shares and the first-order component of all other variables. Repeating this last step one order higher allows us to jointly solve for the first-order component of the difference in portfolio shares and the second-order component of all other variables.

## 3. Local Approximation in Models with Private Information

We now show how the local approximation method needs to be extended to handle private information. An issue in applying a local approximation to a situation of heterogenous agents is that the shocks affecting individual

[5]The only exception is when all agents are identical, so that portfolio shares correspond exactly to relative asset supplies.

[6]This method has been widely applied to open economy DSGE models. Examples are Coeurdacier, Kollmann and Martin (2010), Coeurdacier and Gourinchas (2011), Devereux and Yetman (2010), Ghironi, Lee and Rebucci (2009) and Okawa and van Wincoop (2012).

[7]For both agents the portfolio Euler equations set the expected product of an asset pricing kernel and asset return differentials equal to zero.

agents, such as income shocks or private information, can be larger than aggregate shocks, which can lead to concern about the accuracy. We address this issue more precisely in Section 6, but it should be said from the outset that the accuracy is no worse than under private information as long as we focus on aggregate variables.

The presence of private information raises a host of issues that we address in turn. We first discuss the nature of private signals, and then discuss how to compute expectations of model equations and impose order components. We finally consider the computation of the noise to signal ratio that is a critical element in the investors' signal extraction of information from observed variables.

### *3.1. Nature of Private Signals*

Without loss of generality, we assume for the purpose of this section that the vector of innovations $\varepsilon_{t+1} = \sigma\nu_{t+1}$ is univariate and has a $N(0, \sigma^2)$ distribution. An individual investor $j$ receives a private signal $v_t^j$ at time $t$ of the innovation at time $t+1$. This signal is an imperfect source of information and takes the form

$$v_t^j = \varepsilon_{t+1} + \epsilon_t^j \tag{21}$$

where $\epsilon_t^j$ is the error in agents $j$'s signal. We assume that signal errors add up to zero across all agents: $\int \epsilon_t^j dj = 0$, and that the cross-sectional standard deviation of errors is given by $\sigma_\epsilon$.

As we consider an approximation of the model around $\sigma = 0$, we keep $\sigma_\epsilon$ constant. In other words, we assume that $\sigma_\epsilon$ is zero-order. Conceptually $\sigma_\epsilon$ is a very different parameter than $\sigma$. The former is a measure of the quality of private information, while the latter is a measure of the volatility of exogenous shocks.

The assumption that $\sigma_\epsilon$ is zero-order (independent of $\sigma$) is also needed to ensure a well-behaved solution to the portfolio choice problem. To see this, consider a simple two-period portfolio problem with a riskfree asset. Let $\varepsilon_{t+1}$ be the stochastic component of the return of the risky asset. Investor $j$ formulates her expectation for $\varepsilon_{t+1}$ based on two pieces of information: the publicly known distribution of $\varepsilon_{t+1}$ and the private signal $v_t^j$. The expectation of $\varepsilon_{t+1}$ and its variance are then:

$$E_t^j \varepsilon_{t+1} = \frac{\sigma^2}{\sigma_\epsilon^2 + \sigma^2} v_t^j \quad ; \quad var\left(\varepsilon_{t+1}\right) = \frac{\sigma_\epsilon^2 \sigma^2}{\sigma_\epsilon^2 + \sigma^2} \tag{22}$$

where $E_t^j$ is the expectation of agent $j$. Note that while agents share the same assessment of the variance, they have different expectations.

Consider what would happen if we assumed that $\sigma_\epsilon$ were first-order, i.e. proportional to $\sigma$. The ratio multiplying $v_t^j$ in $E_t^j \varepsilon_{t+1}$ would then be a zero-order constant, independent of $\sigma$. This has two problematic implications. First, a simple mean-variance portfolio allocation implies that the share of the portfolio invested in the risky asset depends on the expected excess return divided by the variance of the excess return. The latter, which is $var(\varepsilon_{t+1})$, is second order (proportional to $\sigma^2$). Differences in the expected excess return across agents are equal to differences in their expectation of $\varepsilon_{t+1}$, which are equal to differences in their private signals $v_t^j$ times a zero-order coefficient. The first-order dispersion of private signals across agents then translates into a first-order dispersion of the expected excess return. As each investor's portfolio share is the ratio of the expected excess return and the variance of the excess return, the dispersion of individual portfolio shares goes to infinity when we take $\sigma$ towards 0. The problem is not present when we assume that $\sigma_\epsilon$ is zero-order (independent from $\sigma$), as the weight on the private signal $v_t^j$ in the expectation of $\varepsilon_{t+1}$ is then second-order.

An additional problem arises in NRE models, where agents also extract information about $\varepsilon_{t+1}$ from the asset price. Agents then have three pieces of information about $\varepsilon_{t+1}$: their private signal, the unconditional distribution and the asset price. As the asset price reflects the investment decisions of all agents, it becomes fully revealing of $\varepsilon_{t+1}$ to the first-order. This is normally avoided in NRE models by introducing a source of noise in asset demand. The impact of the noise on the asset price is however third-order. Shocks to noisy asset supply (or demand) generate changes in the risk premium that are third-order (risk is second and higher order, a change in risk is at least third order), leading to a third-order effect on the asset price. In order for the asset price not to reveal $\varepsilon_{t+1}$ to the first-order, we need the impact of $\varepsilon_{t+1}$ on the asset price to also be third-order. This is not the case if we assume that $\sigma_\epsilon$ is first-order, as the impact of $\varepsilon_{t+1}$ on the asset price is then first-order. The average expectation of $\varepsilon_{t+1}$ depends on the average private signal $\int v_t^i di = \varepsilon_{t+1}$ with a zero-order weight. This means that to the first-order the asset price depends only on $\varepsilon_{t+1}$ and not on the noise (which enters third-order). The first-order solution is then the same as if private information about the asset payoff is replaced by common knowledge of the future payoff. This problem is again avoided by assuming that $\sigma_\epsilon$ is zero-order, so that the weight on the private signal in (22) is second-order.

*3.2. Order Component of Expectation vs Expectation of Order Component*

In models with private information the expressions for $\varphi(x_t, x_{t+1})$ that we derived in Section 2.3 as the sum of its order components continue to hold. Specifically, we still have (10), which is repeated here for convenience:

$$\begin{aligned}
\varphi(x_t, x_{t+1}) &= \varphi(h_0, h_0) + [\varphi_1' x_t(1) + \varphi_2' x_{t+1}(1)] \\
&+ \left[\varphi_1' x_t(2) + \varphi_2' x_{t+1}(2) + \frac{1}{2} x_t(1)' \varphi_{11} x_t(1)\right. \\
&+ \left.\frac{1}{2} x_{t+1}(1)' \varphi_{22} x_{t+1}(1) + x_t(1)' \varphi_{12} x_{t+1}(1)\right] + ...
\end{aligned} \tag{23}$$

Recall that $x_t(s) = g_{s,t}\sigma^s$ and $x_{t+1}(s) = g_{s,t+1}\sigma^s$.

(23) explicitly writes out the zero, first and second-order components of $\varphi(x_t, x_{t+1})$. A major issue with private information is that the expectation of the order $s$ component of $\varphi(x_t, x_{t+1})$ is no longer equal to the order $s$ component of the expectation of $\varphi(x_t, x_{t+1})$. This implies, for example, that setting the first-order component of $E_t^j \varphi(x_t, x_{t+1})$ equal to zero is not the same as setting $\varphi_1' x_t(1) + \varphi_2' E_t x_{t+1}(1) = 0$.

The core of the issue is that with private information the conditional distribution of $g_{s,t+1}$ does depend on $\sigma$. Consider a simple example where $x_{t+1} = \varepsilon_{t+1} = \sigma\nu_{t+1}$, so that $g_{1,t+1} = \nu_{t+1}$ is zero-order. With public information the unconditional distribution of $\nu_{t+1}$ is the only available source of information, and $E_t g_{1,t+1} = 0$, irrespective of $\sigma$.

In the presence of private information however, investor $i$ also relies on her private signal to compute the expectation of $\varepsilon_{t+1}$, which is given by (22). This in turn implies

$$E_t^j g_{1,t+1} = E_t^j \nu_{t+1} = \frac{E_t^j \varepsilon_{t+1}}{\sigma} = \frac{\sigma^2}{\sigma_\epsilon^2 + \sigma^2}\nu_{t+1} + \frac{\sigma}{\sigma_\epsilon^2 + \sigma^2}\epsilon_t^j \tag{24}$$

(24) clearly shows that the expectation of $g_{1,t+1}$ depends on $\sigma$, and thus has several orders. Specifically, the zero-order of $E_t^j g_{1,t+1}$ is zero, its first order is $\left(\epsilon_t^j / \sigma_\epsilon^2\right)\sigma$ and its second-order is $\left(\nu_{t+1}/\sigma_\epsilon^2\right)\sigma^2$. Even though $g_{1,t+1}$ is a zero-order variable, its conditional expectation contains higher order terms.

To see that this implies that the order $s$ component of $E_t^j \varphi(x_t, x_{t+1})$ differs from the expectation of the order $s$ component of $\varphi(x_t, x_{t+1})$, consider the first-order ($s = 1$). The first-order component of $\varphi(x_t, x_{t+1})$ is $\varphi_1' x_t(1) + \varphi_2' x_{t+1}(1)$, whose expectation is

$$\varphi_1' x_t(1) + \varphi_2' E_t^j g_{1,t+1}\sigma \tag{25}$$

This expression is however not limited to first-order terms (it includes higher order terms through $E_t^j g_{1,t+1}$) and therefore differs from the first-order component of $E_t^j \varphi(x_t, x_{t+1})$.

More generally, the order $s$ component of the expectation of $\varphi(x_t, x_{t+1})$ differs from the expectation of the order $s$ component of $\varphi(x_t, x_{t+1})$. The equations (13)-(14), which set the expectation of the first and second order components of $\varphi(x_t, x_{t+1})$ equal to zero, are therefore no longer correct as these are not the first and second-order component of $E_t\varphi(x_t, x_{t+1})$. We therefore need to carefully compute the conditional expectations of the various equations before splitting them into their various orders.

### *3.3. Computing Expectations*

To compute the expectation of $\varphi(x_t, x_{t+1})$, we first write $\varphi(x_t, x_{t+1})$ in a polynomial form as a function of the future innovations $\varepsilon_{t+1}$. We then use the results from signal extraction to compute expectations of terms linear in $\varepsilon_{t+1}$, quadratic in $\varepsilon_{t+1}$, and so on.

Several steps are needed to express $\varphi(x_t, x_{t+1})$ in a polynomial form as a function of $\varepsilon_{t+1}$. We start with the Taylor expansion (8) of $\varphi(x_t, x_{t+1})$. Next we conjecture a polynomial solution of the control variables as a function of the state variables. A quadratic conjecture will be sufficient if the aim is to obtain a second-order solution. Finally, we replace $a_{t+1}$ with $Na_t + \varepsilon_{t+1}$. Then $x_{t+1} = (c_{t+1}, k_{t+1}, a_{t+1})'$ depends on $a_t$, $k_{t+1}$ and $\varepsilon_{t+1}$. The vector $x_t$ only depends on $a_t$ and $k_t$. We substitute these results into the Taylor expansion (8) for $\varphi(x_t, x_{t+1})$. This then allows us to write $\varphi(x_t, x_{t+1})$ as a polynomial function of $\varepsilon_{t+1}$, conditional on the variables $a_t$, $k_t$ and $k_{t+1}$ that are known at time $t$.

The next step is to derive the conditional distribution of $\varepsilon_{t+1}$ from signal extraction in order to compute the expected values of terms that are linear, quadratic and perhaps cubic in the innovations $\varepsilon_{t+1}$. The signal extraction process in NRE models usually relies on three sources of information. The first is the publicly known unconditional distribution of innovations. The second is private signals about future innovations. The last source of information consists of endogenous variables. In asset pricing models the asset price depends on future innovations as agents trade based on their private signals, which average to $\varepsilon_{t+1}$. The asset price then contains information about $\varepsilon_{t+1}$. As pointed out above, NRE models also include an additional source of asset demand, from noise traders for example, to prevent the asset price from fully revealing $\varepsilon_{t+1}$.

Even though macro models are in general non-linear and the asset price will therefore be a non-linear function of $\varepsilon_{t+1}$ and a noise variable, we can still compute the distribution of future innovations from a simple linear signal extraction problem. The reason for this is as follows. We will conjecture and verify that $\varepsilon_{t+1}$ and the noise shock affect asset prices in a jointly linear way through a variable denoted by $h_t$. While $h_t$ in general affects an asset price in a non-linear way, it is itself a linear function of $\varepsilon_{t+1}$ and the noise shock. Agents thus observe $h_t$ (but not its components) from the asset price, controlling for the impact of known state variables. This provides them with another signal of $\varepsilon_{t+1}$ that is in linear form, allowing us to solve a standard linear signal extraction problem.

### *3.4. Imposing Order Components of Equations*

We are now in a position to impose the order components of the model. After computing the expectation of the polynomial in $\varepsilon_{t+1}$, equations take the form

$$f(a_t, k_t, k_{t+1}, h_t, \sigma) = 0 \tag{26}$$

Computing the order components of these equations entails no particular difficulty. For example, setting the first-order component equal to zero implies

$$f_1' a_t(1) + f_2' k_t(1) + f_3' k_{t+1}(1) + f_4 h_t(1) + f_5 \sigma = 0 \tag{27}$$

where $f_i$ is the derivative of element $i$ of $f()$ evaluated at its zero-order component.

As our model focuses on portfolio choice, we cannot use the simple sequential solution outlined in Section 2.4, but instead follow the method discussed in Section 2.5. This delivers a solution of $c_t$ and $k_{t+1}$ as a function of the state variables at various orders. Together with the exogenous forcing process (1) this tells us how the order components of variables evolve over time in response to shocks. However, imposing the order components of model equations as discussed in Section 2.5 does not deliver one key endogenous parameter. This is the zero-order noise-to-signal ratio $\lambda$ that captures the weight of the noise shock relative to $\varepsilon_{t+1}$ in $h_t$.

### *3.5. Computing Noise to Signal Ratio*

We follow the approach of standard NRE models and solve for the noise to signal ratio by imposing asset market equilibrium, equating portfolio demand for assets to asset supply. We impose the first-order component of the asset market clearing conditions. Asset demand reflects the portfolio allocation of the various agents. The easiest way to think about this is again in terms of a simple mean-variance portfolio choice model with two assets. A third-order change in the expected excess return then leads to a first-order portfolio shift as it is divided by the variance of the excess return, which is second-order. Third-order changes in the expected excess return only show up in the third-order component of portfolio Euler equations. That is why we need to impose the third-order component of the average portfolio Euler equation in order to obtain the first-order component of the average portfolio share from a demand perspective. Equating the resulting first-order component of asset demand to the first-order component of asset supply delivers $\lambda$.

## 4. A Simple Noisy Rational Expectation Model

In this section we consider a simple NRE model. It includes the standard assumptions that, while restrictive, allow for a closed-form solution. We first derive the closed-form solution, and then illustrate the approximation method in this setting. It delivers a solution to the asset price that to the zero, first, second and third-order is identical to the closed-form solution.

### *4.1. Building blocks*

The economy is populated by a unit mass of investors that live for one period. An individual investor $j$ solves an optimal portfolio allocation between a risk-free asset, with an exogenous return $r$, and a risky asset. Each share of the risky asset yields a payoff $f$ next period, which is normally distributed with mean $\bar{f}$ and variance $\sigma_f^2$:

$$f = \bar{f} + \varepsilon^f \quad ; \quad \varepsilon^f \sim N(0, \sigma_f^2) \tag{28}$$

Investors have constant absolute risk-aversion preferences. Utility of agent $j$ is $U^j = -Ee^{-c_j}$, where $c_j$ is consumption. Starting with wealth equal to one, consumption next period is:

$$c_j = r + z_j er$$

where $z_j$ is the number of shares of the risky asset purchased by agent $j$, $er = f - rq$ is the excess return on the risky asset and $q$ is the price of one share of the risky asset. The utility of investor $j$ is then:

$$U^j = -\exp\left[-r - z_j E^j er + \frac{1}{2}(z_j)^2 \, var^j(f)\right] \tag{29}$$

Utility maximization leads to a standard mean-variance portfolio allocation that reflects the expected excess return on the risky asset scaled by its variance:

$$z_j = \frac{E^j er}{var^j\left(f\right)} \tag{30}$$

The expectation and variance have a superscript $j$ because investors have different information. Specifically, investor $j$ receives a private signal $v^j$ about the future payoff innovation:

$$v^j = \varepsilon^f + \epsilon^j \quad ; \quad \epsilon^j \sim N(0, \sigma_\epsilon^2) \tag{31}$$

We follow the standard assumption in the NRE literature that the number of investors is sufficiently large for signal errors to cancel out in aggregate: $\int_0^1 \epsilon^j dj = 0$.

The model is closed by imposing market clearing for the risky asset. The asset is in exogenous supply $\bar{b}$. The demand comes from two sources: the utility-maximizing investors, and a random demand $b$ from traders that buy and sell the asset for reasons unrelated to expected payoffs. In the NRE literature these are usually referred to as exogenous noise traders or liquidity traders. We assume $b \sim N(0, \theta\sigma_f^2)$, where $\theta$ measures the variance of the noise relative to the unconditional variance of the payoff. The clearing of the asset market is written as:

$$z = \int_0^1 z_j dj = \bar{b} - b \tag{32}$$

*4.2. Solution*

NRE models are solved in three steps. The first step involves conjecturing an equilibrium asset price. We conjecture that the asset price depends on a constant component, $\bar{q}$, and a combination $h$ of payoff shocks and noise shocks:

$$q = \bar{q} + \alpha h = \bar{q} + \alpha\left(\varepsilon^f + \lambda b\right) \tag{33}$$

where $\bar{q}$, $\alpha$ and $\lambda$ are unknown coefficients. The future payoff innovation $\varepsilon^f$ affects the asset price as agents trade based on their private information and the average private signal is $\varepsilon^f$. The presence of noise shocks $b$ prevents the asset price $q$ from completely revealing $\varepsilon^f$. The coefficient $\lambda$ is the noise to signal ratio that reflects the impact of noise shocks relative to payoff shocks.

The second step of the solution is to compute $E^j\varepsilon^f$ and $var^j(\varepsilon^f)$ by solving a signal extraction problem. There are three sources of information: private signal, public information in the form of the unconditional distribution of $f$ and the asset price. The asset price contains information on the shocks through the combination $h$, so investors observe $h$ (through the asset price), but not its components. We summarize the three signals about the payoff shock $\varepsilon^f$ as follows:

$$\begin{aligned} Y^j &= \begin{pmatrix} 0 \\ v^j \\ h \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \varepsilon^f + \begin{pmatrix} -\varepsilon^f \\ \epsilon^j \\ \lambda b \end{pmatrix} = \iota \varepsilon^f + \epsilon \\ \Sigma &= Var\left(\epsilon\right) = \begin{pmatrix} \sigma_f^2 & 0 & 0 \\ 0 & \sigma_\epsilon^2 & 0 \\ 0 & 0 & \lambda^2 \theta \sigma_f^2 \end{pmatrix} \end{aligned}$$

Using the standard signal extraction technique, the expectation and the variance of the payoff innovation from the perspective of investor $i$ are

$$var\left(\varepsilon^f\right) = \left[\iota' \Sigma^{-1} \iota\right]^{-1} = \frac{1}{\left(\sigma_f^2\right)^{-1} + \left(\sigma_\epsilon^2\right)^{-1} + \left(\lambda^2 \theta \sigma_f^2\right)^{-1}} \tag{34}$$

$$E^j\left(\varepsilon^f\right) = \left[\iota' \Sigma^{-1} \iota\right]^{-1} \iota' \Sigma^{-1} Y^j = a_v v^j + a_h h \tag{35}$$

where:

$$a_v = \frac{var\left(\varepsilon^f\right)}{\sigma_\epsilon^2} \quad ; \quad a_h = \frac{var\left(\varepsilon^f\right)}{\lambda^2 \theta \sigma_f^2} \tag{36}$$

The expected payoff innovation depends positively on the private signal $v^j$ and the signal $h$, with the weight on these signals larger the greater their precision. Both the expectation and the variance of $\varepsilon^f$ are functions of the noise to signal ratio $\lambda$, which remains to be solved.

The last step of the solution imposes market equilibrium. We substitute the moments (34)-(35) into investor $j$'s optimal portfolio (30), aggregate the resulting expression across investors, and then impose the asset market clearing condition (32). This leads to a fixed point problem in the three unknown parameters of the conjectured asset price solution. The solution is:

$$\bar{q} = \frac{\bar{f}}{r} - \frac{\theta \sigma_\epsilon^4 \sigma_f^2}{1 + \theta \sigma_\epsilon^2 \left(\sigma_\epsilon^2 + \sigma_f^2\right)} \frac{\bar{b}}{r} \tag{37}$$

$$\alpha = \frac{1 + \theta \sigma_\epsilon^2 \sigma_f^2}{1 + \theta \sigma_\epsilon^2 \left(\sigma_\epsilon^2 + \sigma_f^2\right)} \frac{1}{r} \tag{38}$$

$$\lambda = \sigma_\epsilon^2 \tag{39}$$

*4.3. Local Approximation Method*

We now apply the local approximation method to the model. In terms of orders, we assume that $\bar{f}$ and $\bar{b}$ are zero order and the shocks $\varepsilon^f$ and $b$ are first-order.

As discussed in Section 3, we also assume that the variance $\sigma_\epsilon^2$ of signal errors is zero order. If instead the signal errors were first-order, and therefore $\sigma_\epsilon^2$ second-order, there would be two problems. First, the cross-sectional variance of portfolio shares goes to infinity when $\sigma_f \to 0$. The portfolio share of investor $j$ is

$$z_j = \frac{\bar{f} - r\left(\bar{q} + \alpha h\right) + a_h h}{var(\varepsilon^f)} + \frac{1}{\sigma_\epsilon^2} v^j \tag{40}$$

The cross-sectional variance of portfolio shares is $var(\epsilon^j)/\sigma_\epsilon^4 = 1/\sigma_\epsilon^2$. If $\sigma_\epsilon^2$ is second-order, proportional to $\sigma_f^2$, then the cross-sectional distribution of portfolio shares explodes to infinity when $\sigma_f \to 0$. This is a problem as the local approximation is around the point where $\sigma_f = 0$.

Second, when $\sigma_\epsilon^2$ is second-order, $\lambda = \sigma_\epsilon^2$ is second-order as well. This means that to the first-order the asset price does not depend on the noise and therefore becomes fully revealing about $\varepsilon^f$. To the first-order the solution to the asset price is then the same as if agents knew the value of $\varepsilon^f$: $q(1) = f/r$.

To avoid these problems, we assume that $\sigma_\epsilon^2$ is zero-order. The cross-sectional distribution of portfolio shares then remains constant even when $\sigma_f \to 0$. Since now the weight $\lambda$ on the noise in the price signal is zero-order, the price is not fully revealing of $\varepsilon^f$ to the first-order.

We now derive the solution through three steps. We write expansions of the equations, then compute expectations, before closing the model by imposing the asset market clearing condition and distinguishing between its various orders.

*4.3.1. Step 1: Expansions of Equations*

There are only two equations in this simple model. The first is the asset market clearing equation (32). The second is the portfolio Euler equation, which shows that the expected discounted excess return on the risky asset is zero:

$$E^j e^{-r - z_j er} er = 0 \tag{41}$$

We are only interested in solving for the zero, first and second-order components of the asset price. In the absence of private information, it would be sufficient to impose the zero, first and second-order components of (41). With dispersed information however, this solution remains conditional on the noise to signal ratio $\lambda$. As discussed in Section 3, computing $\lambda$ requires imposing the third-order component of the average portfolio Euler equation (41) across investors. We therefore need a cubic expansion of the average portfolio Euler equation.

The zero-order component of (41) immediately gives $q(0) = \bar{f}/r$. Taking a cubic expansion of (41) around $q(0)$ and $z_j(0)$, and averaging across all agents, gives:

$$\bar{E}er + m(0)Eer^3 = zEer^2 \tag{42}$$

where $\bar{E}$ is the average expectation across all agents and $m(0) = 0.5 \int z_j(0)^2 dj$. We have also used that the expectations of $er^2$ and $er^3$ are the same for all agents, which is shown below, so that we do not need to index the expectation operator.

*4.3.2. Step 2: Computing Expectations*

Expectations only show up in the average portfolio Euler equation (42), where the expectations of $er$, $er^2$ and $er^3$ enter. As discussed in Section 3, we need to compute the expectations of these variables before we can impose the order components. The excess payoff $er$ depends on the future innovation $\varepsilon^f$. The distribution of $\varepsilon^f$ can be computed from signal extraction.

We conjecture that the asset price $q$ is a (possibly non-linear) function of the combination $h$ of payoff and noise innovations: $h = \varepsilon^f + \lambda b$, so that observing the asset price then reveals the value of $h$. We also conjecture that the noise to signal ratio $\lambda$ is a zero-order parameter that needs to be solved. In general the conjecture for the asset price also depends on publicly observed state variables, but those are absent from our simple model here (they are present in the more general model in the next section). We compute the expectation and variance of $\varepsilon^f$ by solving the exact same signal extraction problem as in Section 4.2, which gives $\bar{E}\varepsilon^f = a_v\varepsilon^f + a_h h$. It also gives the variance of $\varepsilon^f$, which is the same for all agents.

Splitting the coefficients $a_v$ and $a_h$ in (36) into components of different orders, we have (more details are given in Appendix A):

$$\left[\bar{E}\varepsilon^f\right](0) = \left[\bar{E}\varepsilon^f\right](2) = [var(\varepsilon^f)](0) = [var(\varepsilon^f)](1) = 0$$
$$\left[\bar{E}\varepsilon^f\right](1) = \frac{1}{1+\lambda^2\theta}h$$
$$\left[\bar{E}\varepsilon^f\right](3) = \frac{\lambda^2\theta}{\sigma_\epsilon^2\left(1+\lambda^2\theta\right)}\sigma_f^2\left(\varepsilon^f - \frac{h}{1+\lambda^2\theta}\right)$$
$$[var(\varepsilon^f)](2) = \frac{\lambda^2\theta}{1+\lambda^2\theta}\sigma_f^2$$

We use these results to compute $\bar{E}er$, $Eer^2$ and $Eer^3$.

*4.3.3. Step 3: Imposing Order Components of Equations*

We now proceed to imposing the various order components of the average portfolio Euler equation (42) and the market clearing condition (32).

Starting with the first-order component, (32) gives $z(1) = -b$. The first-order component of (42) implies that the first-order expected excess return is zero: $[\bar{E}er](1) = 0$. Using our results this gives the first-order asset price:

$$\left[\bar{E}\varepsilon^f\right](1) - rq(1) = 0 \Rightarrow q(1) = \frac{1}{1+\lambda^2\theta}\frac{1}{r}h$$

Next consider second-order components. (32) implies that $z(2) = 0$. The second-order component of (42) is:

$$[\bar{E}er](2) = z(0)[Eer^2](2) = \bar{b}[Eer^2](2) \tag{43}$$

where we used the zero-order component of (32): $z(0) = \bar{b}$. Using that $Eer^2 = var(er) + (E(er))^2$, we have $[Eer^2](2) = [var(er)](2) + 2[Eer](1)[Eer](2)$. As $[Eer](1) = 0$ for all agents[8], $[Eer^2](2)$ is equal to $[var(er)](2) = [var(\varepsilon^f)](2)$. (43) then gives the second-order asset price:

$$[\bar{E}\varepsilon^f](2) - rq(2) = \bar{b}[var(\varepsilon^f)](2) \Rightarrow q(2) = -\frac{\lambda^2\theta}{1+\lambda^2\theta}\frac{\bar{b}}{r}\sigma_f^2$$

At this point we have solved for the zero-, first- and second-order components of $q$ and $z$. This solution however remains conditional on the noise to signal ratio $\lambda$, to which we now turn.

[8] $\left[E^j er\right](1) = 0$ follows from the first-order component of the portfolio Euler equation (41) for agent $i$.

We solve for $\lambda$ by equating $z(1)$ from the supply side to $z(1)$ from the demand side. The supply side reflects the asset market clearing (32), $z(1) = -b$.[9] Intuitively, an increase in demand $b$ by liquidity traders reduces the remaining net supply of the risky asset. $z(1)$ from the demand side follows by imposing the third-order component of the average portfolio Euler equation (42):

$$z(1) = \frac{[\bar{E}er](3) - m(0)[Eer^3](3)}{[var(er)](2)} \tag{44}$$

We have $[Eer^3](3) = 0$. To see that, we use that $er$ has a normal distribution, so that its third moment can be written as $Eer^3 = (E(er))^3 - 3E(er)var(er)$. The third-order component of this is zero because the first-order component of $E(er)$ is zero. We also have $[\bar{E}er](3) = [\bar{E}\varepsilon^f](3) - rq(3)$. Using the expressions for $[\bar{E}\varepsilon^f](3)$ and $[var(\varepsilon^f)](2)$ in Section 4.3.2, substituting the result into (44) and equating it to $z(1) = -b$ from the supply side, we get

$$\sigma_\epsilon^2 b + \varepsilon^f - \frac{1}{1+\lambda^2\theta}h - \frac{\sigma_\epsilon^2(1+\lambda^2\theta)}{\lambda^2\theta\sigma_f^2}rq(3) = 0 \tag{45}$$

This last equation gives us both $\lambda$ and $q(3)$. Since we have assumed that $q$ is a function of $h$, possibly non-linear, $q(3)$ in general depends on $h$. (45) is then a relationship in $b$, $\varepsilon^f$ and $h$. For this equation to hold, it must be the case that $b$ and $\varepsilon^f$ enter in the same linear combination as in $h$. This immediately implies that $\lambda = \sigma_\epsilon^2$, which is a zero-order constant as conjectured. (45) then implies that

$$q(3) = \frac{\theta^2\sigma_\epsilon^6}{r(1+\theta\sigma_\epsilon^4)^2}\sigma_f^2 h \tag{46}$$

We have now solved for the zero, first, second and third-order components of the asset price and for the noise to signal ratio $\lambda$. It is easily verified that the zero, first, second and third-order components of the exact solution $q = \bar{q} + \alpha h$ are identical to $q(0)$, $q(1)$, $q(2)$, and $q(3)$ from the approximation method.

[9] Note that agents do not know the average portfolio share $z$, and therefore cannot extract $b$ from this. They only know their own portfolio share $z_j$.

## 5. A DSGE Model

We now apply the solution method to a two-country dynamic stochastic general equilibrium model. Agents in each country make saving and portfolio allocation decisions based on public and dispersed private information. Tille and van Wincoop (2012) use the model to analyze the impact of dispersed information on gross and net international capital flows. While the model is more complex than the simple NRE framework of Section 4, we introduce a number of simplifying features that allow us to derive an analytical solution and make it easier to illustrate the method. The description of the model is kept to a minimum, with a full description in Tille and van Wincoop (2012).

### *5.1. Model Description*

There are two countries, Home and Foreign, indexed by $i = H$, $F$. Both produce a single consumption good using capital $K_{i,t}$ and labor $N_{i,t}$:

$$Y_{i,t} = A_{i,t} K_{i,t}^{1-\omega} N_{i,t}^{\omega} \tag{47}$$

The labor input is normalized to unity. The real wage $W_{i,t}$ is the marginal product of labor: $W_{i,t} = \omega A_{i,t} \left(K_{i,t}\right)^{1-\omega}$. Capital accumulation reflects investment $I_{i,t}$ and the depreciation rate $\delta$: $K_{i,t+1} = (1-\delta) K_{i,t} + I_{i,t}$. Capital is built by installment firms that convert the consumption good into capital with a quadratic adjustment cost. Investment is then driven by the Tobin's Q relation:

$$I_{i,t}/K_{i,t} = \delta + \left(Q_{i,t} - 1\right)/\xi \tag{48}$$

where $Q_{i,t}$ is the price of one unit of capital in country $i$, which we refer to as the equity price. The return from purchasing one unit of country $i$ capital at time $t$ consists of the dividend and the capital gain at time $t+1$:

$$R_{i,t+1} = \frac{(1-\omega) A_{i,t+1} \left(K_{i,t+1}\right)^{-\omega} + (1-\delta) Q_{i,t+1}}{Q_{i,t}} \tag{49}$$

Log productivity, $a_{it} = ln(A_{i,t})$, follows an autoregressive process:

$$a_{i,t+1} = \rho a_{i,t} + \varepsilon_{i,t+1} \quad i = H, F \tag{50}$$

where $\varepsilon_{i,t+1}$ has a $N(0, \sigma_a^2)$ distribution and is uncorrelated across countries.

Each agent receives private signals about next period's productivity innovations in both countries. The signals observed by a Home agent $j$ at time $t$ are:

$$v_{j,t}^{H,H} = \varepsilon_{H,t+1} + \epsilon_{j,t}^{H,H} \quad ; \quad \epsilon_{j,t}^{H,H} \sim N\left(0, \sigma_{HH}^2\right) \tag{51}$$

$$v_{j,t}^{H,F} = \varepsilon_{F,t+1} + \epsilon_{j,t}^{H,F} \quad ; \quad \epsilon_{j,t}^{H,F} \sim N\left(0, \sigma_{HF}^2\right) \tag{52}$$

where $\epsilon_{j,t}^{H,H}$ and $\epsilon_{j,t}^{H,F}$ are zero-order idiosyncratic components of the signals. As before, we assume that the errors of the signals average to zero across investors ($\int_0^1 \epsilon_{j,t}^{H,H} dj = \int_0^1 \epsilon_{j,t}^{H,F} dj = 0$). Foreign agents receive a similar set of signals. In each country the variance of the signal's error is $\sigma_{HH}^2$ for the domestic innovation and $\sigma_{HF}^2$ for the foreign innovation. We assume that domestic signals are more precise: $\sigma_{HH}^2 < \sigma_{HF}^2$.

To ensure that equity prices do not fully reveal the future innovations contained in the private signals, we need to introduce a source of noise affecting asset markets that is not directly observed by investors. A standard approach in the NRE literature is to consider noise traders who randomly buy and sell Home and Foreign equity. As investors do not observe these noise trading shocks, they remain uncertain whether a high demand for Home equity for instance is indicative of other investors having positive signals about future payoffs, or whether it is due to higher demand by noise traders. While the presence of noise traders is a tractable device in partial equilibrium NRE models, it would add complexity to our general equilibrium setting as we would need to fully characterize the noise traders, including their income and consumption.

We thus introduce noise shocks in an alternative way by considering a time-varying cost of investing abroad. A Home agent $j$ investing in Foreign equity receives the return $R_{F,t+1}$ times an iceberg cost $e^{-\tau_{Hj,t}} < 1$. Similarly, a Foreign agent $j$ investing in Home equity receives the return $R_{H,t+1}$ times an iceberg cost $e^{-\tau_{Fj,t}} < 1$. These iceberg costs are written as:

$$\tau_{Hj,t} = \tau\left[1 + \varepsilon_t^{\tau} + \xi_{Hj,t}\right] \quad ; \quad \tau_{Fj,t} = \tau\left[1 - \varepsilon_t^{\tau} + \xi_{Fj,t}\right] \tag{53}$$

where $\tau$ is a second-order constant (an iceberg cost on asset returns needs to be at least second-order to ensure a well-behaved portfolio solution). $\varepsilon_t^{\tau}$ is a first-order noise shock with a $N(0, \theta\sigma_a^2)$ distribution. It is the source of noise preventing the asset prices from fully revealing the future productivity innovations. Finally $\xi_{Hj,t}$ and $\xi_{Fj,t}$ are zero-order idiosyncratic terms. We assume that they add up to zero across all investors in a given country: $\int_0^1 \xi_{Hj,t} dj = \int_0^1 \xi_{Fj,t} dj = 0$. (53) implies that the average iceberg costs across investors in the Home and Foreign countries are:

$$\tau_{H,t} = \int_0^1 \tau_{Hj,t} dj = \tau \left[1 + \varepsilon_t^\tau\right] \quad ; \quad \tau_{F,t} = \int_0^1 \tau_{Fj,t} dj = \tau \left[1 - \varepsilon_t^\tau\right]$$

The average of $\tau_{H,t}$ and $\tau_{F,t}$ is thus constant at $\tau$, and their difference reflects the noise shock: $\tau_t^D = \tau_{H,t} - \tau_{F,t} = 2\tau\varepsilon_t^\tau$. An increase in $\varepsilon_t^\tau$ leads to a portfolio shift to Home equity that affects the relative asset price.

While our assumption of stochastic iceberg costs allows us to introduce a source of noise in a tractable way, it introduces an additional source of information, unlike the standard approach of considering noise traders. Specifically, each agent observes her specific iceberg cost before allocating her portfolio, and could potentially use it to infer the value of $\varepsilon_t^\tau$ to some extent. We assume that $\xi_{Hj,t}$ and $\xi_{Fj,t}$ have a very high variance, approaching infinity, so that $\tau_{Hj,t}$ and $\tau_{Fj,t}$ are infinitely noisy signals that cannot be used to infer the level of $\varepsilon_t^\tau$.[10]

We consider an overlapping generations setting where agents live for two periods. This simplifies the portfolio decision by limiting it to one period. Agents supply one unit of labor when young, consume some of their wage income and save the balance in Home and Foreign equity. They consume the proceeds from their investment when old. A young Home agent $j$ at time $t$ maximizes her intertemporal utility of consumption:

$$ln\left(C_{y,t}^{Hj}\right) + \beta E_t^{Hj} ln\left(C_{o,t+1}^{Hj}\right) \tag{54}$$

where $C_{y,t}$ is consumption when young and $C_{o,t+1}$ is consumption when old. The $Hj$ superscript on the expectation denotes that expectations can vary across agents as they are computed using private signals. Agent $j$'s income when young consists of the wage $W_{H,t}$. Her consumption when old is given by the return on her savings, $C_{o,t+1}^{Hj} = (W_{H,t} - C_{y,t}^{Hj}) R_{t+1}^{p,Hj}$. The portfolio return is

$$R_{t+1}^{p,Hj} = z_{Hj,t} R_{H,t+1} + (1 - z_{Hj,t}) e^{-\tau_{Hj,t}} R_{F,t+1} \tag{55}$$

[10] In this model only the relative asset price will contain information about future productivity. The average asset price is driven by world saving. As we will see below, this only depends on current wages and not on expected future productivity. But this is a special feature that follows from our assumption of log utility and can certainly be generalized (see Section 6.1).

where $z_{Hj,t}$ denotes the fraction of wealth invested in Home equity. The optimization problem of a Foreign agent is similar.

The log utility (54) implies that young agents consume a constant fraction of their wage income: $C_{y,t}^{Hj}/W_{H,t} = C_{y,t}^{Fj}/W_{F,t} = (1+\beta)^{-1}$. The optimal allocation of savings between Home and Foreign equity is given by the portfolio Euler equations, which show that agents pick a portfolio that equalizes the expected discounted return on Home and Foreign equity:

$$E_t^{Hj}\left(R_{t+1}^{p,Hj}\right)^{-1}\left(R_{H,t+1} - R_{F,t+1}e^{-\tau_{Hj,t}}\right) = 0 \tag{56}$$

$$E_t^{Fj}\left(R_{t+1}^{p,Fj}\right)^{-1}\left(R_{H,t+1}e^{-\tau_{Fj,t}} - R_{F,t+1}\right) = 0 \tag{57}$$

The clearing of Home and Foreign equity markets equalizes the value of capital in a country to the value of holdings of the country's equity by investors:

$$Q_{H,t}K_{H,t+1} = \beta(1+\beta)^{-1}\left(W_{H,t}z_{H,t} + W_{F,t}z_{F,t}\right) \tag{58}$$

$$Q_{F,t}K_{F,t+1} = \beta(1+\beta)^{-1}\left(W_{F,t}(1-z_{H,t}) + W_{F,t}(1-z_{F,t})\right) \tag{59}$$

where the average portfolio shares invested by Home and Foreign investors in Home equity are denoted $z_{H,t} = \int_0^1 z_{Hj,t}dj$ and $z_{F,t} = \int_0^1 z_{Fj,t}dj$. By Walras' Law we can omit the goods market clearing condition.

*5.2. Solution Method*

While the model is relatively simple and the solution can be derived analytically, there is still a good deal of algebra involved. We therefore only broadly describe the steps involved, leaving some algebra to Appendix B and full algebraic details to a separate Technical Appendix.

The steps in solving the model are the same as those used for the simple setup of Section 4. While the two-country model is more complex and entails more state variables, the solution method parallels the one in Section 4 with only two main differences. The first is that the model is now dynamic, whereas the one in Section 4 is static. Asset returns then include capital gains, and thus depend on future asset price innovations in addition to the future payoff innovations. We therefore need to make a conjecture on the link between the asset prices and the state variables in order to compute expectations of asset returns (as well as higher moments). We discuss this in more detail in Section 5.5 below.

Second, the average portfolio composition across agents differs between the two countries. As discussed in Section 2.5, the zero-order component of the difference in portfolio shares cannot be computed from the zero-order component of model equations. Instead of sequentially imposing the zero, first and second-order components of the equations, we need to impose the order components of equations based on the method developed by Devereux and Sutherland (2010) and Tille and van Wincoop (2010). We briefly reviewed this method in Section 2.5 and will describe it in more detail in the context of our present model in Section 5.6 below.

Apart from these two modifications, the solution method parallels the one described for the simple model of Section 4. After some preliminaries, we describe the method, following the same three broad steps as in Section 4.

*5.3. Some Preliminaries*

Before we expand the model equations, it is useful to condense the set of equations somewhat. First, we remove the consumption of young agents. Our assumption of log utility implies that consumption is proportional to wages. Denoting logs with lower case letters, we have $c^i_{y,t} = ln(\omega/(1+\beta)) + a_{it} + (1-\omega)k_{it}$ for $i = H$, $F$, which solves consumption as a function of state variables without any approximation. Second, we substitute the Tobin's investment equations into the capital accumulation equations to write:

$$e^{k_{i,t+1}-k_{i,t}} = 1 + \left(e^{q_{it}} - 1\right)/\xi \tag{60}$$

The model then consists of 6 equations: the capital accumulation equations (60) of both countries, the asset market clearing conditions (58)-(59) and the portfolio Euler equations (56)-(57). We write these equations as averages of the corresponding Home and Foreign equations, as well as in cross-country differences. We also write the variables as averages and differences. For a variable $x$ we denote the average across countries by $x^A = 0.5(x^H + x^F)$ and the difference across countries by $x^D = x^H - x^F$.

With the exception of portfolio shares, all variables from now on are in logs, denoted with lower case letters. Corresponding to the 6 equations, there are 6 control variables: $k^A_{t+1}$, $k^D_{t+1}$, $z^A_t$, $z^D_t$, $q^A_t$ and $q^D_t$. There are 5 state variables: 4 observed state variables that we write as a vector $S_t = (a^D_t, a^A_t, k^D_t, k^A_t)'$ plus a state variable $h_t = \varepsilon^D_{t+1} + \lambda\tau^D_t/\tau$, which combines the unobserved future relative productivity shock $\varepsilon^D_{t+1}$ and the current relative iceberg cost $\tau_t^D$. A solution of the model involves a mapping from these 5 state variables into the 6 control variables in a way that satisfies the 6 equations.

We have again assumed that unobserved variables affect asset prices only in a joint linear way through $h_t$. This parallels the assumption we made in Section 4. We verify that this conjecture is correct. As the relative asset price $q_t^D$ only depends on $S_t$ and $h_t$, agents learn the value of $h_t$ from the relative asset price.[11] We assume and verify that the noise to signal ratio is zero-order. In a generalization of the model where the noise to signal ratio may change over time, for example because of time variation in the number of noise traders, $\lambda$ has higher order components as well. The method continues to apply though as long as the variables that cause changes in the noise to signal ratio over time are public information. If they are not and $\lambda$ depends on unobserved state variables, then the signal extraction problem is no longer linear as $\lambda$ is multiplied by an unobserved noise shock.

*5.4. Step 1: Quadratic and Cubic Expansions of Equations*

We now turn to the first step towards a solution, which involves Taylor expansions of the equations. The expansions are conducted around the zero-order components of the variables, which are obtained by imposing the zero-order component of the equations. The zero-order components of $a_t^A$, $a_t^D$, $q_t^A$, $q_t^D$ and $k_t^D$ are all zero. Asset market clearing implies that the world portfolio be evenly split across assets: $z^A(0) = 0.5$. Finally, the average capital stock is $k^A(0) = (1/\omega)ln(\beta\omega/(1+\beta))$. As discussed in Section 2.5 , the only variable for which we cannot compute the zero-order component this way is the cross-country difference in average portfolio share, $z_t^D = z_{H,t} - z_{F,t}$. For now we take $z^D(0)$ as an unknown parameter that remains to be solved.

We take cubic Taylor expansions of the portfolio Euler equations and quadratic expansions of the 4 other equations. The cubic approximation of the Euler equations is necessary as we need to solve for the third-order components of these equations, for two reasons. First, the method developed by Devereux and Sutherland (2010) and Tille and van Wincoop (2010) implies that the first-order component of the cross-country difference in the portfolio shares is computed by imposing the third-order component of the *difference*

[11] While we allow the average asset price $q_t^A$ to depend on $h_t$ as well, in equilibrium $q_t^A$ will only depend on $S_t$. See the discussion in footnote 13.

of the portfolio Euler equations. Second, as was the case in Section 4, computing the signal to noise ratio $\lambda$ in $h_t$ relies on the third-order component of the *average* of the portfolio Euler equations.

From now we denote all variables as deviations from their zero-order components. Quadratic expansions of the average and the difference of the capital accumulation equations (60) are:

$$k_{t+1}^A - k_t^A = \frac{1}{\xi} q_t^A + \frac{1}{2}\frac{\xi - 1}{\xi^2}\left(\left(q_t^A\right)^2 + \frac{1}{4}\left(q_t^D\right)^2\right) \quad (61)$$

$$k_{t+1}^D - k_t^D = \frac{1}{\xi} q_t^D + \frac{\xi - 1}{\xi^2} q_t^A q_t^D \quad (62)$$

Quadratic expansions of the average and the difference of the asset market clearing conditions (58)-(59) are:

$$\begin{aligned} q_t^A + k_{t+1}^A &= a_t^A + (1-\omega) k_t^A - 2\left(z_t^A\right)^2 \\ &\quad + \frac{1 - \left(z^D(0)\right)^2}{8}\left(a_t^D + (1-\omega) k_t^D\right)^2 \\ &\quad - z^D(0) z_t^A \left(a_t^D + (1-\omega) k_t^D\right) \end{aligned} \quad (63)$$

$$\begin{aligned} q_t^D + k_{t+1}^D &= z^D(0)\left(a_t^D + (1-\omega) k_t^D\right) + 4 z_t^A \\ &\quad + z_t^D \left(a_t^D + (1-\omega) k_t^D\right) \end{aligned} \quad (64)$$

For the portfolio Euler equations (56)-(57) we take cubic expansions and then average across agents within each country. The average and difference of the resulting portfolio Euler equations across the two countries are

$$z_t^A E_t\left(er_{t+1}\right)^2 = \bar{E}_t^A er_{t+1} + \frac{\tau_t^D}{2} + m(0) E_t\left(er_{t+1}\right)^3 \quad (65)$$

$$\left(z_t^D + z^D(0)\right) E_t\left(er_{t+1}\right)^2 = \bar{E}_t^H er_{t+1} - \bar{E}_t^F er_{t+1} + 2\tau \quad (66)$$

where $er_{t+1} = r_{H,t+1} - r_{F,t+1}$ is the log excess return on Home equity. $\bar{E}_t^H er_{t+1}$ and $\bar{E}_t^F er_{t+1}$ are the average expectations of these excess returns across all Home investors and all Foreign investors, respectively, and $\bar{E}_t^A er_{t+1} = 0.5\left(\bar{E}_t^H er_{t+1} + \bar{E}_t^F er_{t+1}\right)$ is the average across all investors worldwide. $m(0)$ is a zero-order constant that is the same for both countries.[12]

[12] In the Technical Appendix there is one additional term on the right hand side of these equations, which is zero to all relevant orders. We therefore omit it here.

A cubic expansion of the excess return that enters these equations is:

$$er_{t+1} = q_{t+1}^D - q_t^D + \delta_1 x_{t+1}^D + \delta_2 x_{t+1}^A x_{t+1}^D + \frac{\delta_3}{24}(x_{t+1}^D)^3 + \frac{\delta_3}{2}(x_{t+1}^A)^2 x_{t+1}^D \quad (67)$$

where $x_{t+1}^D = a_{t+1}^D - \omega k_{t+1}^D - q_{t+1}^D$, $x_{t+1}^A = a_{t+1}^A - \omega k_{t+1}^A - q_{t+1}^A$ and $\delta_1$, $\delta_2$ and $\delta_3$ are zero-order constants that depend on $\omega$ and $\beta$. (67) shows that the excess return depends on the asset prices at time $t+1$.

### *5.5. Step 2: Computing Expectations*

Expectations only enter in the portfolio Euler equations through the expectations of $er_{t+1}$, $er_{t+1}^2$ and $er_{t+1}^3$. We first compute these expectations and then split them into their different order components. This is done in three parts. We first make conjectures for the asset prices $q_t^D$ and $q_t^A$ as respectively cubic and quadratic functions of the state variables. Substituting the expressions for the future asset prices $q_{t+1}^D$ and $q_{t+1}^A$ that follow from these conjectures into (67), we can write the excess return as a function of future innovations. Second, we compute the distribution of the future innovations from a signal extraction problem. Finally, we apply the moments of the future innovations from the signal extraction problem to compute the expectations of $er_{t+1}$, $er_{t+1}^2$ and $er_{t+1}^3$ and then split these into components of different orders.

#### *5.5.1. Conjecture Asset Prices*

We make the following cubic conjecture for $q_t^D$ and quadratic conjecture for $q_t^A$ as a function of the state variables $S_t$ and $h_t$:

$$\begin{aligned} q_t^D &= \alpha_{qD} S_t + \alpha_{5,qD} h_t + S_t' A_{qD} S_t + \beta_{qD} S_t h_t + \mu_{qD} (h_t)^2 \quad (68) \\ &\quad + cubic_D(S_t, h_t) \\ q_t^A &= \alpha_{qA} S_t + \alpha_{5,qA} h_t + S_t' A_{qA} S_t + \beta_{qA} S_t h_t + \mu_{qA} (h_t)^2 \quad (69) \end{aligned}$$

where $cubic_D(S_t, h_t)$ stands for all 35 cubic terms in the elements of $S_t$ and $h_t$. Since $q_{t+1}^A$ is multiplied in the excess return expression (67) (through $x_{t+1}^A$) with terms that are linear and quadratic in other control and state variables, a quadratic approximation of the average asset price is sufficient for a cubic approximation of the excess return.

It is useful to write the state variables $S_{t+1}$ as the sum of variables known at time $t$ and future innovations: $S_{t+1} = (\rho a_t^D, \rho a_t^A, k_{t+1}^D, k_{t+1}^A)' + (\varepsilon_{t+1}^D, \varepsilon_{t+1}^A, 0, 0)'$. Using the conjectures (68)-(69) we write $q_{t+1}^D$ and $q_{t+1}^A$ as respectively cubic and quadratic expressions of variables known at time $t$ and future innovations $\varepsilon^D_{t+1}$, $\varepsilon^A_{t+1}$ and $h_{t+1}$. Substituting this into (67), we write the excess return as a cubic expression of variables known at time $t$ and the future innovations. The expectations of the terms involving future innovations are computed from signal extraction, to which we now turn.

*5.5.2. Signal Extraction*

Agents have no information on the future innovation $h_{t+1}$ beyond its unconditional distribution with mean zero and variance $2(1+2\lambda^2\theta)\sigma_a^2$, and the fact that it is uncorrelated with productivity innovations at time $t+1$.

Home and Foreign investors infer the future productivity innovations, $\varepsilon_{H,t+1}$ and $\varepsilon_{F,t+1}$, using three sources of information: the unconditional distribution of innovations, private signals and the relative asset price $q^D_t$. Through the relative asset price agents learn $h_t = \varepsilon^D_{t+1} + \lambda\tau^D_t/\tau$.

Consider agent $j$ in the Home country (the problem of a Foreign agent is solved along similar lines). The vector of productivity innovations is $\xi_{t+1} = (\varepsilon_{H,t+1}, \varepsilon_{F,t+1})'$. The Home agent $j$ has a vector of signals $Y^{j,H}_t = \left(h_t, v^{H,H}_{j,t}, v^{H,F}_{j,t}, 0, 0\right)'$ that is linked to $\xi_{t+1}$ through a matrix $X^{j,H}$ and is affected by shocks $v^{j,H}$:

$$Y^{j,H}_t = \begin{pmatrix} 1 & -1 \\ 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} \xi_{t+1} + \begin{pmatrix} \lambda\tau^D_t/\tau \\ \epsilon^{H,H}_{j,t} \\ \epsilon^{H,F}_{j,t} \\ -\varepsilon^H_{t+1} \\ -\varepsilon^F_{t+1} \end{pmatrix} \equiv X^{j,H}\xi_{t+1} + v^{j,H}_t$$

The expectation and variance of $\xi_{t+1}$ by agent $j$ are given by:

$$E^{j,H}_t\left[\xi_{t+1}\right] = V^{j,H}_t\left[\xi_{t+1}\right]\left(X^{j,H}\right)'\left(R^{j,H}\right)^{-1}Y^{j,H}_t \quad (70)$$

$$V^{j,H}_t\left[\xi_{t+1}\right] = \left[\left(X^{j,H}\right)'\left(R^{j,H}\right)^{-1}\left(X^{j,H}\right)\right]^{-1} \quad (71)$$

where $R^{j,H}$ is variance-covariance matrix of $v^{j,H}_t$, which is diagonal with $diag\left(R^{j,H}\right) = \left(4\lambda^2\theta\sigma_a^2, \sigma^2_{H,H}, \sigma^2_{H,F}, \sigma_a^2, \sigma_a^2\right)'$. A more detailed presentation of the coefficients is given in Appendix B.

We use (70) to write the expectations $\varepsilon^D_{t+1}$ and $\varepsilon^A_{t+1}$ as functions of $h_t$ and the private signals. Taking the average across agents within each country, these become functions of $h_t$, $\varepsilon^D_{t+1}$ and $\varepsilon^A_{t+1}$ as the idiosyncratic components of private signals average to zero. The same is the case for expectations of quadratic and cubic terms in $\varepsilon_{t+1}^{D}$ and $\varepsilon_{t+1}^{A}$. The coefficients that multiply $h_t$, $\varepsilon_{t+1}^{D}$ and $\varepsilon_{t+1}^{A}$ in these expectations are functions of $\sigma_a^2$ and generally have components of different orders. We use these results to split the expectations of linear, quadratic and cubic terms in the future innovations $\varepsilon_{t+1}^{D}$ and $\varepsilon_{t+1}^{A}$ into components of different orders.

*5.5.3. Excess Return Moments*

We use our results to compute the expectations of $er_{t+1}$, $er_{t+1}^2$ and $er_{t+1}^3$. As discussed above, we can write the excess return as a cubic expression of variables known at time $t$ and the future innovations $\varepsilon_{t+1}^{D}$, $\varepsilon_{t+1}^{A}$ and $h_{t+1}$. Using the results from the signal extraction, we write $\bar{E}_t^H er_{t+1}$ and $\bar{E}_t^F er_{t+1}$ as a cubic function of variables known at time $t$ and the future innovations $\varepsilon_{t+1}^{D}$ and $\varepsilon_{t+1}^{A}$. The same is the case for expectations of $er_{t+1}^2$ and $er_{t+1}^3$.

A couple of points are worth making about these expectations. First, the expectations of $er_{t+1}^2$ and $er_{t+1}^3$ are the same for all agents (up to third-order terms) and only depend on variables publicly known at time $t$. Second, $\bar{E}_t^H er_{t+1}$ and $\bar{E}_t^F er_{t+1}$ depend on the future innovations $\varepsilon_{t+1}^{D}$ and $\varepsilon_{t+1}^{A}$, but only linearly and only in the third-order component. Finally, this third-order component of the expectation of $er_{t+1}$ is the only term that differs across average Home and Foreign investors. This difference results from the higher precision of signals on domestic innovations ($\sigma_{HH}^2 < \sigma_{HF}^2$). Specifically:

$$\left[\bar{E}_t^H er_{t+1} - \bar{E}_t^F er_{t+1}\right](3) = \mu_{er,1}(0)\frac{4\lambda^2\theta}{1+2\lambda^2\theta}\left(\frac{1}{\sigma_{HH}^2} - \frac{1}{\sigma_{HF}^2}\right)\sigma_a^2\varepsilon_{t+1}^{A} \quad (72)$$

where $\mu_{er,1}(0) > 0$ is a function of the known parameters. When future world productivity rises, agents from both countries believe that productivity will rise more in their own country as they have better information about that. Holding $\varepsilon_{t+1}^{D}$ fixed, this then implies that the expectation of the excess return on Home equity rises for Home investors and falls for Foreign investors.

*5.6. Step 3: Imposing Equilibrium*

We now impose the equations at various orders to compute the solution. This is itself a three-step process. The first two steps follow Devereux and Sutherland (2010) and Tille and van Wincoop (2010). Step 1 involves imposing the second-order component of the cross-country difference in portfolio Euler equations (66) combined with first-order components of the 5 other equations. This delivers $z^D(0)$ and the first-order components of the so-called "other variables": $q_t^D(1)$, $q_t^A(1)$, $z_t^A(1)$, $k_{t+1}^D(1)$ and $k_{t+1}^A(1)$. These first-order solutions are a function of $S_t(1)$ and $h_t$. Step 2 involves imposing the third-order component of the cross-country difference in portfolio Euler equations (66) combined with second-order components of the 5 other equations. This delivers $z_t^D(1)$ and the second-order component of the "other variables": $q_t^D(2)$, $q_t^A(2)$, $z_t^A(2)$, $k_{t+1}^D(2)$ and $k_{t+1}^A(2)$. These second-order components are a function of $S_t(1)$, $S_t(2)$ and $h_t$.

The last step follows from the presence of dispersed information. Analogous to Section 3, we compute the noise to signal ratio $\lambda$ by equating the first-order component of the average portfolio share $z_t^A$ from the perspective of asset supply (first-order component of the difference in asset market clearing conditions (64)) to that from the perspective of portfolio demand. The latter follows by imposing the third-order component of the average portfolio Euler equation (65).

### 5.6.1. *First-Order of Other Variables and Zero-Order Difference in Portfolio Shares*

The first-order solutions of $q_t^A$ and $k_{t+1}^A$ are easily computed from the first-order component of the average capital accumulation equation (61) and the average asset market clearing equation (63):

$$q_t^A(1) = \frac{\xi}{1+\xi}\left[a_t^A(1) - \omega k_t^A(1)\right]$$
$$k_{t+1}^A(1) = \frac{1}{1+\xi}\left[a_t^A(1) + (1+\xi-\omega)\, k_t^A(1)\right]$$

The first-order component of the difference in capital accumulation equations (62) and the difference in asset market clearing conditions (64) gives

$$k_{t+1}^D(1) = k_t^D(1) + \frac{1}{\xi}q_t^D(1) \tag{73}$$

$$4z_t^A(1) = \frac{1+\xi}{\xi}q_t^D(1) + k_t^D(1) - z^D(0)(a_t^D + (1-\omega)k_t^D(1)) \tag{74}$$

This gives the solution for $k_{t+1}^D(1)$ and $z_t^A(1)$ conditional on the first-order component of the relative asset price, $q_t^D(1)$.

From the conjecture (68) we have

$$q_t^D(1) = \alpha_{qD}(0)S_t(1) + \alpha_{5,qD}(0)h_t \tag{75}$$

We can find the coefficients $\alpha_{qD}(0)$ and $\alpha_{5,qD}(0)$ by imposing the first-order component of the average portfolio Euler equation (65), which is

$$\left[\bar{E}_t^A er_{t+1}\right](1) = 0 \tag{76}$$

Computing the expected excess return as described in the previous subsection, we get $[\bar{E}_t^A er_{t+1}](1) = \alpha_{er}(0)S_t(1) + \alpha_{5,er}(0)h_t$, where $\alpha_{er}(0)$ and $\alpha_{5,er}(0)$ are functions of the parameters $\alpha_{qD}(0)$ and $\alpha_{5,qD}(0)$. Setting the first-order component of the expected excess return equal to zero then implies $\alpha_{er}(0) = \alpha_{5,er}(0) = 0$, from which we compute $\alpha_{qD}(0)$ and $\alpha_{5,qD}(0)$. The latter is a function of the signal to noise ratio $\lambda$ that remains to be computed. This solution for $q_t^D(1)$ then gives the solution of $k_{t+1}^D(1)$ and $z_t^A(1)$ as a function of $S_t(1)$ and $h_t$ from (73)-(74).

Finally, we impose the second-order component of the cross-country difference in the portfolio Euler equation (66). This gives

$$z^D(0) = \frac{2\tau}{\left[E_t er_{t+1}^2\right](2)} \tag{77}$$

where we used the finding that the second-order component of the expected squared excess returns is the same for all investors worldwide. $\left[E_t er_{t+1}^2\right](2)$ is in turn computed from the first-order solution for the relative asset price (75).[13]

#### 5.6.2. *Second-Order of Other Variables and First-Order Difference in Portfolio Shares*

We now repeat all of this one order higher. The second-order solutions of $q_t^A$ and $k_{t+1}^A$ are computed from the second-order component of the average capital accumulation equation (61) and the average asset market clearing

[13] In our particular model $[E_t er_{t+1}^2](2)$ does not depend on $z^D(0)$ as the first-order component of the relative asset price (and therefore the excess return) does not depend on $z^D(0)$. In a more general model though, $[E_t er_{t+1}^2](2)$ does depend on $z^D(0)$ (see Tille and van Wincoop (2010) for an example). In that case $z^D(0)$ needs to be computed as a fixed point problem from (77).

equation (63):

$$
\begin{aligned}
q_t^A(2) &= -\frac{\xi\omega}{1+\xi}k_t^A(2) + S_t(1)'A_{qA}(0)S_t(1) \\
&\quad +\beta_{qA}(0)S_t(1)h_t + \mu_{qA}(0)h_t^2 \\
k_{t+1}^A(2) &= \frac{1+\xi-\omega}{1+\xi}k_t^A(2) + S_t(1)'A_{kA}(0)S_t(1) \\
&\quad +\beta_{kA}(0)S_t(1)h_t + \mu_{kA}(0)h_t^2
\end{aligned}
$$

where the 4x4 matrices $A_{qA}(0)$ and $A_{kA}(0)$ are functions of known parameters, and the 1x4 vectors $\beta_{qA}(0)$ and $\beta_{kA}(0)$ and the scalars $\mu_{qA}(0)$ and $\mu_{kA}(0)$ are functions of known parameters and the noise to signal ratio $\lambda$.

The second-order component of the difference in capital accumulation equations (62) and the difference in asset market clearing conditions (64) gives

$$
k_{t+1}^D(2) = k_t^D(2) + \frac{1}{\xi}q_t^D(2) + \frac{\xi-1}{\xi^2}q_t^A(1)q_t^D(1) \tag{78}
$$

$$
\begin{aligned}
4z_t^A(2) &= q_t^D(2) + k_{t+1}^D(2) - 2z^D(0)(1-\omega)k_t^D(2) \\
&\quad -z_t^D(1)\left[a_t^D(1) + (1-\omega)k_t^D(1)\right]
\end{aligned} \tag{79}
$$

We have already solved for $q_t^A(1)$ and $q_t^D(1)$. This then gives the solution for $k_{t+1}^D(2)$ conditional on $q_t^D(2)$ and the solution of $z_t^A(2)$ conditional on $q_t^D(2)$ and $z_t^D(1)$.

From the conjecture (68) we have

$$
\begin{aligned}
q_t^D(2) &= \alpha_{qD}(0)S_t(2) + \alpha_{qD}(1)S_t(1) + \alpha_{5,qD}(1)h_t \\
&\quad +S_t(1)'A_{qD}(0)S_t(1) + \beta_{qD}(0)S_t(1)h_t + \mu_{qD}(0)h_t^2
\end{aligned} \tag{80}
$$

We have already solved for $\alpha_{qD}(0)$. We can find the coefficients $\alpha_{qD}(1)$, $\alpha_{5,qD}(1)$, $A_{qD}(0)$, $\beta_{qD}(0)$ and $\mu_{qD}(0)$ by imposing the second-order component of the average portfolio Euler equation (65), which is

$$
\left[\bar{E}_t^A er_{t+1}\right](2) = 0 \tag{81}
$$

From (67) we write the expected excess return as:

$$
\begin{aligned}
[\bar{E}_t^A er_{t+1}](2) &= \alpha_{er}(0)S_t(2) + \alpha_{er}(1)S_t(1) + \alpha_{5,er}(1)h_t \\
&\quad +S_t(1)'A_{er}(0)S_t(1) + \beta_{er}(0)S_t(1)h_t + \mu_{er}(0)h_t^2
\end{aligned}
$$

where the coefficients are functions of the parameters in the expression (80) for $q_t^D(2)$. Setting the second-order component of the expected excess return equal to zero implies $\alpha_{er}(0) = 0$, $\alpha_{er}(1) = 0$, $\alpha_{5,er}(1) = 0$, $A_{er}(0) = 0$, $\beta_{er}(0) = 0$ and $\mu_{er}(0) = 0$. From this we can compute the parameters in the expression for $q_t^D(2)$. The vector $\alpha_{qD}(0)$ is the same as we had already computed before.

This solution for $q_t^D(2)$ then gives the solution of $k_{t+1}^D(2)$ from (78). The solution of $z_t^A(2)$ in (79) still depends on $z_t^D(1)$, which we compute by imposing third-order component of the cross-country difference in the portfolio Euler equation (66):

$$z_t^D(1) = \frac{\left[\bar{E}_t^H er_{t+1}\right](3) - \left[\bar{E}_t^F er_{t+1}\right](3)}{\left[E_t\left(er_{t+1}\right)^2\right](2)} - z^D(0)\frac{\left[E_t\left(er_{t+1}\right)^2\right](3)}{\left[E_t\left(er_{t+1}\right)^2\right](2)} \tag{82}$$

As discussed above, the second and third-order component of $E_t er_{t+1}^2$ are the same for all agents worldwide. Computing this expectation as described above, the last term in (82) becomes a linear function of $a_t^A$ and $k_t^A$, with coefficients that depend on known parameters and the noise to signal ratio $\lambda$.

(72) gives the third-order component of the difference in the expected excess return between Home and Foreign investors, which shows up in the numerator of the first term in (82). It is proportional to $\varepsilon_{t+1}^A$. $[E_t er_{t+1}^2](2)$ in the denominator only depends on known parameters and $\lambda$. (82) then gives a solution for $z_t^D(1)$ as a linear function of $a_t^A$, $k_t^A$ and $\varepsilon_{t+1}^A$. Together with the solution for $q_t^D(2)$, this then also gives the solution for $z_t^A(2)$ from (79).

At this point the only parameter that remains to be solved is the noise to signal ratio $\lambda$, to which we now turn.

*5.6.3. Noise to Signal Ratio*

As in Section 4, we compute the noise to signal ratio by equating the first order component of the average portfolio share, $z_t^A(1)$, from the asset supply side to that from the asset demand side. The former has already been solved by imposing the difference in asset market clearing conditions and is given by (74). Using the solution for $q_t^D(1)$, (74) gives $z_t^A(1)$ as a function of $a_t^D(1)$, $k_t^D(1)$ and $h_t$.

The average portfolio share from the demand side follows by imposing third-order component of the average of portfolio Euler equation (65). This gives

$$z_t^A(1) = \frac{\left[\bar{E}_{t+1}^A er_{t+1}\right](3) + 0.5\tau_t^D}{\left[E_t(er_{t+1})^2\right](2)} \tag{83}$$

where we used that $\left[E_t(er_{t+1})^3\right](3) = 0$. Using the expressions for the excess returns and our results from the signal extraction, $\left[\bar{E}_{t+1}^A er_{t+1}\right](3)$ is equal to

$$\mu_{er,1}(0)\frac{\sigma_{H,H}^2 + \sigma_{H,F}^2}{\sigma_{H,H}^2\sigma_{H,F}^2}\frac{\lambda^2\theta}{1+2\lambda^2\theta}\sigma_a^2\varepsilon_{t+1}^D + joint_t \tag{84}$$

where $joint_t$ includes terms in $S_t$ and $h_t$. A full expression is in the Technical Appendix.

We now equate the expression for $z_t^A(1)$ from the supply and demand sides. This gives us a solution for both $\lambda$ and $q_t^D(3)$, but our main interest is in $\lambda$. Note that in the expression for $z_t^A(1)$ from the supply side, $\varepsilon_{t+1}^D$ and $\tau_t^D$ only enter in a jointly linear way through $h_t$ (which affects $q_t^D(1)$). On the demand side they enter jointly through $h_t$ (through the term $joint_t$ in (84)) and individually. In order for an equilibrium to exist we therefore need to make sure that $\varepsilon_{t+1}^D$ and $\tau_t^D$ enter the numerator of (83) in the same joint way as in $h_t$. This leads to an implicit solution for $\lambda$ that is zero-order as conjectured:

$$\mu_{er,1}(0)\frac{\sigma_{H,H}^2 + \sigma_{H,F}^2}{\sigma_{H,H}^2\sigma_{H,F}^2}\frac{2\lambda^2\theta}{1+2\lambda^2\theta}\sigma_a^2 = \frac{\tau}{\lambda} \tag{85}$$

Equating $z_t^A(1)$ from the supply to the demand side then amounts to setting the coefficients of a cubic expression in $S_t$ and $h_t$ equal to zero. Using (68), this yields $\alpha_{qD}(2)$, $\alpha_{5,qD}(2)$, $A_{qD}(1)$, $\beta_{qD}(1)$, $\mu_{qD}(1)$ and zero-order components of parameters associated with the cubic terms in the conjecture for $q_t^D$. This gives $q_t^D(3)$. For most purposes however, first and second-order components are sufficient.

## 6. Aggregation and Accuracy

This section first discusses aggregation issues that arise in extensions of the model that lead to differences in wealth across agents. After that we discuss numerical accuracy by considering Euler equation errors.

*6.1. Aggregation in a More General Model*

A common problem in models with heterogeneous agents and incomplete markets is that often the entire wealth distribution becomes a state variable. Aggregation issues do not arise in our model because wealth is identical across agents thanks to the OLG setting and the log utility of consumption. But they do arise in more general versions of our model where wealth does differ across agents. This happens when we consider non-log preferences as saving (and therefore wealth) depends on the expected portfolio return, which itself depends on private signals. It is also the case when agents have infinite horizons.[14] Their wealth then depends on past portfolio returns, which reflect past portfolio allocation choices, and thus past private signals. But we will argue that even under such extensions the aggregation is quite straightforward and therefore does not pose any particular problem for the method.

We write wealth of agent $j$ in country $H$ as $M_{Hj,t}$. Total demand for Home assets by Home agents is then $\int_0^\infty z_{Hj,t} M_{Hj,t}$. This enters the market clearing equation for the Home asset. The aggregation problem is that $z_{Hj,t}$ and $M_{Hj,t}$ are both $j$-specific in general. This problem is relevant when computing the zero, first and second-order components of $\int_0^\infty z_{Hj,t} M_{Hj,t}$ that enter the respective components of the market clearing condition (the method does not use higher orders of the market clearing condition). With regards to portfolio shares the disagreement among the investors only shows up in the zero-order component, which depends on the private signal errors $\epsilon_{j,t}^{H,H}$ and $\epsilon_{j,t}^{H,F}$, as shown in the Technical Appendix.

Disagreement can affect wealth in different ways. When we deviate from log utility, wealth reflects the saving decisions that depend on expected portfolio returns. The disagreement across agents in expectations of portfolio returns is second-order because of the second-order weight on the private signals. It is proportional to $\sigma_a^2 \epsilon_{j,t}^{H,H}$ and $\sigma_a^2 \epsilon_{j,t}^{H,F}$ for current saving decisions, and lags of this for past saving decisions. First note that since this disagreement is second-order, there is no aggregation problem if one only wishes to solve the first-order solution of the model, for which the first-order component of the market clearing condition is needed.

[14] When assuming infinite horizons, we need to make an additional assumption to make sure that wealth has a zero-order component. The two standard ways to do this is to assume either a positive probability of death or Uzawa preferences.

Another approach ensures that there is no aggregation problem even when considering the second-order component. This occurs when saving decisions are made prior to portfolio decisions (for instance in a two-members household) and are therefore based on expectations from the previous period, which depend on past signal errors that are uncorrelated with current signal errors. In that case the cross sectional distribution of $z_{Hj,t}$ and $M_{Hj,t}$ are uncorrelated.

When saving and portfolio decisions are made in the same period, it follows from the results so far that the idiosyncratic second-order component of $z_{Hj,t}M_{Hj,t}$ depends on a term of the form

$$a_1\sigma_a^2\left(\epsilon_{j,t}^{H,H}\right)^2 + a_2\sigma_a^2\left(\epsilon_{j,t}^{H,F}\right)^2 + a_3\sigma_a^2\epsilon_{j,t}^{H,F}\epsilon_{j,t}^{H,H} \tag{86}$$

with the $a_i$ being zero-order constants. Aggregating this across agents is straightforward and is simply equal to $\sigma_a^2(a_1\sigma_{HH}^2 + a_2\sigma_{HF}^2)$. So clearly, aggregation does not pose any significant problem.

The aggregation issue is no more complicated when agents have infinite lives and therefore current wealth also depends on past private signals through past portfolio decisions. Note first that there is no aggregation problem at all in this case if we assume, as we did in this paper, that agents receive private signals about next period's productivity innovation. The idiosyncratic components of past portfolio decisions depend on past signal errors, which are uncorrelated with the current signal errors that affect $z_{Hj,t}$.

Aggregation issues only arise if we assume that agents receive signals about productivity innovations several periods ahead. For example, consider that agents receive a signal of the productivity innovation two periods ahead: $v_{j,t-1}^{H,H} = \varepsilon_{H,t+1} + \epsilon_{j,t-1}^{H,H}$ and similarly for the Foreign productivity innovation. The signal $v_{j,t-1}^{H,H}$ will affect portfolio choice $z_{Hj,t-1}$, which affects the portfolio return and therefore wealth at time $t$. The same signal also affects $z_{Hj,t}$ as this signal is still relevant at time $t$ because it is about the productivity innovation at $t+1$. In this case differences across agents show up in the zero-order component of $z_{Hj,t}$ in the form $\epsilon_{j,t-1}^{H,H}$ and the first-order component of $M_{Hj,t}$ in the form $er_t(1)\epsilon_{j,t-1}^{H,H}$ (and similar for signal errors for Foreign productivity). Aggregation is again straightforward as

$$\int_0^1 er_t(1)\left(\epsilon_{j,t-1}^{H,H}\right)^2 = \sigma_{HH}^2 er_t(1)$$

We have abstracted here from one other difference in the portfolios of agents, which is the idiosyncratic component of their cost of investing abroad. But this does not generate any aggregation problems as the current portfolio choice $z_{Hj,t}$ depends on the current idiosyncratic component of this cost, while in extensions of the model discussed above wealth would depend on past idiosyncratic components of this cost. The two are uncorrelated, so that this does not generate a cross-sectional relationship between $z_{Hj,t}$ and $M_{Hj,t}$. Of course, the noise can also be modeled in more standard ways simply in the form of exogenous noise traders, in which case idiosyncratic noise does not affect $z_{Hj,t}$ and $M_{Hj,t}$ at all.

*6.2. Accuracy of the Solution*

The solution method is a local approximation method. This naturally leads to the question of how large the approximation errors are and particularly how much dispersed information affects these errors. We assess this by looking at the Euler equation errors. We briefly describe our approach and leave full details to the Technical Appendix. As we go only up to a first-order solution for portfolio shares, we rely on the first-order solution for all variables to compute the Euler errors.

We consider Euler equation errors for the Home and Foreign assets as well as for the excess return. The only Euler equation in the model is for the excess return:

$$E_t^{Hj}\left(R_{t+1}^{p,Hj}\right)^{-1}\left(R_{H,t+1}-R_{F,t+1}e^{-\tau_{Hj,t}}\right)=0 \tag{87}$$

We multiply the left hand side by $R(0)$ so that the excess return Euler error can be interpreted in terms of an equivalent riskfree excess return.

We can write down separate Euler equations for the Home and Foreign assets:

$$\beta E_t^{Hj}\frac{C_t}{C_{t+1}}R_{H,t+1}=1 \quad ; \quad \beta E_t^{Hj}\frac{C_t}{C_{t+1}}e^{-\tau_{Hj,t}}R_{F,t+1}=1 \tag{88}$$

Following the usual practice in the literature, we write errors of these two Euler equations in terms of consumption, where an error of 0.01 can be thought of as a one dollar mistake for each hundred dollars spent. The errors are

$$Home \quad : \quad \frac{C_t - \left[\beta E_t^{Hj} \frac{R_{H,t+1}}{C_{t+1}}\right]^{-1}}{C_t} = 1 - \left[E_t^{Hj} \frac{R_{H,t+1}}{R_{t+1}^{p,H}}\right]^{-1} \tag{89}$$

$$Foreign \quad : \quad \frac{C_t - \left[\beta E_t^{Hj} \frac{e^{-\tau^{Hj,t}} R_{F,t+1}}{C_{t+1}}\right]^{-1}}{C_t} = 1 - \left[E_t^{Hj} \frac{e^{-\tau^{Hj,t}} R_{F,t+1}}{R_{t+1}^{p,H}}\right]^{-1} \tag{90}$$

We compute the errors over the entire ergodic state by simulating the model over 100,000 periods. We compute the largest absolute Euler error as well as the root mean squared error (RMSE). In order to compute the expectations we discretize the conditional normal distributions of $\varepsilon_{H,t+1}$, $\varepsilon_{F,t+1}$, $\varepsilon_{t+2}^D$ and $\tau_{t+1}^D$, with details given in the Technical Appendix. The mean and standard deviations of the conditional distributions are preserved and results change little when a finer discrete distribution is used.

We compare the results to a common knowledge model where the agents have common signals of next period's productivity innovations. The signals for all agents (Home and Foreign) are then $v_t^H = \varepsilon_{H,t+1} + \epsilon_t^H$ and $v_t^F = \varepsilon_{F,t+1} + \epsilon_t^F$. The signal errors are normally distributed, uncorrelated, and have a variance of $\sigma_v^2$. Full details of the solution to this public information model can be found in the Technical Appendix.

Table 1 reports the results. All errors are times $10^{-4}$. The first three columns use the benchmark parameterization in Tille and van Wincoop (2012), which is discussed in detail in that paper. $\sigma_v^2$ is set such that the public information model delivers the same conditional variance of the excess return as the private information model. The final three columns report results when $\tau$ is set equal to 0. In that case there is no home bias in that the average share Home agents invest in the Home country is 0.5 to the zero-order. Under the benchmark parameterization it is 0.865.

Under dispersed private information results are reported both when private signal errors are drawn from the distribution of these errors (under the column “individual agent”) and for a representative agent whose private signal is the average signal across all agents (signal error is zero, under the column "representative agent"). Naturally one can expect the Euler errors to be larger in the former case because of the large agent-specific “shocks” in the form of private signal errors. But such agent-specific shocks are of less interest when our focus is on the solution for aggregate variables. These are not affected by the signal errors of individual agents as these errors aggregate to zero. We therefore consider the results under the "representative agent" column to be more meaningful.

The first point that emerges is that the errors are quite small. Under the benchmark parameterization the largest Euler error for the excess return is only 0.00023 in terms of a riskfree excess return. For the Home (Foreign) return the Euler error corresponds to $2.30 ($3.50) mistake on every $10,000 spent. These numbers are smaller than Euler errors reported in most of the literature, for example about a factor 10 smaller than in Aruoba et.al. (2006) or Evans and Hnatkovska (2012).

In general the magnitude of the Euler errors depends on the form of the pricing kernel and the volatility of the asset returns. In our model the pricing kernel only depends on the portfolio return. In more general models it also depends on non-traded wealth (labor income) and leisure, which can lead to larger Euler errors. The standard deviation of the asset returns is 1.1%. This is not unreasonable given that the asset is a claim on the entire capital stock, not a residual claim like corporate equity. While the standard deviation of productivity shocks is chosen to match output volatility, we find that doubling the standard deviation of the shocks and therefore asset returns will roughly quintuple the magnitude of the errors in both the public information model and the representative agent case of the dispersed information model.

Perhaps of more interest in the context of the present paper is a comparison between the errors in the public and private information models. The largest absolute errors in the private information model are very similar to those in the public information model when we use the representative agent in the private information model whose private signal is the average across all agents. The method therefore does not generate larger errors than in a standard common knowledge model. The picture is different when we consider a specific agent whose private signal errors are randomly drawn from the distribution. As discussed above, this generates more noise due to large agent specific shocks in the form of private signal errors, which in turn generates larger Euler errors. The errors are still small though. But more importantly, such agent specific shocks have little relevance when we are concerned about the accuracy of the solution in terms of the aggregate variables.

A final point to make is that under the benchmark parameterization the error for the Home return is much smaller than the Foreign return for both the public information model and the representative agent case in the information dispersion model. The reason for this is portfolio home bias. This is most easily understood by considering the extreme of perfect home bias, where $z_{Hj,t}$ is always 1. In that case the portfolio return is simply the reciprocal of the Home return and the Home error is always zero. In the last three columns we show that this difference disappears when $\tau = 0$, so the the average share invested in the Home country by Home agents is one half (perfect diversification) to the zero-order.

## 7. Conclusion

We develop a method for applying local approximation techniques to portfolio choice DSGE models with dispersed private information. This method allows for the inclusion of dispersed information in DSGE models, while it had so far been mostly confined to simple linear NRE models in finance. Our method combines existing local approximation methods that are widely used to solve macro models with public information with methods that are used to solve linear NRE models with dispersed private information in finance. We first apply the method to a simple linear NRE model for which a closed form solution exists, and show that the zero, first, second and third-order components of the solution using our technique are the same as in the closed form solution. We then apply the method to a more complex two-country DSGE model.

While we have purposefully kept the DSGE model relatively simple in order to better illustrate the local approximation method, there are no clear obstacles to applying the same method to larger scale models with more state and control variables and more sources of private information. We have also discussed extensions where agents have infinite horizons, savings is affected by private information and agents have information about fundamentals further into the future. This gives rise to aggregation issues, but this does not complicate the method when the distribution of signal errors across agents is known. Signal extraction may involve more signals when agents receive private information about asset payoffs further into the future, but this also does not fundamentally alter the method itself.

## A. Signal extraction: the simple model

This Appendix computes order components of $\bar{E}\varepsilon^f = a_v\varepsilon^f + a_h h$ and $var(\varepsilon^f)$ in the simple model of Section 4. Signal extraction gives the general expressions for $var\left(\varepsilon^f\right)$, $a_v$ and $a_h$. These are functions of $\sigma_f^2$ and thus have different orders. We compute the orders by expanding the coefficients with respect to $\sigma_f$ around $\sigma_f = 0$. For instance:

$$a_h = a_h|_{\sigma_f=0} + \left.\frac{\partial a_h}{\partial \sigma_f}\right|_{\sigma_f=0} \sigma_f + \frac{1}{2}\left.\frac{\partial^2 a_h}{(\partial \sigma_f)^2}\right|_{\sigma_f=0} \sigma_f^2 + \frac{1}{3}\left.\frac{\partial^3 a_h}{(\partial \sigma_f)^3}\right|_{\sigma_f=0} \sigma_f^3 + ...$$

The first term on the right-hand side is the zero-order component, the second term the first-order component, and so on. These expansions show that $a_v$ has a second-order component, as does $var\left(\varepsilon^f\right)$, and $a_h$ has zero- and second-order components:

$$\begin{aligned} a_v(2) &= \frac{\lambda^2\theta}{\sigma_\epsilon^2\left(1+\lambda^2\theta\right)}\sigma_f^2 \quad ; \quad var\left(\varepsilon^f\right)(2) = \frac{\lambda^2\theta}{1+\lambda^2\theta}\sigma_f^2 \\ a_h(0) &= \frac{1}{1+\lambda^2\theta} \quad ; \quad a_h(2) = -\frac{\lambda^2\theta}{\sigma_\epsilon^2\left(1+\lambda^2\theta\right)^2}\sigma_f^2 \end{aligned}$$

All other components are zero: $a_v(0) = a_v(1) = a_v(3) = 0$, $var\left(\varepsilon^f\right)(0) = var\left(\varepsilon^f\right)(1) = var\left(\varepsilon^f\right)(3) = 0$ and $a_h(1) = a_h(3) = 0$.

Using these results and that $h(1) = h$, we have

$$\begin{aligned} \left[\bar{E}\varepsilon^f\right](0) &= 0 \\ \left[\bar{E}\varepsilon^f\right](1) &= a_v(0)\varepsilon^f + a_h(0)h = \frac{1}{1+\lambda^2\theta}h \\ \left[\bar{E}\varepsilon^f\right](2) &= a_v(1)\varepsilon^f + a_h(1)h = 0 \\ \left[\bar{E}\varepsilon^f\right](3) &= a_v(2)\varepsilon^f + a_2(2)h = \frac{\lambda^2\theta\sigma_f^2}{\sigma_\epsilon^2\left(1+\lambda^2\theta\right)}\left[\varepsilon^f - \frac{h}{1+\lambda^2\theta}\right] \end{aligned}$$

## B. Signal extraction: the general model

The coefficients in the inferences of a Home investor $j$ (70)-(71) are given by:

$$
\begin{aligned}
\begin{pmatrix} E_t^{j,H}\varepsilon_{H,t+1} \\ E_t^{j,H}\varepsilon_{F,t+1} \end{pmatrix} &= \begin{pmatrix} \alpha^{j,H}_{\varepsilon H,h} & \alpha^{j,H}_{\varepsilon H,vH} & \alpha^{j,H}_{\varepsilon H,vF} \\ \alpha^{j,H}_{\varepsilon F,h} & \alpha^{j,H}_{\varepsilon F,vH} & \alpha^{j,H}_{\varepsilon F,vF} \end{pmatrix} \begin{pmatrix} h_t \\ v^{H,H}_{j,t} \\ v^{H,F}_{j,t} \end{pmatrix} \\
V_t^{j,H}\left(\xi_{t+1}\right) &= \frac{\sigma_a^2}{V} \begin{pmatrix} \frac{1}{4\lambda^2\theta} + \frac{\sigma_a^2}{\sigma_{H,F}^2} + 1 & \frac{1}{4\lambda^2\theta} \\ \frac{1}{4\lambda^2\theta} & \frac{1}{4\lambda^2\theta} + \frac{\sigma_a^2}{\sigma_{H,H}^2} + 1 \end{pmatrix}
\end{aligned}
$$

where:

$$
\begin{aligned}
V &= \left(\frac{\sigma_a^2}{\sigma_{H,H}^2} + 1\right)\left(\frac{\sigma_a^2}{\sigma_{H,F}^2} + 1\right) + \frac{1}{4\lambda^2\theta}\left(\frac{\sigma_a^2}{\sigma_{H,F}^2} + \frac{\sigma_a^2}{\sigma_{H,H}^2} + 2\right) \\
\alpha^{j,H}_{\varepsilon H,h} &= \frac{1}{V}\left(\frac{\sigma_a^2}{\sigma_{H,F}^2} + 1\right)\frac{1}{4\lambda^2\theta} \\
\alpha^{j,H}_{\varepsilon H,vH} &= \frac{1}{V}\left(\frac{\sigma_a^2}{\sigma_{H,F}^2} + 1 + \frac{1}{4\lambda^2\theta}\right)\frac{\sigma_a^2}{\sigma_{H,H}^2} \\
\alpha^{j,H}_{\varepsilon H,vF} &= \frac{1}{V}\frac{1}{4\lambda^2\theta}\frac{\sigma_a^2}{\sigma_{H,F}^2} \\
\alpha^{j,H}_{\varepsilon F,h} &= -\frac{1}{V}\left(\frac{\sigma_a^2}{\sigma_{H,H}^2} + 1\right)\frac{1}{4\lambda^2\theta} \\
\alpha^{j,H}_{\varepsilon F,vH} &= \frac{1}{V}\frac{1}{4\lambda^2\theta}\frac{\sigma_a^2}{\sigma_{H,H}^2} \\
\alpha^{j,H}_{\varepsilon F,vF} &= \frac{1}{V}\left(\frac{\sigma_a^2}{\sigma_{H,H}^2} + 1 + \frac{1}{4\lambda^2\theta}\right)\frac{\sigma_a^2}{\sigma_{H,F}^2}
\end{aligned}
$$

The inferences for a Foreign investor are computed along similar lines. The inferred variance is the same across all investors: $V_t^{j,F}\left(\xi_{t+1}\right) = V_t^{j,H}\left(\xi_{t+1}\right) = V_t\left(\xi_{t+1}\right)$. The coefficients in the expectations are:

$$
\begin{aligned}
\alpha^{j,F}_{\varepsilon H,h} &= -\alpha^{j,H}_{\varepsilon F,h} \quad ; \quad \alpha^{j,F}_{\varepsilon F,h} = -\alpha^{j,H}_{\varepsilon H,h} \\
\alpha^{j,F}_{\varepsilon H,vH} &= \alpha^{j,H}_{\varepsilon F,vF} \quad ; \quad \alpha^{j,F}_{\varepsilon F,vH} = \alpha^{j,H}_{\varepsilon H,vF} \\
\alpha^{j,F}_{\varepsilon H,vF} &= \alpha^{j,H}_{\varepsilon F,vH} \quad ; \quad \alpha^{j,F}_{\varepsilon F,vF} = \alpha^{j,H}_{\varepsilon H,vH}
\end{aligned}
$$

We now turn to the zero-, first- and second- order of the various coefficients. The inferred variance only has second-order terms:

$$\left[V_t\left(\xi_{t+1}\right)\right](2)=\frac{1}{2\left(1+2\lambda^2\theta\right)}\begin{pmatrix} 1+4\lambda^2\theta & 1 \\ 1 & 1+4\lambda^2\theta \end{pmatrix}\sigma_a^2$$

The coefficients on the asset price $h_t$ have zero- and second-order terms:

$$\begin{aligned}
\left[\alpha_{\varepsilon H,h}^{j,H}\right](0) &= -\left[\alpha_{\varepsilon F,h}^{j,H}\right](0)=\frac{1}{2\left(1+2\lambda^2\theta\right)} \\
\left[\alpha_{\varepsilon H,h}^{j,H}\right](2) &= \frac{\sigma_{H,H}^2-\left(1+4\lambda^2\theta\right)\sigma_{H,F}^2}{4\left(1+2\lambda^2\theta\right)^2\sigma_{H,H}^2\sigma_{H,F}^2}\sigma_a^2 \\
\left[\alpha_{\varepsilon F,h}^{j,H}\right](2) &= \frac{\left(1+4\lambda^2\theta\right)\left(\sigma_{H,H}^2\right)-\sigma_{H,F}^2}{4\left(1+2\lambda^2\theta\right)^2\sigma_{H,H}^2\sigma_{H,F}^2}\sigma_a^2
\end{aligned}$$

The coefficients on the private signals only have second-order terms:

$$\begin{aligned}
\left[\alpha_{\varepsilon H,vH}^{j,H}\right](2) &= \frac{1+4\lambda^2\theta}{2\left(1+2\lambda^2\theta\right)}\frac{\sigma_a^2}{\sigma_{H,H}^2} \quad ; \quad \left[\alpha_{\varepsilon H,vF}^{j,H}\right](2)=\frac{1}{2\left(1+2\lambda^2\theta\right)}\frac{\sigma_a^2}{\sigma_{H,F}^2} \\
\left[\alpha_{\varepsilon F,vH}^{j,H}\right](2) &= \frac{1}{2\left(1+2\lambda^2\theta\right)}\frac{\sigma_a^2}{\sigma_{H,H}^2} \quad ; \quad \left[\alpha_{\varepsilon F,vF}^{j,H}\right](2)=\frac{1+4\lambda^2\theta}{2\left(1+2\lambda^2\theta\right)}\frac{\sigma_a^2}{\sigma_{H,F}^2}
\end{aligned}$$

Using the various orders of the coefficients, we compute the various order of the expected innovations. While the signal from the asset price $h_t$ only has a first-order component, the private signals include both a zero-order component through the idiosyncratic signal, and a first-order component through the common term reflecting the true innovation.

The zero-order component of expected innovations are zero for all agents. All investors also agree on the first-order component of expected innovations, which reflects the asset price:

$$\begin{aligned}
&\left[E_t^{j,H}\varepsilon_{H,t+1}\right](1)=\left[E_t^{j,F}\varepsilon_{H,t+1}\right](1) \\
= & -\left[E_t^{j,H}\varepsilon_{F,t+1}\right](1)=-\left[E_t^{j,F}\varepsilon_{F,t+1}\right](1)=\frac{1}{2\left(1+2\lambda^2\theta\right)}h_t
\end{aligned}$$

The second-order component of expected innovations reflects the idisosyncratic components of private signals, and thus averages to zero in each country:

$$\left[E_t^{j,H}\varepsilon_{H,t+1}\right](2) = \left[\left(1+4\lambda^2\theta\right)\frac{\epsilon_{j,t}^{H,H}}{\sigma_{H,H}^2}+\frac{\epsilon_{j,t}^{H,F}}{\sigma_{H,F}^2}\right]\tilde{\sigma}^2$$

$$\left[E_t^{j,H}\varepsilon_{F,t+1}\right](2) = \left[\frac{\epsilon_{j,t}^{H,H}}{\sigma_{H,H}^2}+\left(1+4\lambda^2\theta\right)\frac{\epsilon_{j,t}^{H,F}}{\sigma_{H,F}^2}\right]\tilde{\sigma}^2$$

$$\left[E_t^{j,F}\varepsilon_{H,t+1}\right](2) = \left[\left(1+4\lambda^2\theta\right)\frac{\epsilon_{j,t}^{F,H}}{\sigma_{H,F}^2}+\frac{\epsilon_{j,t}^{F,F}}{\sigma_{H,H}^2}\right]\tilde{\sigma}^2$$

$$\left[E_t^{j,F}\varepsilon_{F,t+1}\right](2) = \left[\frac{\epsilon_{j,t}^{F,H}}{\sigma_{H,F}^2}+\left(1+4\lambda^2\theta\right)\frac{\epsilon_{j,t}^{F,F}}{\sigma_{H,H}^2}\right]\tilde{\sigma}^2$$

where: $\tilde{\sigma}^2 = \sigma_a^2\left[2\left(1+2\lambda^2\theta\right)\right]^{-1}$. The third-order component of expected innovations reflects the asset price and the common component of private signals. It differs between Home and Foreign agents as each put more weight on the signals on domestic innovations that are more precise:

$$\left[E_t^{j,H}\varepsilon_{H,t+1}\right](3) = \left[\frac{\sigma_{H,H}^2-\left(1+4\lambda^2\theta\right)\sigma_{H,F}^2}{2\left(1+2\lambda^2\theta\right)\sigma_{H,H}^2\sigma_{H,F}^2}h_t+\left(1+4\lambda^2\theta\right)\frac{\varepsilon_{H,t+1}}{\sigma_{H,H}^2}+\frac{\varepsilon_{F,t+1}}{\sigma_{H,F}^2}\right]\tilde{\sigma}^2$$

$$\left[E_t^{j,H}\varepsilon_{F,t+1}\right](3) = \left[\frac{\left(1+4\lambda^2\theta\right)\sigma_{H,H}^2-\sigma_{H,F}^2}{2\left(1+2\lambda^2\theta\right)\sigma_{H,H}^2\sigma_{H,F}^2}h_t+\frac{\varepsilon_{H,t+1}}{\sigma_{H,H}^2}+\left(1+4\lambda^2\theta\right)\frac{\varepsilon_{F,t+1}}{\sigma_{H,F}^2}\right]\tilde{\sigma}^2$$

$$\left[E_t^{j,F}\varepsilon_{H,t+1}\right](3) = \left[-\frac{\left(1+4\lambda^2\theta\right)\sigma_{H,H}^2-\sigma_{H,F}^2}{2\left(1+2\lambda^2\theta\right)\sigma_{H,H}^2\sigma_{H,F}^2}h_t+\left(1+4\lambda^2\theta\right)\frac{\varepsilon_{H,t+1}}{\sigma_{H,F}^2}+\frac{\varepsilon_{F,t+1}}{\sigma_{H,H}^2}\right]\tilde{\sigma}^2$$

$$\left[E_t^{j,F}\varepsilon_{F,t+1}\right](3) = \left[-\frac{\sigma_{H,H}^2-\left(1+4\lambda^2\theta\right)\sigma_{H,F}^2}{2\left(1+2\lambda^2\theta\right)\sigma_{H,H}^2\sigma_{H,F}^2}h_t+\frac{\varepsilon_{H,t+1}}{\sigma_{H,F}^2}+\left(1+4\lambda^2\theta\right)\frac{\varepsilon_{F,t+1}}{\sigma_{H,H}^2}\right]\tilde{\sigma}^2$$

We compute the expectations of cross products of innovations using the definition of the covariance, namely: $[Exy](2) = [cov(x,y)](2)+[Ex](1)[Ey](1)$. All investors have the same second-order component of expected cross-products:

$$\left[E_t^{j,H}\left(\varepsilon_{H,t+1}\right)^2\right](2) = \left[E_t^{j,H}\left(\varepsilon_{H,t+1}\right)^2\right](2)$$

$$= \frac{1}{4\left(1+2\lambda^2\theta\right)^2}\left(h_t\right)^2+\frac{1+4\lambda^2\theta}{2\left(1+2\lambda^2\theta\right)}\sigma_a^2$$

$$\left[E_t^{j,H}\varepsilon_{H,t+1}\varepsilon_{F,t+1}\right](2) = -\frac{1}{4\left(1+2\lambda^2\theta\right)^2}\left(h_t\right)^2+\frac{1}{2\left(1+2\lambda^2\theta\right)}\sigma_a^2$$

The third-order component of expected cross-products of innovations reflects the idisosyncratic components of private signals, and thus averages to zero in each country:

$$
\begin{aligned}
\left[E_t^{j,H}\left(\varepsilon_{H,t+1}\right)^2\right](3) &= \left[\left(1+4\lambda^2\theta\right)\frac{\epsilon_{j,t}^{H,H}}{\sigma_{H,H}^2}+\frac{\epsilon_{j,t}^{H,F}}{\sigma_{H,F}^2}\right]\tilde{\sigma}^2 h_t \\
\left[E_t^{j,H}\left(\varepsilon_{F,t+1}\right)^2\right](3) &= -\left[\frac{\epsilon_{j,t}^{H,H}}{\sigma_{H,H}^2}+\left(1+4\lambda^2\theta\right)\frac{\epsilon_{j,t}^{H,F}}{\sigma_{H,F}^2}\right]\tilde{\sigma}^2 h_t \\
\left[E_t^{j,H}\varepsilon_{H,t+1}\varepsilon_{F,t+1}\right](3) &= \left[\frac{\epsilon_{j,t}^{H,F}}{\sigma_{H,F}^2}-\frac{\epsilon_{j,t}^{H,H}}{\sigma_{H,H}^2}\right]\frac{\lambda^2\theta\sigma_a^2}{\left(1+2\lambda^2\theta\right)^2}h_t \\
\left[E_t^{j,F}\left(\varepsilon_{H,t+1}\right)^2\right](3) &= \left[\left(1+4\lambda^2\theta\right)\frac{\epsilon_{j,t}^{F,H}}{\sigma_{H,F}^2}+\frac{\epsilon_{j,t}^{F,F}}{\sigma_{H,H}^2}\right]\tilde{\sigma}^2 h_t \\
\left[E_t^{j,F}\left(\varepsilon_{F,t+1}\right)^2\right](3) &= -\left[\frac{\epsilon_{j,t}^{F,H}}{\sigma_{H,F}^2}+\left(1+4\lambda^2\theta\right)\frac{\epsilon_{j,t}^{F,F}}{\sigma_{H,H}^2}\right]\tilde{\sigma}^2 h_t \\
\left[E_t^{j,F}\varepsilon_{H,t+1}\varepsilon_{F,t+1}\right](3) &= \left[\frac{\epsilon_{j,t}^{F,F}}{\sigma_{H,H}^2}-\frac{\epsilon_{j,t}^{F,H}}{\sigma_{H,F}^2}\right]\frac{\lambda^2\theta\sigma_a^2}{\left(1+2\lambda^2\theta\right)^2}h_t
\end{aligned}
$$

We finally compute the expectations of cubic products using $\left[E\left(x^3\right)\right](3) = \left(\left[Ex\right](1)\right)^3 + 3\left(\left[Ex\right](1)\right)\left[var\left(x\right)\right](2)$ and $\left[E\left(yx^2\right)\right](3) = 2\left[cov\left(x,y\right)\right](2)\left[Ex\right](1) + \left[Ey\right](1)\left[E\left(x^2\right)\right](2)$. All agents share the same expectations for the third-order components of cubic products:

$$
\begin{aligned}
\left[E_t^{j,H}\left(\varepsilon_{H,t+1}\right)^3\right](3) &= -\left[E_t^{j,H}\left(\varepsilon_{F,t+1}\right)^3\right](3) \\
&= \left[\frac{1}{2\left(1+2\lambda^2\theta\right)}h_t\right]^3 + 3\frac{1}{2\left(1+2\lambda^2\theta\right)}h_t\frac{1+4\lambda^2\theta}{2\left(1+2\lambda^2\theta\right)}\sigma_a^2 \\
\left[E_t^{j,H}\left(\varepsilon_{H,t+1}\right)^2\varepsilon_{F,t+1}\right](3) &= -\left[E_t^{j,H}\varepsilon_{H,t+1}\left(\varepsilon_{F,t+1}\right)^2\right](3) \\
&= -\left[\frac{1}{2\left(1+2\lambda^2\theta\right)}h_t\right]^3 + \frac{1}{2\left(1+2\lambda^2\theta\right)}h_t\frac{1-4\lambda^2\theta}{2\left(1+2\lambda^2\theta\right)}\sigma_a^2
\end{aligned}
$$

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| | Largest Absolute Errors (x$10^{-4}$) | | | | | |
|---|---|---|---|---|---|---|
| | Benchmark | | | $\tau = 0$ | | |
| | Dispersed Information | | Public Information | Dispersed Information | | Public Information |
| | individual agent | representative agent | | individual agent | representative agent | |
| Excess return | 2.3 | 1.7 | 2.2 | 2.3 | 1.8 | 2.1 |
| Home return | 3.2 | 0.2 | 0.2 | 2.3 | 0.6 | 0.7 |
| Foreign return | 3.5 | 0.9 | 1.1 | 2.1 | 0.6 | 0.7 |
| | Root mean Squared Error (x$10^{-4}$) | | | | | |
| | Benchmark | | | $\tau = 0$ | | |
| | Dispersed Information | | Public Information | Dispersed Information | | Public Information |
| | individual agent | representative agent | | individual agent | representative agent | |
| Excess return | 0.3 | 0.2 | 0.4 | 0.3 | 0.2 | 0.3 |
| Home return | 0.3 | 0.02 | 0.03 | 0.12 | 0.06 | 0.09 |
| Foreign return | 0.4 | 0.1 | 0.2 | 0.12 | 0.06 | 0.09 |

Note: All errors are times $10^{-4}$. They are computed based on 100,000 simulations of the model. The excess return error corresponds to Eq. (70) times $R(0)$. It is measured in terms of an equivalent riskfree excess return. The errors for the Home and Foreign returns are measured by respectively Eq. (72) and Eq. (73). An error of 1, which means $10^{-4}$, corresponds to a \$1 mistake on \$10,000 spent. For the dispersed information model the table reports results both for an individual agent $j$ in country $H$, with the errors in the agents's signals drawn from the distribution of these errors, and a representative agents case where the errors in the agent's signals are zero (the average error across all agents is always zero). The first three columns of numbers are based on the parameterization in Tille and van Wincoop (2012), where the zero-order component of the average share invested in domestic equity is 0.865. The last three columns assume instead that $\tau = 0$, so that the zero-order component of the average share invested in domestic equity is 0.5 (perfectly diversified).

Table 1: Euler Equation Errors