

# International Capital Flows under Dispersed Private Information<sup>1</sup>

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## **Abstract**

It is well established that private information is critical to our understanding of asset prices. In this paper we argue that it also affects international capital flows and use a simple two-country DSGE model to illustrate its impact. We show that private information (i) increases the volatility of both net and gross capital flows, (ii) leads to a high correlation between capital inflows and outflows, (iii) leads to a disconnect of capital flows from observed macro fundamentals and (iv) implies that capital flows contain information about the future macro fundamentals. We also show that dispersed information affects capital flows both through asset prices and directly, so that the impact on flows is not just the mirror image of the impact on prices.

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# 1 Introduction

As with all asset trade, international capital flows require some form of heterogeneity. If all investors were identical, higher demand for an asset would only drive up its price, without any flows.<sup>1</sup> The assumption of identical investors is however unrealistic. First, investors live in different countries, so that their non-asset incomes are differently impacted by shocks. Second, investors from different countries hold different portfolios as various frictions lead to a bias towards domestic assets. Models of international capital flows commonly include these basic forms of heterogeneity, but abstract from one of the most important sources of heterogeneity, namely differences across investors in information and expectations. This heterogeneity in information sets is however likely to be important. For instance, Broner et.al. (2013) emphasize that differences in expectations between domestic and foreign investors are critical for understanding the strong positive correlation between capital inflows and outflows, i.e. the fact that when U.S. investors are buying more foreign assets, foreign investors tend to buy more U.S. assets.

In this paper we investigate the impact of heterogeneous information on international capital flows. There is extensive evidence of heterogeneous information among agents, with for example a significant cross section dispersion of survey forecasts of asset prices, exchange rates and macroeconomic variables. The large asset market trading volume and the close connection between asset prices and order flow are further evidence of the significant role of dispersed information.<sup>2</sup> A vast literature has documented the many implications of this information heterogeneity for asset prices, such as market bubbles and crashes, asset price volatility and disconnect from observed fundamentals.<sup>3</sup> However, the implications of dispersed information for international capital flows are not yet well understood.

Albuquerque, Bauer and Schneider (2007, 2009) and Brennan and Cao (1997) explicitly consider the impact of heterogeneous information on international capital flows. While this provides some important insights, the analysis is limited to linear

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<sup>1</sup>See Forbes and Warnock (2012), who make exactly this point when discussing the importance of heterogeneity for understanding the impact of push and pull factors on international capital flows.

<sup>2</sup>Wang (1994) investigates the impact of dispersed information on trading volume. Evans and Lyons (2002) show that most short-run exchange rate volatility is related to order flow, which aggregates private information. See Hasbrouck (1991) for related evidence for equity prices.

<sup>3</sup>See Brunnermeier (2001) for a survey.

partial equilibrium settings. This has several drawbacks. First, these models are not suited to study aggregate international capital flows because there is a riskfree asset in infinite supply in an unspecified location. Capital flows refer only to the assets that are in finite supply, which is affected by shifts between assets in finite supply and infinite supply that play no role when focusing on aggregate capital flows. Second, these models abstract from saving and investment. Net capital flows, which correspond to the current account, i.e. saving minus investment, are then zero by construction. Third, the absence of investment in these models implies that there is no general equilibrium feedback mechanism between portfolio flows, the capital stock and asset returns that depends on the marginal product of capital.

In this paper we investigate the implications of dispersed information for gross and net international capital flows in a general equilibrium setting. This has several advantages. It allows us to consider aggregate flows instead of a subset of them. It also allows us to explicitly take into account important feedback mechanisms. Portfolio shifts across countries affect relative asset prices, which affect expected returns, which in turn impact portfolio flows. Expected returns, asset returns risk and capital flows are all determined jointly in the model. Finally, by adopting a two-country non-linear DSGE model we connect more closely to the existing general equilibrium literature on international capital flows, particularly the recent literature that has introduced endogenous portfolio choice into open economy DSGE settings<sup>4</sup>. The main difference is that this literature abstracts from information heterogeneity. Our contribution thus brings together the literature on dispersed information in partial equilibrium settings with the literature on portfolio choice in open economy DSGE models.

We focus on three implications of heterogeneous information for capital flows that have received significant attention in the asset price literature: the disconnect from observed fundamentals, information content about future fundamentals and volatility.<sup>5</sup> We show that, just like for asset prices, dispersed information leads

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<sup>4</sup>See for instance Devereux and Sutherland (2010) and Tille and van Wincoop (2010a).

<sup>5</sup>A good example of asset price disconnect from observed fundamentals is the October 1987 stock market crash, which saw a 20% drop in the stock price without any obvious news. Cutler et.al. (1989) find that most of the largest stock prices changes in 1946-1987 are hard to relate to public news. Roll (1988), French and Roll (1986) and Romer (1993) provide further evidence on this disconnect for equity prices.

to a disconnect between capital flows and observed fundamentals and implies that capital flows contain information about future fundamentals.<sup>6</sup> We also show that private information increases the volatility of capital flows. Finally, we show that dispersed information leads to a higher correlation between capital inflows and outflows, an effect that reflects different expectations between Home and Foreign investors.

We consider three versions of the model that only differ in their information structure. In each version agents make saving, investment and portfolio decisions. The model is driven by standard productivity shocks. The first version, which we refer to as the standard model, assumes that agents receive no signals about future productivity innovations. This is a useful starting point as the existing literature on international capital flows relies on models of this type.<sup>7</sup> The second version introduces private signals about next period's productivity innovations, and we refer to it as the private signals model. The last version only differs in that we replace the private signals about future productivity with publicly observed signals about future productivity, and we refer to it as the public signals model.

We also introduce another feature standard in noisy rational expectations (NRE) asset pricing models, namely non-informational trade (noise). When combined with private information, this noise prevents the asset price from completely revealing the private information. The NRE asset pricing literature has shown that the combination of the information content of asset prices due to private information and even small non-informational trades (e.g. noise trade or liquidity trade) can lead to a very large impact of non-informational trades on asset prices.<sup>8</sup> For example, a higher asset price resulting from noise trade can rationally cause all agents to infer that others have private information about better future fundamentals. This leads all agents to be more optimistic about future fundamentals even if no one has favorable signals about them.

The paper is related to a small literature that introduces NRE asset pricing features into open economy models. These include Albuquerque, Bauer and Schneider

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<sup>6</sup>Consistent with the information content of capital flows, State Street Corporation sells real-time portfolio flow data to investors that use them for trading purposes. State Street also uses the portfolio flow data to make FX trading recommendations.

<sup>7</sup>For some recent papers on gross and net capital flows in two-country DSGE models see Devereux and Sutherland (2010), Evans and Hnatkovska (2013) and Tille and van Wincoop (2010a,b).

<sup>8</sup>See for example Bacchetta and van Wincoop (2006), Gennotte and Leland (1990) and Romer (1993).

(2007, 2009), Bacchetta and van Wincoop (2004, 2006), Brennan and Cao (1997), Gehrig (1993) and Veldkamp and van Nieuwerburgh (2009). These papers focus on a variety of issues, ranging from exchange rate puzzles to international portfolio home bias and the relationship between asset returns and portfolio flows. Together they show that information dispersion within and across countries can tell us a lot about a wide range of stylized facts related to international asset prices and portfolio allocation. However, as pointed out above, the linear partial equilibrium nature of such models has serious limitations in analyzing aggregate capital flows.<sup>9</sup>

The paper is organized as follows. Section 2 describes the model. Section 3 presents the general approach of the solution method and Section 4 applies it to our specific model. We discuss the determinants of capital flows in Section 5, stressing the role of private information. Section 6 provides a numerical illustration and Section 7 concludes.

## 2 The Model

We consider a two-country DSGE model that we purposely keep very simple. There is just one numeraire good that is produced in the Home and Foreign countries, with production affected by country-specific productivity shocks. There are two assets, one for each country. We adopt a simple overlapping generations setup to simplify consumption and portfolio decisions. This section first describes the features that are common to the three versions of the model, and then presents the information structure and non-informational trade that differs across them.

### 2.1 Production, Investment and Assets

The good is produced using a constant returns to scale technology in labor and capital:

$$Y_{i,t} = A_{i,t} K_{i,t}^{1-\omega} N_{i,t}^{\omega} ; \quad i = H, F \quad (1)$$

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<sup>9</sup>Mertens (2011) considers a non-linear DSGE open economy model with dispersed information, as we do, in contrast to the other partial equilibrium open economy NRE models listed above. He develops a solution technique that has many similarities to our solution method. His focus is however on asset prices rather than capital flows as he assumes complete financial markets. By contrast, our emphasis is on gross and net capital flows and their various drivers.

where  $H$  and  $F$  denote the Home and Foreign country respectively.  $Y_i$  is the output in country  $i$ ,  $A_i$  is a country-specific exogenous stochastic productivity term,  $K_i$  is the capital input and  $N_i$  the labor input that we normalize to unity. Log productivity follows an autoregressive process:

$$a_{i,t+1} = \rho a_{i,t} + \varepsilon_{i,t+1} \quad i = H, F$$

where  $\varepsilon_{i,t+1}$  has a  $N(0, \sigma_a^2)$  distribution that is known by all agents. Innovations are uncorrelated across countries.

The dynamics of the capital stock reflects depreciation at a rate  $\delta$  and investment  $I_{i,t}$ :

$$K_{i,t+1} = (1 - \delta) K_{i,t} + I_{i,t} \quad i = H, F \quad (2)$$

The wage rate in country  $i$  reflects the marginal productivity of labor:

$$W_{i,t} = \omega A_{i,t} (K_{i,t})^{1-\omega} \quad i = H, F \quad (3)$$

Capital is supplied by a competitive installment firm. In period  $t$  it produces  $I_{i,t}$  units of new capital and sells them at a price  $Q_{i,t}$ . The production entails a quadratic adjustment cost, and producing  $I_{i,t}$  units of capital good requires the following amount in units of the consumption good:

$$I_{i,t} + \frac{\xi}{2} \frac{(I_{i,t} - \delta K_{i,t})^2}{K_{i,t}} \quad (4)$$

Profit maximization by the installment firm implies a standard Tobin's Q relation:

$$\frac{I_{i,t}}{K_{i,t}} = \delta + \frac{Q_{i,t} - 1}{\xi} \quad (5)$$

Home and Foreign assets are claims on a unit of Home and Foreign capital, respectively. The price of the country  $i$  asset is equal to the cost of purchasing one unit of capital from the installment firm,  $Q_{i,t}$ . An agent purchasing the asset at the end of period  $t$  gets a dividend of  $(1 - \omega)Y_{i,t+1}/K_{i,t+1}$  in period  $t + 1$ , and can sell the remaining  $1 - \delta$  units of capital at a price  $Q_{i,t+1}$ . The return on the country  $i$  asset is:

$$R_{i,t+1} = \frac{(1 - \omega) A_{i,t+1} (K_{i,t+1})^{-\omega} + (1 - \delta) Q_{i,t+1}}{Q_{i,t}} \quad i = H, F \quad (6)$$

## 2.2 Consumption and Portfolio Choice

We adopt an OLG structure where agents live for two periods and only work in the first period. In each country, each generation consists of a unit mass of individual agents that we index by  $j$ . A young Home agent at time  $t$  maximizes

$$\ln \left( C_{y,t}^{Hj} \right) + \beta E_t \ln \left( C_{o,t+1}^{Hj} \right) \quad (7)$$

where  $C_{y,t}^{Hj}$  is consumption when young and  $C_{o,t+1}^{Hj}$  is consumption when old.

We assume that agents face an iceberg cost of investing abroad. Specifically, a Home agent  $j$  investing in the Foreign asset at time  $t$  only gets a return  $e^{-\tau_{Hj,t}} R_{F,t+1}$  with  $\tau_{Hj,t} \geq 0$ . Similarly, a Foreign agent  $j$  receives a return  $e^{-\tau_{Fj,t}} R_{H,t+1}$  when investing in the Home asset with  $\tau_{Fj,t} \geq 0$ . The costs vary across time and agents, as well as across countries when averaged across agents. Iceberg costs serve two purposes. First, they are a standard device in the literature to generate portfolio home bias. Second, they serve as the source of noise in the private signals model that prevents asset prices from fully revealing aggregate information. A more detailed discussion of this aspect and the specific form of  $\tau_{Hj,t}$  and  $\tau_{Fj,t}$  is provided below.

The Home agent maximizes the utility in equation (7) subject to the budget constraint and portfolio return,  $R_{t+1}^{p,Hj}$ :

$$\begin{aligned} C_{o,t+1}^{Hj} &= (W_{H,t} - C_{y,t}^{Hj}) R_{t+1}^{p,Hj} \\ R_{t+1}^{p,Hj} &= z_{Hj,t} R_{H,t+1} + (1 - z_{Hj,t}) e^{-\tau_{Hj,t}} R_{F,t+1} \end{aligned} \quad (8)$$

where  $z_{Hj,t}$  is the fraction of wealth invested in the Home asset.

The optimization is characterized by a consumption Euler equation that determines the consumption-saving choice and a portfolio Euler equation that determines the allocation of savings. The first implies that consumption and saving are constant fractions of labor income, which is identical across agents in a given country,

$$C_{y,t}^{Hj} = (1 + \beta)^{-1} W_{H,t} \quad ; \quad S_{y,t}^{Hj} = W_{H,t} - C_{y,t}^{Hj} = (1 + \beta)^{-1} \beta W_{H,t} \quad (9)$$

The portfolio Euler equation is given by:

$$E_t^H \left( R_{t+1}^{p,Hj} \right)^{-1} (R_{H,t+1} - e^{-\tau_{Hj,t}} R_{F,t+1}) = 0 \quad (10)$$



Equation (10) shows that the investor chooses a portfolio that equalizes the expected discounted return (the expected product of the asset pricing kernel and asset return) across assets. The asset pricing kernel is the marginal utility of future consumption, which is inversely proportional to the return on the agent's portfolio.

Foreign agents face an analogous decision problem with portfolio return

$$R_{t+1}^{p,Fj} = z_{Fj,t} e^{-\tau_{Fj,t}} R_{H,t+1} + (1 - z_{Fj,t}) R_{F,t+1} \quad (11)$$

The corresponding optimality conditions for a Foreign agent are:

$$C_{y,t}^{Fj} = (1 + \beta)^{-1} W_{F,t} \quad ; \quad S_{y,t}^{Fj} = (1 + \beta)^{-1} \beta W_{F,t} \quad (12)$$

$$E_t^F \left( R_{t+1}^{p,Fj} \right)^{-1} \left( e^{-\tau_{Fj,t}} R_{H,t+1} - R_{F,t+1} \right) = 0 \quad (13)$$

## 2.3 Asset Market Clearing

Financial wealth in country  $i$  is equal to saving by the young, which we denote by  $W_{i,t}^F = (1 + \beta)^{-1} \beta W_{i,t}$ . The clearing of asset markets requires that the value of capital in a country be equal to the worldwide demand by all young agents for that country's asset:

$$Q_{H,t} K_{H,t+1} = W_{H,t}^F z_{H,t} + W_{F,t}^F z_{F,t} \quad (14)$$

$$Q_{F,t} K_{F,t+1} = W_{H,t}^F (1 - z_{H,t}) + W_{F,t}^F (1 - z_{F,t}) \quad (15)$$

where  $z_{H,t} = \int_0^1 z_{Hj,t} dj$  and  $z_{F,t} = \int_0^1 z_{Fj,t} dj$  denote the average portfolio shares among agents in the respective countries. We can omit the world goods market clearing condition due to Walras law.

In contrast to standard NRE models, which only focus on one asset market and assume infinite supply in others, our analysis encompasses the clearing of all asset markets. This general equilibrium approach then allows us to discuss overall capital flows, which is done in Section 5 below.

## 2.4 Information Structure and Non-Informational Trade

We complete our presentation of the model by describing the information structure and non-informational trade, where the three versions of the model differ. In the *standard* model all agents only know the unconditional distribution of productivity

innovations  $\varepsilon_{i,t+1}$ . In the *private signals* model each agent also receives private signals about next period's productivity innovations. In addition there is noise trade due to time-variation in the cost of investing abroad ( $\tau_{Hj,t}$  and  $\tau_{Fj,t}$ ) that is described in more detail below. The *public signals* model only differs from the private signals model in that all agents receive the same signals on future innovations.

### 2.4.1 Private Signals Model

The private signals model is the main focus of the paper. It contains the two key elements of NRE models: private information about future fundamentals and noise that prevents asset prices from revealing the private information.

Each agent receives private signals about next period's productivity innovations. The signals observed by Home agent  $j$  about respectively the Home and Foreign innovations are:

$$v_{j,t}^{H,H} = \varepsilon_{H,t+1} + \epsilon_{j,t}^{H,H} \quad \epsilon_{j,t}^{H,H} \sim N(0, \sigma_{HH}^2) \quad (16)$$

$$v_{j,t}^{H,F} = \varepsilon_{F,t+1} + \epsilon_{j,t}^{H,F} \quad \epsilon_{j,t}^{H,F} \sim N(0, \sigma_{HF}^2) \quad (17)$$

Each signal consists of the true innovation and a stochastic error uncorrelated across agents.<sup>10</sup> Similarly, agent  $j$  in the Foreign country observes the signals:

$$v_{j,t}^{F,H} = \varepsilon_{H,t+1} + \epsilon_{j,t}^{F,H} \quad \epsilon_{j,t}^{F,H} \sim N(0, \sigma_{HF}^2) \quad (18)$$

$$v_{j,t}^{F,F} = \varepsilon_{F,t+1} + \epsilon_{j,t}^{F,F} \quad \epsilon_{j,t}^{F,F} \sim N(0, \sigma_{HH}^2) \quad (19)$$

As is standard in NRE models, we assume that the errors of the signals average to zero across agents in a given country ( $\int_0^1 \epsilon_{j,t}^{H,H} dj = \int_0^1 \epsilon_{j,t}^{H,F} dj = 0$ ).<sup>11</sup>

Our setup is symmetric as the cross-sectional dispersion of signals on domestic productivity is the same for agents in the two countries, and so is the dispersion of signals on productivity abroad. We allow for an information asymmetry

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<sup>10</sup>As there is a large number of agents in each country,  $\sigma_{HH}^2$  and  $\sigma_{HF}^2$  also represent the cross-sectional dispersion of signals on domestic and foreign productivity innovations.

<sup>11</sup>While it is standard to assume that average signal errors are zero in NRE models, it is perhaps not terribly realistic. If we instead assumed that there is a common component to signal errors across agents, the average private signal would equal the future productivity innovation plus the common signal error. Since agents trade based on these signals, such an extension would imply that asset prices and capital flows are affected by common errors in private signals. This would generate an additional source of disconnect from observed macro fundamentals.

with agents receiving more precise signals about shocks in their own country than abroad:  $\sigma_{HH}^2 \leq \sigma_{HF}^2$ . A substantial literature has documented such information differences, with local agents having more reliable information than foreign agents.<sup>12</sup>

A central ingredient of NRE models is the presence of unobserved noise trade or non-informational asset trade, which prevents the asset price from revealing aggregate private information about a future fundamental. This usually takes the form of exogenous sales and purchases of assets by "noise" or "liquidity" traders who do not optimize their portfolio.<sup>13</sup> While this device is tractable in partial equilibrium NRE models, it would add complexity to our general equilibrium setting as we would need to fully characterize the noise traders, including their income and consumption.

To keep the model simple, we introduce noise in an alternative way, specifically through the cost of investing abroad  $\tau_{Hj,t}$  and  $\tau_{Fj,t}$ . These costs are written as:

$$\tau_{Hj,t} = \tau [1 + \varepsilon_t^\tau + \xi_{Hj,t}] \quad ; \quad \tau_{Fj,t} = \tau [1 - \varepsilon_t^\tau + \xi_{Fj,t}] \quad (20)$$

Here  $\tau$  is a constant and  $\varepsilon_t^\tau$  is a noise shock with a  $N(0, \theta\sigma_a^2)$  distribution. It is the source of noise that prevents asset prices from fully revealing the future productivity innovations. Finally,  $\xi_{Hj,t}$  and  $\xi_{Fj,t}$  are idiosyncratic terms that add up to zero across all agents in a given country:  $\int_0^1 \xi_{Hj,t} dj = \int_0^1 \xi_{Fj,t} dj = 0$ . Our specification of the iceberg costs in equation (20) implies that their average values across agents in the Home and Foreign countries are  $\tau_{H,t} = \tau [1 + \varepsilon_t^\tau]$  and  $\tau_{F,t} = \tau [1 - \varepsilon_t^\tau]$ . The average of  $\tau_{H,t}$  and  $\tau_{F,t}$  is therefore constant at  $\tau$  and their difference

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<sup>12</sup>See for example Bae, Stulz and Tan (2008), who document that earnings forecasts are more precise for local than foreign analysts. Leuz, Lins and Warnock (2009) provide evidence that agency problems are better monitored by locals. Ahearne et.al. (2004) find that home bias of U.S. investors relative to other countries is significantly reduced when the stock of foreign countries is traded on centralized exchanges. This reduces information barriers as a result of the regulatory and accounting burden imposed on such foreign firms. Portes and Rey (2005) find that "the geography of information is the main determinant of the pattern of international (financial) transactions", documenting the effect of a variety of informational frictions on cross-border equity flows. Kang and Stulz (1997) document that investors tend to invest in foreign firms for which information barriers are lower (large firms with good accounting performance, low unsystematic risk and low leverage).

<sup>13</sup>Some papers have introduced the noise endogenously in various forms of hedge trade and liquidity trade. See for example Bacchetta and van Wincoop (2006), Dow and Gorton (1995), Spiegel and Subrahmanyam (1992) and Wang(1994).

reflects the noise shock:  $\tau_t^D = \tau_{H,t} - \tau_{F,t} = 2\tau\varepsilon_t^\tau$ . An increase in  $\varepsilon_t^\tau$  leads to a portfolio shift by all agents to the Home asset as it becomes relatively more expensive for Home agents to invest abroad than for Foreign agents.

The role of the idiosyncratic terms  $\xi_{Hj,t}$  and  $\xi_{Fj,t}$  is to prevent information revelation. Recall that an agent  $j$  observes her own cost  $\tau_{Hj,t}$  when making her portfolio decision at time  $t$ . Without any idiosyncratic shock, she would observe  $\varepsilon_t^\tau$ , which would then not act as an unobservable noise shock. Even though the agent's cost is an imperfect signal of the noise shock, she could potentially use it as a source of information to form inferences about the noise shock, which would complicate our signal extraction problem. To rule this out, we assume that  $\xi_{Hj,t}$  and  $\xi_{Fj,t}$  have a cross-sectional dispersion approaching infinity so that  $\tau_{Hj,t}$  and  $\tau_{Fj,t}$  are infinitely noisy signals of  $\tau_{H,t}$  and  $\tau_{F,t}$ . This assumption can be relaxed but simplifies the analysis.<sup>14</sup>

## 2.4.2 Public Signals Model

In the public signals model all agents receive the same signals about future productivity:

$$v_t^H = \varepsilon_{H,t+1} + \epsilon_t^H \quad ; \quad v_t^F = \varepsilon_{F,t+1} + \epsilon_t^F \quad (21)$$

The errors  $\epsilon_t^H$  and  $\epsilon_t^F$  each have a  $N(0, \sigma_v^2)$  distribution and are uncorrelated. The public signals model has in common with the standard model that expectations are only conditioned on publicly available information. The difference is that the public information set is somewhat expanded in the public signals model.

Public signals are the only feature that differs between the private and public signals models. Noise or non-informational trade is assumed to be the same as in the private information model. This allows us to highlight the impact of private information about future fundamentals by comparing the two versions of the model as all other aspects of the two models are identical.

## 3 Local Approximation Method

In this section we summarize the system of equations and discuss at a general level the issues that arise when deriving a local approximation. In the next section

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<sup>14</sup>See Bacchetta and van Wincoop (2006) for a similar assumption.

we then illustrate the local approximation method in the context of our model. Given space constraints, we do not get into all algebraic details. For those we refer the reader to the companion paper Tille and van Wincoop (2014) and its Technical Appendix. Introducing both portfolio choice and private information into a DSGE model requires us to extend familiar local approximation methods applied to models where these features are not present. This also requires us to be more precise than usual about what exactly is meant by local approximation and introduce explicit notation related to the precision of the approximation as captured by the order components of variables and equations. This notation is also important for understanding the solution for asset prices and capital flows discussed in subsequent sections.

### 3.1 System of Equations

The model gives an immediate solution for the wage rate and consumption of the young as a function of state variables. In terms of logs, denoted by lower case letters, we have  $w_{it} = \ln(\omega) + a_{i,t} + (1-\omega)k_{it}$  and  $c_{y,t}^i = \ln(\omega/(1+\beta)) + a_{i,t} + (1-\omega)k_{i,t}$ . An expression for investment follows immediately from the Tobin's Q equation (5). We compact the system by substituting it into the capital accumulation equation (2). The system then becomes:

$$E_t^{H,j} e^{-r_{t+1}^{p,Hj}} (e^{r_{H,t+1}} - e^{r_{F,t+1} - \tau_{Hj,t}}) = 0 \quad (22)$$

$$E_t^{F,j} e^{-r_{t+1}^{p,Fj}} (e^{r_{H,t+1} - \tau_{Fj,t}} - e^{r_{F,t+1}}) = 0 \quad (23)$$

$$e^{q_{H,t} + k_{H,t+1}} = \frac{\beta\omega}{1+\beta} (z_{H,t} e^{a_{H,t} + (1-\omega)k_{H,t}} + z_{F,t} e^{a_{F,t} + (1-\omega)k_{F,t}}) \quad (24)$$

$$e^{q_{F,t} + k_{F,t+1}} = \frac{\beta\omega}{1+\beta} ((1 - z_{H,t}) e^{a_{H,t} + (1-\omega)k_{H,t}} + (1 - z_{F,t}) e^{a_{F,t} + (1-\omega)k_{F,t}}) \quad (25)$$

$$e^{k_{i,t+1} - k_{i,t}} = 1 + \frac{1}{\xi} (e^{q_{i,t}} - 1) \quad i = 1, 2 \quad (26)$$

$$a_{i,t+1} = \rho a_{i,t} + \varepsilon_{i,t+1} \quad i = 1, 2 \quad (27)$$

where the log rates of return on portfolios and assets are:

$$e^{r_{t+1}^{p,Hj}} = z_{Hj,t} e^{r_{H,t+1}} + (1 - z_{Hj,t}) e^{r_{F,t+1} - \tau_{Hj,t}} \quad (28)$$

$$e^{r_{t+1}^{p,Fj}} = z_{Fj,t} e^{r_{H,t+1} - \tau_{Fj,t}} + (1 - z_{Fj,t}) e^{r_{F,t+1}} \quad (29)$$

$$e^{r_{i,t+1}} = (1 - \omega) e^{a_{i,t+1} - \omega k_{i,t+1} - q_{i,t}} + (1 - \delta) e^{q_{i,t+1} - q_{i,t}} \quad (30)$$

Equations (22) are (23) are the portfolio Euler equations, equations (24) and (25) are the asset market clearing equations, equations (26) are the capital accumulation equations and equations (27) are the productivity processes. The private or public signals in Section 2.4 only come into play in computing expectations in the portfolio Euler equations.

After substituting equations (28)-(30) in equations (22) and (23) we have a system of four equations per country, which are used to solve for  $q_{it}$ ,  $z_{it}$ ,  $k_{i,t+1}$  and  $a_{i,t+1}$  as a function of time  $t$  state variables and innovations. It will be convenient to take averages and differences of these four equations across countries and correspondingly solve for the average and cross-country difference of  $q_{it}$ ,  $z_{it}$ ,  $k_{i,t+1}$  and  $a_{i,t+1}$ . We denote an average with the superscript A and the difference with superscript D, so that for example  $q_t^A = 0.5(q_{H,t} + q_{F,t})$  and  $q_t^D = q_{H,t} - q_{F,t}$ . For the portfolio Euler equations (22)-(23) we first take the average across agents within a country and then the average across countries or the difference between countries.

### 3.2 Order Components

Assuming that all model innovations vary proportionally when  $\sigma_a$  changes, one can think of local approximation methods as an approximation around  $\sigma_a = 0$ . It is useful to normalize model innovations in a way that is independent of  $\sigma_a$ . We denote these scaled innovations whose distribution is independent of  $\sigma_a$  by  $\nu_t$ , with  $\nu_t = (\varepsilon_t^A/\sigma_a, \varepsilon_t^D/\sigma_a, \varepsilon_t^\tau/\sigma_a)'$  in our model.<sup>15</sup>

Let  $x_t$  be a vector of variables to be solved for. In our model  $x_t$  includes averages and differences of  $q_{it}$ ,  $z_{it}$  and  $k_{i,t+1}$ , all known at time  $t$ . We write the precise solution for  $x_t$  as a function of  $\sigma_a$ , conditional on the model innovations  $\nu$  at time  $t$  and earlier. Assuming that this is a differentiable function of  $\sigma_a$ , it can be written as a Taylor expansion around  $\sigma_a = 0$ :

$$x_t = g_0 + \sum_{s=1}^{\infty} g_{s,t} \sigma_a^s \quad (31)$$

where  $g_0$  and  $g_{s,t}$  are vectors that depend on innovations  $\nu$  at time  $t$  and earlier. We refer to  $g_0$  as the order zero component of  $x_t$  and  $g_{s,t} \sigma_a^s$  as the order  $s$  component. We denote these as respectively  $x(0)$  and  $x_t(s)$ . We can thus write a variable as the

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<sup>15</sup>The variance of  $\varepsilon_t^\tau/\sigma_a$  is  $\theta^2$ , which is assumed not to depend on  $\sigma_a$ .

sum of all of its order components:  $x_t = x(0) + \sum_{s=1}^{\infty} x_t(s)$ . Model innovations  $\varepsilon_t^A$ ,  $\varepsilon_t^D$  and  $\varepsilon_t^r$  are always first-order as they are the product of  $\sigma_a$  and the normalized innovations. Examples of second-order terms are  $\sigma_a^2$  or  $\varepsilon_{H,t}^2$  or  $\varepsilon_{H,t}\varepsilon_{F,t}$ . An example of a third-order term is  $\sigma_a^2\varepsilon_{H,t}$ .

We also compute the order components of model equations. These equations generally take the form  $E_t\varphi(x_t, x_{t+1}) = 0$ . Of course there is no expectation operator when all variables are known at time  $t$ , such as in the asset market clearing conditions equations (24) and (25). Since  $x_t$  and  $x_{t+1}$  are functions of  $\sigma_a$ , the expression  $E_t\varphi(x_t, x_{t+1})$  is generally a function of  $\sigma_a$  as well, which can be written as a Taylor expansion around  $\sigma_a = 0$ . We can then solve for  $x_t$  as a function of  $\sigma_a$  by setting all the order components of  $E_t\varphi(x_t, x_{t+1})$  equal to zero.

### 3.3 Local Approximation with Public Information and Without Portfolio Choice

In standard models with public information and without portfolio choice the order components of variables can be computed as follows. We first solve  $x(0)$  by imposing the zero-order component of equations:  $\varphi(x(0), x(0)) = 0$ . We then derive a Taylor expansion of the equations around  $x_t = x(0)$  and then substitute  $x_t = x(0) + \sum_{s=1}^{\infty} x_t(s)$ . As shown in Tille and van Wincoop (2014), with public information the order component of the expectation of  $\varphi(x_t, x_{t+1})$  corresponds exactly to the expectation of the order component of  $\varphi(x_t, x_{t+1})$ .

Consider the first-order component as an illustration. When we take a Taylor expansion of  $\varphi(x_t, x_{t+1})$  and substitute  $x_t = x(0) + \sum_{s=1}^{\infty} x_t(s)$ , it is immediate that the first-order component is  $\varphi'_1 x_t(1) + \varphi'_2 x_{t+1}(1)$ , where  $\varphi'_i$  is the derivative of  $\varphi$  with respect to element  $i$ , evaluated at  $x_t = x_{t+1} = x(0)$ . Setting the first-order component of  $E_t\varphi(x_t, x_{t+1})$  equal to zero then simply implies setting

$$\varphi'_1 x_t(1) + \varphi'_2 E_t x_{t+1}(1) = 0 \tag{32}$$

This is a familiar linear equation that is used to find an approximate linear solution of a model. We can sequentially solve higher order components of variables by imposing higher order components of equations.<sup>16</sup>

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<sup>16</sup>Usually the solution is given as a mapping from state variables to  $x_t$  rather than in the form of equation (31). But the two are exactly the same. For example, a first-order solution  $x_t(1) = a_t^D$  is the same as  $x_t(1) = g_{1,t}\sigma_a$ , with  $g_{1,t} = \sum_{i=0}^{\infty} \rho^i \nu_{t-i}$ , where  $\nu_t = \varepsilon_t^D/\sigma_a$ .

### 3.4 Local Approximation with Portfolio Choice and Private Information

The introduction of portfolio choice complicates local approximation methods even without private information. Devereux and Sutherland (2010) and Tille and van Wincoop (2010a) show how to adjust local approximation techniques to solve such models. The key aspect is that we can no longer solve the order components of the variables by sequentially imposing the order components of equations. In particular, portfolio shares depend on risk, which is of order 2 and higher. This affects even the zero-order portfolio share. The solution, applied to a two-country model, involves two steps. The first step involves computing the zero-order component of the difference across countries in portfolio shares jointly with the first-order component of the other variables, using the second-order component of difference across countries in portfolio Euler equations and the first-order component of the other equations. The second step repeats this one order higher. Dependent on the question at hand, the second step may not be needed. Here we will need it though in order to compute the first-order component of portfolio home bias, which affects capital flows.

Additional complications arise in the presence of private information. In Tille and van Wincoop (2014) we show that the order  $s$  component of the expectation of  $\varphi(x_t, x_{t+1})$  is no longer equal to the expectation of the order  $s$  component of  $\varphi(x_t, x_{t+1})$ . We therefore need to first compute expectations before imposing order components. This involves several steps. First we take a Taylor expansion of  $\varphi(x_t, x_{t+1})$ . We then conjecture a solution of  $x_t$  as a polynomial function of the state variables (a quadratic one is sufficient for a second-order solution). Next, we substitute the exogenous forcing processes such as equation (27) to write  $\varphi(x_t, x_{t+1})$  as a polynomial function of future innovations and variables known at time  $t$ . Once we have written an expression as a polynomial of future innovations we can compute expectations using results from signal extraction. Expectations differ across agents because of private information. Only after we have computed expectations can we impose the order components of equations.

The signal extraction problem is a central element of the model. An individual relies on three sources of information: the unconditional distribution of innovations, her private signals given by equations (16)-(19) about future innovations, and asset prices. As discussed in the next section, the relative asset price  $q_t^D$  de-



depends on the future relative productivity innovation  $\varepsilon_{t+1}^D$  and the noise innovation  $\varepsilon_t^\tau$  through a composite state variable  $h_t$ :

$$h_t = \varepsilon_{t+1}^D + 0.5\lambda\varepsilon_t^\tau \quad (33)$$

Through the relative asset price agents learn about  $h_t$  and therefore receive a noisy signal about  $\varepsilon_{t+1}^D$ .  $\lambda$  is a zero-order coefficient that captures the noise to signal ratio. It is solved as a function of model parameters by imposing asset market equilibrium. This requires employing the third-order component of the portfolio Euler equations and is discussed further in Section 4.3 below. Note that  $h_t$  is not a state variable in the standard and public signals models. When all information is public, asset prices do not contain any additional information.

A couple of points are worth making about the signals given by equations (16)-(19) in the private signals model. We assume that the idiosyncratic components of the private signals (such as  $\epsilon_{j,t}^{H,H}$  or  $\epsilon_{j,t}^{H,F}$ ) are zero-order. In other words, their distribution does not depend on  $\sigma_a$ . Their variance  $\sigma_{H,H}^2$  and  $\sigma_{H,F}^2$  is therefore held constant as we consider a local approximation around  $\sigma_a = 0$ . If we assumed instead that the idiosyncratic components were first-order, the cross-sectional dispersion of portfolio shares would go to infinity as  $\sigma_a \rightarrow 0$ . Intuitively, first-order idiosyncratic signals would lead to first-order disagreement across agents about expected returns, which would be high relative to the second-order variance of asset returns. As portfolio shares reflect the ratio between expected returns and the variance of returns, they would go to infinity as  $\sigma_a \rightarrow 0$ . A similar issue arises with respect to the average iceberg cost  $\tau$ . We assume that it is second-order, as otherwise it would lead to first-order differences between Home and Foreign agents in expected returns. Notice that since  $\tau$  is second-order,  $\tau_t^D = 2\tau\varepsilon_t^\tau$  is third-order.<sup>17</sup>

We now turn to the application of the approach to our specific model, focusing on the first-order solution for the control variables. This solution nonetheless requires employing the zero, first, second and third-order components of various equations.

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<sup>17</sup>In the public signals model we assume that the signals errors  $\epsilon_t^H$  and  $\epsilon_t^F$  are first order, so  $\sigma_v$  is proportional to  $\sigma_a$ . This is not inconsistent with the assumption in the private signals model though. First, the signal errors in the private signal model measure something else, which is disagreement among agents. Second, based on the solution from the signal extraction problem we find that average expectational errors about future productivity in the private signals model are first-order as well.

## 4 Solution of the Model

Before we turn to capital flows, we first discuss the solution of the control variables as a function of the state variables. The control variables are  $q_t^A$ ,  $q_t^D$ ,  $k_{t+1}^A$ ,  $k_{t+1}^D$ ,  $z_t^A$  and  $z_t^D$ . In the private signals model the state variables are  $S_t = (a_t^D, a_t^A, k_t^D, k_t^A)'$  and  $h_t$ . In the standard model  $S_t$  contains all state variables. In the public signals model  $S_t$  is expanded to include  $v_t^A$  and  $v_t^D$  as additional state variables. We follow the two-step methodology developed by Devereux and Sutherland (2010) and Tille and van Wincoop (2010a). In the first step we solve for the zero-order component of  $z_t^D$  jointly with the first-order component of the other control variables by combining the second-order component of the difference in the portfolio Euler equations with the first-order components of the other equations. In the private signals model, this solution remains conditional on the noise-to-signal ratio  $\lambda$ . In the second step, we solve for the first-order component of  $z_t^D$ , which we need to derive a first-order solution for capital flows. This is done by combining the third-order component of the difference in the portfolio Euler equations with the second-order components of the other equations. In the private signals model one additional step is needed. We use the third-order component of the average of the portfolio Euler equations to compute  $\lambda$ . Due to space constraints, we focus on the main aspects and leave many of the algebraic details to the Technical Appendix.

### 4.1 First Step of the Solution

In all three versions of the model, the first step of the solution combines the second-order component of the difference in the portfolio Euler equations with the first-order component of the other equations:

$$z^D(0) = \frac{2\tau}{[E_t(er_{t+1})^2]}(2) \quad (34)$$

$$[\bar{E}_t^A er_{t+1}](1) = 0 \quad (35)$$

$$q_t^A(1) + k_{t+1}^A(1) = a_t^A(1) + (1 - \omega)k_t^A(1) \quad (36)$$

$$k_{t+1}^A(1) - k_t^A(1) = \xi^{-1}q_t^A(1) \quad (37)$$

$$q_t^D(1) + k_{t+1}^D(1) = 4z_t^A(1) + z^D(0)[a_t^D(1) + (1 - \omega)k_t^D(1)] \quad (38)$$

$$k_{t+1}^D(1) - k_t^D(1) = \xi^{-1}q_t^D(1) \quad (39)$$

Here  $er_{t+1} = r_{H,t+1} - r_{F,t+1}$  denotes the excess return. Equation (34) follows

from the second-order component of the cross-country difference in the portfolio Euler equations (22)-(23). Intuitively, the average cost of investing abroad tilts all portfolios towards domestic assets, leading to a relatively high share of the Home asset in the Home agents' portfolio, and a low share in the Foreign agents' portfolios. Note that the zero-order component  $z^D(0)$  is the ratio of two second-order components (in the numerator and denominator). The variance of the excess return in the denominator is computed using the first-order solution of the other variables. Equation (35) follows from the first-order component of the average of portfolio Euler equations (22)-(23) across all agents, with  $\bar{E}_t^A$  denoting the average expectation across all (Home and Foreign) agents. Equation (35) shows that the average expected excess return is zero to the first-order.<sup>18</sup> Intuitively, the equilibrium expected excess return can only differ from zero due to a risk premium, which only has second- and higher order terms.

Equations (36) and (37) are the first-order components of respectively the average of asset market clearing conditions equations (24) and (25) and the average capital accumulation equation (26). These are used to solve for  $k_{t+1}^A(1)$  and  $q_t^A(1)$  as a function of state variables:

$$q_t^A(1) = \frac{\xi}{1 + \xi} (a_t^A(1) - \omega k_t^A(1)) \quad (40)$$

$$k_{t+1}^A(1) = \frac{1}{1 + \xi} a_t^A + \left(1 - \frac{\omega}{1 + \xi}\right) k_t^A(1) \quad (41)$$

The solution for the average asset price is the same under all versions of the model and is not affected by private information. Intuitively, the global asset price is driven by global asset demand, which in turn depends on global saving. Global saving is proportional to global labor income and therefore does not depend on signals about future productivity.<sup>19</sup> Even when agents have information about the future world productivity innovation  $\varepsilon_{t+1}^A$  through private signals, this does not affect the average asset price because there are no other assets to sell (or buy) in order to buy (sell) the Home and Foreign assets that have a higher expected return. Since world saving also does not change, the world demand for the two assets remains unaffected and therefore the average asset price does not change.

<sup>18</sup>The same equation holds not just for the average, but for all agents.

<sup>19</sup>This is because young agents consume a constant fraction of their labor income, as seen in (9), which is a result of the assumed log utility of consumption. In a more general version without log utility, saving will depend on the expected portfolio return, which depends on signals about future productivity.

Equations (38) and (39) are the first-order components of respectively the difference across countries in asset market clearing conditions (24)-(25) and the difference across countries in the capital accumulation equations (26). They are used to solve for  $k_{t+1}^D(1)$  and  $z_t^A(1)$ . The solution is a function of the state variables as well as  $q_t^D(1)$ , which remains to be solved as a function of the state variables.

To solve for  $q_t^D$ , we conjecture a polynomial solution as a function of the state variables. A linear conjecture is sufficient as we are only interested here in the first-order component of  $q_t^D$ :

$$q_t^D = \alpha_{qD} S_t + \alpha_{5,qD} h_t \quad (42)$$

Since  $h_t$  is not a state variable in the standard and public signals models,  $\alpha_{5,qD}$  is non-zero only in the private signals model. To derive the first-order component of the expected excess return, it is sufficient to use a linear Taylor expansion:  $er_{t+1} = (1 - \delta_1)q_{t+1}^D - q_t^D + \delta_1(a_{t+1}^D - \omega k_{t+1}^D)$ , where  $\delta_1 = (1 - \omega)(1 + \beta)/[(1 - \omega)(1 + \beta) + (1 - \delta)\beta\omega]$ . After substituting equation (42) for periods  $t$  and  $t + 1$ , equation (37), equation (39), and  $a_{t+1}^D = \rho a_t^D + \varepsilon_{t+1}^D$ , the excess return becomes a linear function of state variables at time  $t$  and future innovations  $\varepsilon_{t+1}^D$ ,  $\varepsilon_{t+1}^A$  and  $h_{t+1}$ .

We compute  $E_t er_{t+1}$  by computing the expectation of  $h_{t+1}$ ,  $\varepsilon_{t+1}^D$  and  $\varepsilon_{t+1}^A$ . The expectation of  $h_{t+1}$  is zero. In the standard model the expectation of  $\varepsilon_{t+1}^D$  and  $\varepsilon_{t+1}^A$  is also zero as agents only know their unconditional distribution. In the public signals model their expectation is computed based on a simple signal extraction problem, using their unconditional distribution and the public signals:

$$E_t \varepsilon_{t+1}^D = \frac{\sigma_a^2}{\sigma_a^2 + \sigma_v^2} v_t^D \quad E_t \varepsilon_{t+1}^A = \frac{\sigma_a^2}{\sigma_a^2 + \sigma_v^2} v_t^A \quad (43)$$

In the private signals model we also solve a signal extraction problem to compute the expectation of  $\varepsilon_{t+1}^D$  and  $\varepsilon_{t+1}^A$ . Agents now have three sources of information: the unconditional distribution of innovations, private signals, and the public signal  $h_t$ . The latter is inferred from the relative asset price  $q_t^D$ , controlling for the other state variables  $S_t$  in equation (42). The solution of the signal extraction problem is described in Appendix A. For instance, the expectation of the  $t + 1$  Home productivity innovation by agent  $j$  in the Home country is

$$E_t^{j,H} \varepsilon_{H,t+1} = \alpha_{\varepsilon H,h}^{j,H} h_t + \alpha_{\varepsilon H,vH}^{j,H} v_{j,t}^{H,H} + \alpha_{\varepsilon H,vF}^{j,H} v_{j,t}^{H,F} \quad (44)$$

where the coefficients  $\alpha_{\varepsilon H, h}^{j, H}$ ,  $\alpha_{\varepsilon H, vH}^{j, H}$  and  $\alpha_{\varepsilon H, vF}^{j, H}$  are functions of model parameters and have components of various orders. The average of equation (44) across Home investors is proportional to  $h_t$  and the future productivity innovations, as the average of the private signals equals the future productivity innovations. This is how the future productivity innovations affect the relative asset price at time  $t$ .

Using the results from signal extraction, we compute the expectation of  $er_{t+1}$  and then take its first-order component, which is linear in period  $t$  state variables. Setting all coefficients on the state variables equal to zero in equation (35) then gives the parameters  $\alpha_{qD}$  and  $\alpha_{5, qD}$  in equation (42). A shock that raises expected future dividends in the Home country relative to the Foreign country to a first-order leads to a first-order increase in the relative price of the Home asset. Otherwise the expected return would be higher for the Home asset to the first-order, which violates equation (35). For example, a rise in  $a_t^D$  leads to a first-order increase in  $q_t^D$  as the persistence of dividends implies higher future expected relative Home dividends. Similarly, a rise in  $\varepsilon_{t+1}^D$  in the private signals model leads to a first-order increase in  $q_t^D$  (through  $h_t$ ) because it leads to a first-order increase in the expected relative dividend next period. Using the results from signal extraction (see Appendix A), we have  $[E_t \varepsilon_{t+1}^D](1) = h_t / (1 + 2\lambda^2\theta)$ .

It should also be noted that in equilibrium a shock that raises expected relative dividends in the Home country does not automatically make the Home asset more attractive. The higher expected dividend is exactly offset by a higher relative price, so that expected returns remain the same to the first-order. As we will see below, the impact of expected returns on portfolio allocation is therefore more subtle and is driven by the third-order component in expected returns. To the first-order there is no difference in equilibrium expected returns.

Using the results from signal extraction we can also compute the variance of the excess return, which enters in equation (34). At this point we have solved for the zero-order difference in portfolio shares  $z^D(0)$  and the first-order component of all other variables. In the private signals model however, this solution is conditional on the noise to signal ratio  $\lambda$ , which enters through  $h_t$  and through the coefficients in the expectation of future innovations in equation (44).

## 4.2 Second Step of the Solution

The second step of the solution parallels the first, but one order higher. In all three models it is needed to derive the first-order component of  $z_t^D$ , which affects the first-order component of capital flows. It is computed jointly with the second-order component of the other control variables from the third-order component of the difference in portfolio Euler equations and the second-order component of the other equations. We leave the full details of this solution to the Technical Appendix as it involves significant algebra, and limit the discussion to the expression for  $z_t^D(1)$  as it affects capital flows. It is derived from the third-order component of the cross-country difference in the portfolio Euler equations (22)-(23):

$$z_t^D(1) = \frac{[\bar{E}_t^H er_{t+1}](3) - [\bar{E}_t^F er_{t+1}](3)}{[var_t(er_{t+1})](2)} - \frac{[var_t(er_{t+1})](3)}{[var_t(er_{t+1})](2)} \quad (45)$$

where  $\bar{E}_t^H$  and  $\bar{E}_t^F$  refer to the average expectation of respectively Home and Foreign agents.

The home bias  $z_t^D(1)$  increases when Home agents expect a larger excess return of the Home asset than Foreign agents do, or when the variance of the excess return falls. The latter reduces the scope for diversification. Notice that the expression for the first-order component of  $z_t^D$  involves a ratio of a third-order term in the numerator and second-order term in the denominator. The home bias  $z_t^D$  changes over time due to third-order changes in the expected excess return of Home relative to Foreign agents, as well as third-order changes in the variance of the excess return. The third-order component of the expectations and variance in the numerator can be derived only after also solving for the second-order component of all other variables, which requires imposing the second-order component of all other equations. We omit the algebraic details and focus on the form these expressions take and intuition behind them.

The cross-country difference in the expected excess return (third-order component) is naturally zero in the standard and public signals models, but not in the model with private signals. Specifically, the third-order component of the difference in the expected excess return is equal to the third-order component of the difference across countries in the expectation of  $\varepsilon_{t+1}^D$ , times a zero-order constant that depends on model parameters. Using the results from signal extraction, we

show in Appendix A that

$$[\bar{E}_t^H \varepsilon_{t+1}^D] (3) - [\bar{E}_t^F \varepsilon_{t+1}^D] (3) = \frac{\sigma_{H,F}^2 - \sigma_{H,H}^2}{\sigma_{H,H}^2 \sigma_{H,F}^2} \frac{2\lambda^2 \theta}{1 + 2\lambda^2 \theta} \sigma_a^2 \varepsilon_{t+1}^A \quad (46)$$

Under our assumption that  $\sigma_{HH}^2 < \sigma_{HF}^2$ , agents receive more precise signals about their domestic asset market. A positive global innovation leads all agents to expect that productivity will increase by more in their own country as they have more precise information about their own productivity through their private signals. All agents then expect the return on their own country's asset to rise relative to that of the other country, which leads to increased home bias  $z_t^D(1)$ .

In the standard and private signals model the expression for the third-order component of the variance of the excess return takes the form

$$[var_t(er_{t+1})] (3) = \sigma_a^2 (\eta_1 a_t^A (1) + \eta_2 k_t^A (1)) \quad (47)$$

where  $\eta_i$  are zero-order coefficients that depend on model parameters. In the public signals model it also depends on  $v_t^D$  and  $v_t^A$ . Risk is thus time-varying even without private information (see also Tille and van Wincoop (2010a)). In order to get a sense of where this time-varying variance comes from, a second-order solution of the other variables leads to an expression for the excess return that has the second-order term  $a_t^A \varepsilon_{t+1}^D$  as well as a linear term  $\varepsilon_{t+1}^D$ . The variance of the excess return then depends on the covariance of these two terms, which is  $a_t^A var(\varepsilon_{t+1}^D)$ . In the standard model this is equal to  $2\sigma_a^2 a_t^A$ . This is a third-order term that is time-varying in the state variable  $a_t^A$ .

In principle one could repeat this second step even one order higher to solve for  $z_t^D(2)$  jointly with the third-order component of the other variables. This would allow us to compute the second-order component of capital flows. However, this would involve an excessive amount of algebra as it requires computing the fourth-order component of the difference in portfolio Euler equations and the third-order component of all other equations. We therefore restrict ourselves to the first-order solution for capital flows in the next section.

### 4.3 Noise to Signal Ratio

As in NRE models, the noise to signal ratio  $\lambda$  is solved by imposing asset market equilibrium. This is done here by equating  $z_t^A(1)$  from the asset supply perspective

to that from the asset demand perspective. The solution from the asset supply perspective is given by the asset market clearing condition in terms of cross-country differences (38). After substituting (39) this gives

$$z_t^A(1) = \frac{1}{4} \left( 1 + \frac{1}{\xi} \right) q_t^D(1) + \frac{1}{4} (1 - (1 - \omega)z^D(0)) k_t^D(1) - \frac{1}{4} z^D(0) a_t^D(1) \quad (48)$$

This is the average portfolio share that brings asset markets into balance for given values of asset prices and capital stocks. Note that it depends on  $q_t^D(1)$ , which affects the relative supply both directly and through its impact on relative investment.

The demand perspective focuses on the portfolio choice of investors. We compute  $z_t^A(1)$  by imposing the third-order component of the average of portfolio Euler equations (22)-(23) across all investors:

$$z_t^A(1) = \frac{[\bar{E}_t^A er_{t+1}] (3) + \tau \varepsilon_t^\tau}{[var_t(er_{t+1})] (2)} \quad (49)$$

The average fraction invested in the Home asset rises when the expected excess return on the Home asset rises to the third-order or when the friction of investing abroad increases for Home relative to Foreign agents. Just like for  $z_t^D(1)$ , it is a relatively small third-order change in the expected excess return that affects the average portfolio share to the first-order. This is because the expected excess return is divided by the variance of the excess return, which is second-order.

We solve for  $\lambda$  by setting equation (48) equal to equation (49). In particular, we need to make sure that  $\varepsilon_{t+1}^D$  and  $\varepsilon_t^\tau$  directly enter equation (49) in the same way as in the supply expression equation (48), where they enter through  $h_t = \varepsilon_{t+1}^D + 0.5\lambda\varepsilon_t^\tau$  that affects  $q_t^D(1)$ . Note that in the portfolio equation (49) the noise  $\varepsilon_t^\tau$  enters directly, while  $\varepsilon_{t+1}^D$  enters through  $[\bar{E}_t^A er_{t+1}] (3)$ .

#### 4.4 Impact of Private Information on the Solution

As should be clear from the discussion above, there are significant differences in the solution of the model with private signals relative to the two versions of the model where all information is public. Our main interest in this paper is on the impact of private information on capital flows, to which we turn in the next section. But some brief comments are in order about the impact of private information on the control variables  $q_t^A$ ,  $q_t^D$ ,  $k_{t+1}^A$ ,  $k_{t+1}^D$ ,  $z_t^A$  and  $z_t^D$ . The key difference is that in the



private information model they depend on the unobservable noise and productivity shocks  $\varepsilon_t^\tau$ ,  $\varepsilon_{t+1}^D$  and  $\varepsilon_{t+1}^A$ , which do not affect the first-order solution in the standard and public signals models.

First consider asset prices. We have seen that the average asset price  $q_t^A$  only depends on observed state variables  $a_t^A$  and  $k_t^A$ . It is not affected by private information, although that is only because of the assumed log-utility. The relative asset price  $q_t^D$  in the private information model is quite different than in the two versions of the model with public information. In particular, it depends on the unobservables  $\varepsilon_{t+1}^D$  and  $\varepsilon_t^\tau$  through  $h_t$ , which is only a state variable in the model with private information.

The impact of  $\varepsilon_{t+1}^D$  simply reflects the private information that agents have about future productivity innovations. The private signals themselves average to future productivity innovations, which is why they affect equilibrium asset prices. The impact of the noise shock  $\varepsilon_t^\tau$  is more subtle. In the model without private information it affects the relative asset price only to the third-order. It has the same effect as an exogenous shift in the relative asset supply, which leads to a change in the equilibrium risk-premium. But such a change in risk is third-order and has a third-order effect on the relative asset price. It therefore does not show up in the first-order solution.

In the private signals model by contrast, noise shocks have a first-order impact on asset prices through  $h_t$ . This is due to an amplification effect that is well known in the NRE literature, which Bacchetta and van Wincoop (2006) refer to as “rational confusion”. It occurs because agents use asset prices as a source of information about future innovations. A positive noise shock  $\varepsilon_t^\tau$  leads to a portfolio shift towards the Home asset, which raises its relative price. But because the relative asset price contains information due to private signals, agents then rationally increase their expectation of relative future Home productivity. This in turn leads to a further increase in the relative asset price, amplifying the impact of the initial noise shock. In terms of orders, the amplification implies an increase in the impact from third-order to first-order.<sup>20</sup>

The control variables  $k_{t+1}^D$ ,  $z_t^A$  and  $z_t^D$  are also only affected by unobservables

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<sup>20</sup>Gennotte and Leland (1990) show that such amplification effects can be very large in practice. They provide evidence that during the U.S. stock market crash of October 19, 1987, the impact of non-informational trade on the U.S. stock price was amplified by a factor greater than 100 as a result of the information content of the stock price.

in the model with private information.  $k_{t+1}^D$  depends on the relative asset price  $q_t^D$  through the Tobin's Q equation (39) and therefore also depends on the unobservables  $\varepsilon_{t+1}^D$  and  $\varepsilon_t^\tau$  in the private signals model.  $z_t^D$  depends on the unobservable  $\varepsilon_{t+1}^A$ , which we have seen affects the differences across countries in the expected excess return. This feature again only applies with private information as all agents have the same expectations in the standard and public signals models.

Finally, the solution of  $z_t^A$  is given in equation (48) as a function of observed state variables and the relative asset price. Only in the private information model does it depend on the unobservables  $\varepsilon_{t+1}^D$  and  $\varepsilon_t^\tau$  through the relative asset price. From a demand perspective one might think that even without private information  $z_t^A$  depends to the first-order on noise shocks, as a rise in  $\varepsilon_t^\tau$  leads to a first-order shift to the Home asset (see equation (49)). But this leads to a third-order increase in the relative Home asset price, which lowers the expected excess return on the Home asset to a third-order. We thus get an exactly offsetting first-order shift away from the Home asset (see again equation (49)), so that the net first-order effect is zero. It must be that way as there is no first-order change in the relative supply in the absence of private information. This point clearly highlights the importance of using a general equilibrium framework.

To sum up, our results indicate that not only prices, but also quantities are significantly affected by private information. We now turn to another key quantity, capital flows.

## 5 Capital Flows

Gross capital outflows are defined as purchases of the Foreign asset by Home agents while gross capital inflows are purchases of the Home asset by Foreign agents. We first derive expressions for the first-order components of capital inflows and outflows that depend on wealth accumulation (national saving), asset return expectations and asset return risk. We then relate these terms to observed state variables and unobserved innovations. We highlight the role of private information in affecting capital flows through the various terms.

## 5.1 Expressions for Capital Flows

As shown in Appendix B for capital outflows, using standard balance of payments accounting we can relate first-order capital flows to changes in saving and portfolio shares:

$$outflows_t(1) = (1 - z_H(0))s_t^H(1) - [\Delta z_{H,t}(1) - \Delta \tilde{z}_t(1)] \quad (50)$$

$$inflows_t(1) = (1 - z_H(0))s_t^F(1) + [\Delta z_{F,t}(1) - \Delta \tilde{z}_t(1)] \quad (51)$$

where capital flows are normalized by steady state financial wealth and  $\Delta$  is the first difference operator ( $\Delta z_{H,t}(1) = z_{H,t}(1) - z_{H,t-1}(1)$ ).

The first term on the right hand side of equations (50) and (51) represents portfolio growth. This component of capital flows occurs when changes in financial wealth are allocated across assets in line with the steady state portfolio shares (zero-order component). A first-order change in financial wealth in country  $i$  is reflected in national saving  $s_t^i(1)$ .

The second term on the right hand side of equations (50) and (51) represents portfolio reallocation, and reflects changes in portfolio shares. A key point is that a change in portfolio shares does not necessarily translate into capital flows. Specifically, asset price movements directly impact portfolio shares by changing the value of holdings, without any capital flows taking place. For instance, an increase in the relative Home asset price  $q_t^D(1)$  automatically raises the portfolio share of the Home asset. We denote such valuation effects, which are the same for Home and Foreign agents, by  $\Delta \tilde{z}_t(1)$ :

$$\Delta \tilde{z}_t(1) = \frac{1 - (z^D(0))^2}{4} \Delta q_t^D(1) \quad (52)$$

Controlling for this effect, a reduction of the share that Home agents allocate to the Home asset (a negative  $\Delta z_{H,t}(1) - \Delta \tilde{z}_t(1)$ ) leads them to purchase the Foreign asset, which translates into positive capital outflows. Similarly, an increased share allocated to the Home asset by Foreign agents (a positive  $\Delta z_{F,t}(1) - \Delta \tilde{z}_t(1)$ ) leads to positive capital inflows as they purchase the Home asset. Equations (50) and (51) are rewritten in terms of averages and differences of portfolio shares as:

$$outflows_t(1) = (1 - z_H(0))s_t^H(1) - [\Delta z_t^A(1) - \Delta \tilde{z}_t(1)] - 0.5\Delta z_t^D(1) \quad (53)$$

$$inflows_t(1) = (1 - z_H(0))s_t^F(1) + [\Delta z_t^A(1) - \Delta \tilde{z}_t(1)] - 0.5\Delta z_t^D(1) \quad (54)$$

An increase in home bias ( $\Delta z_t^D(1) > 0$ ) lowers both capital outflows (Home agents sell the Foreign asset to purchase the Home asset) and capital inflows (Foreign agents purchase the Foreign asset sold by Home agents). Holding home bias constant, an increase in the average share invested in the Home asset implies larger capital inflows and smaller capital outflows.

Changes in the average portfolio share invested in the Home asset that are not due to valuation effects,  $\Delta z_t^A(1) - \Delta \tilde{z}_t(1)$ , are associated with a specific component of the third-order expected excess return. This component, which we denote by  $\Delta [\bar{E}_t^A er_{t+1}] (3)^{IS}$ , reflects changes in savings and investment:

$$\Delta z_t^A(1) - \Delta \tilde{z}_t(1) = \frac{\Delta [\bar{E}_t^A er_{t+1}] (3)^{IS}}{[var_t(er_{t+1})] (2)} \quad (55)$$

In order to derive equation (55), we start with the first-order component of the difference in the asset market clearing equations (24) and (25):

$$\Delta z_t^A(1) - \Delta \tilde{z}_t(1) = \frac{1}{4} (\Delta i_t^D(1) - z^D(0) s_t^D(1)) \quad (56)$$

where  $i_t^D(1) = k_{t+1}^D(1)$  and  $s_t^D(1)$  are the first order components of the cross-country differences in investment and national saving, respectively, which are presented in more detail below. An increase in the left-hand side of equation (56) reflects a rise in the relative quantity of Home to Foreign assets held by investors that is associated with portfolio reallocation. The clearing of asset markets requires one of the two counterparts on the right-hand side of equation (56) to increase. First, the relative Home asset supply can increase through higher Home relative investment  $i_t^D(1)$ , which raises the relative Home capital stock. Second, a rebalancing of demand away from the Home asset can occur through a drop in Home relative saving  $s_t^D(1)$ , which reduces demand for the Home asset in the presence of home bias in portfolios ( $z^D(0) > 0$ ).

Combining the first difference of equation (49) with equation (56), and recalling that  $[var_t(er_{t+1})] (2)$  is constant, we write the change in the third-order component of the average expected excess return as:

$$\begin{aligned} \Delta [\bar{E}_t^A er_{t+1}] (3) &= -\tau \Delta \varepsilon_t^\tau + [var_t(er_{t+1})] (2) \Delta \tilde{z}_t(1) \\ &\quad + \frac{[var_t(er_{t+1})] (2)}{4} (\Delta i_t^D(1) - z^D(0) s_t^D(1)) \end{aligned} \quad (57)$$

We refer to the last term on the right hand side as  $\Delta [\bar{E}_t^A er_{t+1}] (3)^{IS}$ . Equation (55) then follows from equation (56). It is only this last component of the expected excess return that affects capital flows.

The interpretation of the three factors on the right-hand side of equation (57) is as follows. A higher relative friction of investing in the Home asset ( $\Delta \varepsilon_t^T < 0$ ) lowers the demand for the Home asset. A higher equilibrium expected excess return on the Home asset is then needed to raise demand and clear the asset market. Notice however that this does not lead to capital flows, as the change in the expected excess return simply brings asset demand equation (49) back to its initial value. The second term captures a higher relative supply of the Home asset due to the higher relative price, which impacts  $\Delta \tilde{z}_t(1)$ . To clear markets there needs to be a rise in the relative demand for the Home asset, which is accomplished by a higher equilibrium expected excess return on the Home asset. This also does not generate any capital flows as a rise in the relative asset price automatically leads to a portfolio shift towards the Home asset through valuation effects, without any need for capital flows. The third and final term corresponds to  $\Delta \bar{E}_t^A er_{t+1}(3)^{IS}$  and reflects investment and saving. Either an increase in Home relative investment  $i_t^D$  or drop in Home relative saving  $s_t^D$  leads the relative supply of the Home asset to be larger than relative demand. Asset markets then clear through a rise in the expected excess return on the Home asset, which leads Home and Foreign investors to shift funds towards the Home asset. It is only this part of the expected excess return that affects capital flows.

Substituting equation (45) and equation (55) into equations (53) and (54), we derive four drivers of capital flows:

$$\begin{aligned} outflows_t(1) = & -\frac{1}{2} \frac{\Delta [\bar{E}_t^H er_{t+1}](3) - \Delta [\bar{E}_t^F er_{t+1}](3)}{[var_t(er_{t+1})](2)} + (1 - z_H(0)) s_t^H(1) \\ & - \frac{\Delta \bar{E}_t^A er_{t+1}(3)^{IS}}{[var_t(er_{t+1})](2)} + \frac{z^D(0) \Delta [var_t(er_{t+1})](3)}{2 [var_t(er_{t+1})](2)} \end{aligned} \quad (58)$$

$$\begin{aligned} inflows_t(1) = & -\frac{1}{2} \frac{\Delta [\bar{E}_t^H er_{t+1}](3) - \Delta [\bar{E}_t^F er_{t+1}](3)}{[var_t(er_{t+1})](2)} + (1 - z_H(0)) s_t^F(1) \\ & + \frac{\Delta \bar{E}_t^A er_{t+1}(3)^{IS}}{[var_t(er_{t+1})](2)} + \frac{z^D(0) \Delta [var_t(er_{t+1})](3)}{2 [var_t(er_{t+1})](2)} \end{aligned} \quad (59)$$

The first term on the right hand side represents the difference between Home and Foreign agents in the expected excess return. The second term captures capital flows due to portfolio growth. The third term is associated with capital flows due to changes in the average expected excess return. The last term captures capital flows due to time-varying risk.

We will discuss each of these terms below and the role of private information in impacting these terms. But before doing so it is useful also to point out what is not in these terms. Any information that raises the expectation of relative Home dividends to the first-order (increase in expected relative Home productivity) does not directly impact capital flows by raising the expected excess return on the Home asset. Also, a rise in  $\varepsilon_t^\tau$ , which leads to a first-order portfolio shift towards the Home asset, might be expected to generate larger capital inflows and smaller capital outflows. But it is not that simple. Because of the general equilibrium nature of the model, such shocks only impact capital flows in more subtle ways.

This is related to some of the discussion in Section 4. For example an increase in  $\varepsilon_{t+1}^D$  raises the expected relative Home productivity in the private information model to the first-order. This in turn leads to a higher relative price of the Home asset so that to the first-order expected returns remain equal in equilibrium. Instead, capital flows are impacted in more subtle ways as the change in the relative asset price affects saving and investment, which also impact the third-order expected excess return. We discuss this below.

As also discussed in Section 4, without private information a rise in  $\varepsilon_t^\tau$  generates a third-order increase in the Home relative asset price. This lowers the expected excess return to the third-order, which leads to a first-order portfolio shift back to the Foreign asset. It must be so as there is no first-order change in the relative supply of the Home asset. As we will see, the only way that  $\varepsilon_t^\tau$  can affect capital flows is again more subtle and operates through the asset price, which is impacted to the first-order only in the private information model due to rational confusion.

As mentioned in the introduction, Albuquerque, Bauer and Schneider (2007, 2009) and Brennan and Cao (1997) also derive capital flows in open economy models with dispersed information. But these are partial equilibrium linear NRE models that differ substantially from standard DSGE macro models. This leads to different expressions for capital flows. There is no saving and investment in these models. Therefore the portfolio growth component is completely absent. In addition the component associated with changes in the average expected excess return is also absent as this term depends on changes in saving and investment. The last term, associated with time-varying risk, also does not exist because risk is constant in these models due to their linear nature. The only other term left is associated with differences in expected returns across countries. This term does exist in these papers, although they focus on a different set of implications than

we do below.

In addition capital flows in these papers also depend on a possible shift between the risky assets and a riskfree asset that is in infinite supply. In Albuquerque, Bauer and Schneider (2007) there is also an asset with an exogenous stochastic return that is in infinite supply. The infinite supply of some assets is clearly a partial equilibrium feature. Capital flows are accordingly defined in a narrower way that only includes flows involving the risky assets that are in finite supply. But since these models are not general equilibrium, this narrow measure of capital flows is affected by a shift between the assets that are in fixed supply and those that are in infinite supply. This does not happen in our general equilibrium framework.

## 5.2 The four drivers of capital flows and dispersed information

We now turn to a discussion of each of the four drivers of capital flows on the right hand side of equations (58) and (59). We focus on the economic intuition and contrast the role of observed  $(a_t^D, a_t^A, k_t^D, k_t^A)$  and unobserved  $(\varepsilon_{t+1}^D, \varepsilon_{t+1}^A, \varepsilon_t^\tau)$  fundamentals. Several contributions to the empirical literature on capital flows distinguish between so-called push and pull factors (Forbes and Warnock 2012). Push factors are developments in global markets, which in the context of our model correspond to the worldwide variables  $a_t^A, k_t^A$  and  $\varepsilon_{t+1}^A$ . Pull factors relate to shocks that are specific to the country receiving capital flows. In our model, these correspond to the cross-country differences,  $a_t^D, k_t^D, \varepsilon_{t+1}^D$  and  $\varepsilon_t^\tau$  (holding the worldwide variables unchanged).

We now show how private information makes capital flows sensitive to the unobserved push and pull factors. This leads to a disconnect of capital flows from publicly observed fundamentals, a predictive content of capital flows for future fundamentals, and increased volatility of capital flows. In Figure 1 we graphically display the channels through which the unobservables impact capital flows.

### Differences in Expected Returns across Countries

The first term on the right hand side of equations (58) and (59) is specific to the private signals model. It represents the difference in the average expected excess return by Home agents relative to Foreign agents. This term is zero in the standard and public signals models. In the private signals model it is proportional

to the difference between the expectations of  $\varepsilon_{t+1}^D$  of Home agents (averaged across agents) and the expectation of Foreign agents. This difference in expectations is proportional to  $(\sigma_{H,F}^2 - \sigma_{H,H}^2) \sigma_a^2 \varepsilon_{t+1}^A$  as shown by equation (46).

A push factor in the form of improved future global productivity leads all agents to infer a relative increase of productivity in their own country when domestic signals are more precise ( $\sigma_{H,F}^2 - \sigma_{H,H}^2 > 0$ ). This leads to increased portfolio home bias. Agents then sell assets abroad to purchase the domestic asset, and both capital inflows and outflows drop by an equal amount. This push factor leaves net capital flows unchanged, but lowers both capital inflows and outflows. Gross capital flows (the sum of inflows and outflows) therefore fall. This channel is represented on the right hand side of the chart in Figure 1.

Note that there are no offsetting general equilibrium effects as the demand for Home and Foreign assets does not change. A rise in  $\varepsilon_{t+1}^A$  raises the demand for the Home asset by Home agents, while it lowers the demand for the Home asset by the Foreign agents by an equal amount. Similarly, the demand for the Foreign asset by Home agents drops, while the demand for the Foreign asset by Foreign agents rises by an equal amount. The ownership of the assets is thus reallocated across the agents, without any need for asset price changes to clear markets. Indeed, to the first-order neither  $q_t^D$  nor  $q_t^A$  are changed. This impact of private information on capital flows is therefore not linked to asset prices, so flows and prices are not mere mirror images of each other.

### Portfolio Growth

The second term on the right hand side of equations (58) and (59) represents portfolio growth, which measures outflows and inflows when Home and Foreign saving are invested abroad at the steady state portfolio share. The portfolio growth component therefore depends on saving. Expressions for Home and Foreign saving are derived in Appendix B. In terms of worldwide averages, saving only depends on observed state variables:

$$s_t^A(1) = \Delta a_t^A(1) + (1 - \omega) \Delta k_t^A(1) - \Delta q_t^A(1) \quad (60)$$

The observed state variables  $a_t^A$  and  $k_t^A$  affect average wages and therefore world saving by the current young generation. As these same variables lagged by one period affect dissaving by the current old generation, aggregate saving depends on the first difference of  $a_t^A$  and  $k_t^A$ . In addition the dissaving by the old generation is



affected by the average asset price. A higher asset price implies higher wealth, and thus larger dissaving through a wealth effect. As shown in Section 4, the average asset price itself also depends on the public information variables  $a_t^A$  and  $k_t^A$ .

The cross-country difference in saving is:

$$s_t^D(1) = \Delta a_t^D(1) + (1 - \omega)\Delta k_t^D(1) - z^D(0)\Delta q_t^D(1) \quad (61)$$

It depends not only on the observed state variables  $a_t^D$  and  $k_t^D$ , which affect relative wages and saving by the young agents, but also on the relative asset price  $q_t^D$ , again through a wealth effect affecting old agents. As long as there is positive portfolio home bias ( $z^D(0) > 0$ ), an increase in the relative price of the Home asset raises the wealth of the old generation in the Home country relative to that in the Foreign country. This wealth effect raises relative consumption in the Home country and lowers relative saving.

The unobserved pull factors  $\varepsilon_{t+1}^D$  and  $\varepsilon_t^\tau$  impact capital flows through the relative asset price. An increase in either  $\varepsilon_{t+1}^D$  or  $\varepsilon_t^\tau$  raises the relative price of the Home asset, which lowers Home saving and raises Foreign saving. Through portfolio growth this leads to lower capital outflows and higher capital inflows, and thus a drop in net capital outflows. Gross capital flows (sum of inflows and outflows) do not change as the drop in outflows is equal to the increase in inflows. This effect is illustrated in Figure 1 through the relative asset price-saving-portfolio growth channel.

In the standard and public signals models the expressions for average and relative saving are the same, but the relative asset price only depends on the publicly observed fundamentals  $a_t^D$ ,  $k_t^D$  and  $v_t^D$ .

### Average Expected Excess Return

The third term on the right hand side of equations (58) and (59) represents capital flows due to changes in the average expected excess return that is associated with changes in relative saving and investment,  $\Delta [\bar{E}_t^A er_{t+1}] (3)^{IS}$ . We have seen that it is proportional to  $i_t^D(1) - z^D(0)s_t^D(1)$ . We have already shown that relative saving depends on the relative asset price. The same is the case for relative investment through a standard Tobin's Q expression (from equation (39)):

$$i_t^D(1) = \xi^{-1}q_t^D(1) \quad (62)$$

The unobserved pull factors  $\varepsilon_{t+1}^D$  and  $\varepsilon_t^\tau$  impact capital flows again through the relative asset price. An increase in either  $\varepsilon_{t+1}^D$  or  $\varepsilon_t^\tau$  raises the relative asset price,

which lowers relative Home saving and raises relative Home investment. Both lead to an excess relative supply of the Home asset. The expected excess return on the Home asset then rises to clear asset markets through a portfolio shift towards the Home asset. Capital outflows drop, capital inflows rise, net capital outflows fall, and gross capital flows remain unchanged. This effect is illustrated in Figure 1 through the relative asset price-saving/investment-expected return channel. This effect is again specific to private information, as in the standard and public signals models  $\Delta \bar{E}_t^A er_{t+1}(3)^{IS}$  only depends on  $a_t^D$ ,  $k_t^D$  and  $v_t^D$ .

### Time-Varying Risk

The last term in equations (58) and (59) represents capital flows due to changes in the variance of the excess return, captured by its third-order component. There is a trade-off in the model between home bias due to the cost of investing abroad and the desire to reduce risk through diversification. An increase in the variance of the excess return raises the scope for portfolio diversification and leads to an increase in both capital inflows and outflows.

We saw in section 4 (equation (47)) that in the standard and private signals model the third-order component of the variance of the excess return is proportional to  $\sigma_a^2 a_t^A$  and  $\sigma_a^2 k_t^A$ . In the public signals model it also depends on  $v_t^D$  and  $v_t^A$ . In all versions of the model therefore the variance of the excess return only depends on publicly observed push factors.

## 5.3 Summary of the Impact of Private Information

We have shown that in the private information model capital flows are affected by the unobserved macro fundamentals  $\varepsilon_{t+1}^D$ ,  $\varepsilon_{t+1}^A$  and  $\varepsilon_t^\tau$  through the first three drivers in equations (58) and (59). This disconnects capital flows from publicly observed macro fundamentals, just as is the case for asset prices. This is not the case in the standard and public signals models where capital flows only depend on the publicly observed state variables, including  $v_t^D$  and  $v_t^A$  in the public signals model. The impact of the unobserved fundamentals also increases capital flow volatility in the private information model. Moreover, capital flows contain information about the future fundamentals  $\varepsilon_{t+1}^D$  and  $\varepsilon_{t+1}^A$  that is not contained in the public information set. Net capital flows contain information about  $\varepsilon_{t+1}^D$  while gross capital flows

contain information about  $\varepsilon_{t+1}^A$ .<sup>21</sup>

It should also be emphasized that private information impacts capital flows in different ways than asset prices, so the two do not merely mirror each other. We have seen that  $\varepsilon_{t+1}^D$  and  $\varepsilon_t^\tau$  only affect capital flows through asset prices. Asset prices affect saving and investment, which in turn affect portfolio growth and the expected excess return. On the other hand  $\varepsilon_{t+1}^A$  only affects capital flows and has no first-order effect on asset prices. More generally, to the extent that private information leads to portfolio shifts that do not affect aggregate demand for the assets, it affects capital flows but not asset prices.

## 5.4 Amplified Role of Innovations due to Signals

Our discussion so far focuses on the impact of private information through the unobserved macro fundamentals. But information about future innovations has the additional effect of increasing the impact of public information on capital flows, which further increases their volatility.

We illustrate this by comparing the standard and private signals models, starting with gross capital flows. In the standard model gross capital flows depend positively on  $\varepsilon_t^A$ , as higher productivity raises Home and Foreign saving and boosts both capital inflows and outflows through portfolio growth.<sup>22</sup> In the private signals model, the rise in  $\varepsilon_t^A$  also affects capital flows through the cross-country difference in the third-order components of expected excess return (the first term on the right-hand side of equations (58) and (59)). Specifically, the rise in  $\varepsilon_t^A$  enters agents' signals at  $t - 1$ . As discussed above, all agents expect a relative productivity boom in their country, leading to a contraction in both outflows and inflows at  $t - 1$ . As capital flows depend on the first-difference of expectations,

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<sup>21</sup>We have not used this information content of capital flows as a source of information of investors when computing expectations of  $\varepsilon_{t+1}^D$  and  $\varepsilon_{t+1}^A$ . In practice the difficulty is that capital flows are observed both with substantial delay and with noise, which limits the use to investors. We could allow investors in our model to observe capital flows with noise. This makes no difference for net capital flows as the relative asset price contains the same information. For gross capital flows it allows investors to obtain another piece of information about  $\varepsilon_{t+1}^A$ . But as long as the information is noisy, it does not reveal  $\varepsilon_{t+1}^A$  and therefore does not qualitatively change our results for gross flows either. It just adds another source of noise.

<sup>22</sup>There is a slight offsetting effect as a rise in  $\varepsilon_t^A$  lowers the variance of the excess return, which reduces inflows and outflows. But we find this effect to be very small, independent of the parameterization.

$\Delta [\bar{E}_t^H er_{t+1}] (3) - \Delta [\bar{E}_t^F er_{t+1}] (3)$ , which is proportional to  $\varepsilon_{t+1}^A - \varepsilon_t^A$ , this contraction of capital flows is followed by an opposite boom at time  $t$ . This amplification channel through the expected return differential is entirely a result of private information.

We now turn to net capital flows, which depends positively on  $\varepsilon_t^D$  in the standard model. An increase in  $\varepsilon_t^D$  raises relative Home wages and therefore saving, leading to net capital outflows both through the portfolio growth and average expected return channels.<sup>23</sup> In the private signals model, the rise in  $\varepsilon_t^D$  leads to an additional increase in net capital outflows. As it enters agents' signals at  $t - 1$ , it raises the relative asset price  $q_{t-1}^D$ . This in turn lowers relative Home saving and raises relative Home investment at  $t - 1$ , which reduces net capital outflows through the portfolio growth and average expected return channels. This is reversed at time  $t$  (as capital flows are affected by  $\Delta q_t^D$  instead of  $q_t^D$  itself), leading to higher net capital outflows. The amplified impact of  $\varepsilon_t^D$  on net capital outflows thus operates through the private signals and their impact on the relative asset price.

Our discussion stresses the amplified effect of productivity shocks in the private signals model relative to the standard model. Some amplification is also present in the public signals model relative to the standard model. It however only reflects the second of the two mechanisms presented here, as the first one relies on disagreements between Home and Foreign agents that are absent in the public signals model.

## 6 Numerical Illustration

In this section we complement our qualitative analysis with a numerical illustration of the three variants of the model. This shows that the impact of private information can be quantitatively large even within the context of our simple model. We also discuss how the impact of private information is affected by some of the key parameters of the model.

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<sup>23</sup>There is an offsetting effect through the relative asset price, which depends positively on  $a_t^D$  and therefore on  $\varepsilon_t^D$ . A higher relative price lowers relative Home saving and raises relative Home investment. But we find that only for extreme parameter assumptions does this offset the effect through relative wages.

## 6.1 Calibration

For parameters unrelated to information dispersion we either choose standard values or match basic model moments for 6 industrialized countries over the period 1977 to 2006 (the G7 minus Italy). We set the rate of depreciation at  $\delta = 0.1$ , the time discount rate at  $\beta = 0.95$  and the labor share at  $\omega = 0.7$ . We set  $\rho = 0.99$  as it is hard to distinguish between  $\rho$  close to 1 and exactly 1 and the unit root case cannot be rejected by the data (e.g. Baxter and Crucini, 1995). We set  $\sigma_a = 0.017$  so that the standard deviation of output growth in the model is equal to the average standard deviation of annual real GDP growth for the 6 countries, which is 1.7%. We set the adjustment cost parameter  $\xi = 2.5$  in order to match the standard deviation of annual real investment growth relative to the standard deviation of annual real GDP growth. This ratio is 2.8 in the data when averaged across the 6 countries.

We choose the average cost  $\tau$  of investment abroad in order to match the observed portfolio home bias in the data. The standard measure of portfolio home bias is

$$1 - \frac{\text{share of foreign equity in portfolio of domestic agents}}{\text{share of foreign equity in world portfolio}}$$

This corresponds to  $z^D(0)$  in the steady state of our model, which depends on  $\tau$ . Fidora, Fratzscher and Thimann (2007) report this measure of home bias for a wide range of countries based on 2001-2003 data. This includes 5 of our industrialized countries (all but Canada). The average measure of home bias for those 5 countries is 0.73. They also report a measure of home bias for debt securities, which is virtually identical. We therefore set the cost  $\tau$  of investment abroad such that the zero-order component of  $z^D(0)$  in the model is equal to 0.73.<sup>24</sup>

The remaining parameters relate to information dispersion. They are the standard deviations of the errors of the private signals  $\sigma_{HH}$  and  $\sigma_{HF}$  and the parameter  $\theta$  that measures the volatility of the noise. Since it is hard to calibrate these parameters we illustrate their impact over a very wide range. In the benchmark parameterization we set  $(\sigma_{HH} + \sigma_{HF})/2 = 0.21$ ,  $\sigma_{HF}/\sigma_{HH} = 1.5$  and  $\theta = 100$ . In Appendix C we discuss some motivation for setting  $(\sigma_{HH} + \sigma_{HF})/2 = 0.21$ . It generates a standard deviation of the cross-sectional distribution of expected asset prices, scaled by the unconditional variance of asset price changes, which matches

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<sup>24</sup>This implies that both countries invest a fraction 0.865 in the domestic asset.

survey data for the United States and Japan. In the public signals model we set the variance  $\sigma_v^2$  of the signal errors such that the conditional variance of the excess return (second-order component) is the same as in the private information model.

## 6.2 Simulation Results

We simulate the model over 100,000 periods to produce capital flow volatility, the information content of capital flows as well as the disconnect between capital flows and observed fundamentals. These moments, along with the correlation between capital inflows and outflows, are reported in Table 1 for all three versions of the model along with the empirical moments for the 6 industrialized countries. In both the model and the data capital flows are divided by GDP and HP(10) filtered. For net capital flows we report volatility in the data based both on capital flows and the current account, with the latter being more reliable.

Even though our calibration is not based on any capital flow data, the top half of Table 1 shows that the private signals model generates a volatility of capital flows and a correlation between inflows and outflows that is broadly consistent with the data. The model and the data show similar values for the standard deviation of capital inflows and outflows (about 3%) and of gross capital flows (just below 6%). Net capital flows are a bit more volatile in the data (0.7% versus 0.5% in the model), and the correlation between capital inflows and outflows is somewhat lower in the data, albeit still very high (0.89 versus 0.99 in the model).

The bottom half of Table 1 is based on unfiltered capital flows from the model, and illustrates the role of private information. There is a sizable disconnect between capital flows and public information as these account for only half of the variance of capital flows. The model also generates a substantial information content of both gross and net capital flows. One can measure this as the  $R^2$  of a regression of  $\varepsilon_{t+1}^A$  and  $\varepsilon_{t+1}^D$  on respectively gross and net flows. This gives respectively 0.5 and 0.34. Note that publicly observed fundamentals  $S_t$  in the private information model have no explanatory power for  $\varepsilon_{t+1}^A$  and  $\varepsilon_{t+1}^D$ , so this is entirely a result of private information.

These results from the private signals model stand in sharp contrast to those from the standard and public signals models. First, while the volatility of capital flows is higher in the public signals model than in the standard model, it is much higher in the presence of private information, especially for gross flows. Second,

the standard and public signals model generate a strongly negative correlation between capital inflows and outflows. This largely reflects the time-variation in the expected excess return, as a higher expected excess return on Home assets raises capital inflows and reduces capital outflows. This has been a source of criticism by Broner et.al. (2013) of capital flow models with public information. By contrast, the large positive correlation between outflows and inflows in the private signals model is due to the first term in equations (58) and (59). The disagreement between Home and Foreign agents about expected returns can lead Home agents to switch to Foreign assets and Foreign agents to Home assets, which generates the positive co-movement between capital inflows and outflows.

Third, capital flows are not disconnected from public information in the standard and public signals models. Finally, they have no information content about future fundamentals in these two models. By information content we refer to an increase in the  $R^2$  when adding capital flows to regressions of the future fundamentals on current publicly observed fundamentals. In the public signals model capital flows have predictive power for  $\varepsilon_{t+1}^A$  and  $\varepsilon_{t+1}^D$ , but this is only through publicly observed fundamentals  $v_t^D$  and  $v_t^A$ .

Table 2 sheds further light on the difference between the private signals model and the other two versions with public information. It breaks down the volatility of capital flows associated with the 4 components of capital flows in equations (58) and (59). It is clear from Table 2 that the high capital flow volatility in the private signals model is driven by the cross-country difference in expected excess returns, a term that is zero in the models with public information. While net capital flows are not affected by this difference, they are nonetheless more volatile in the private signals model due to volatility in portfolio growth and the average expected excess return.

The results show that private information can potentially have a very large impact on international capital flows in terms of their volatility, correlation, disconnect from observables and predictive content. However, we want to emphasize again that this is only an illustration specific to our simple model, and one should not draw any conclusions about the quantitative impact of private information on capital flows more generally. We discuss some ways to quantify this impact empirically in the conclusion.

### 6.3 Sensitivity to Dispersed Information Parameters

We now consider the sensitivity of our results to the three parameters related to private information, namely information dispersion  $(\sigma_{HH} + \sigma_{HF})/2$  (depicted in Figure 2), information asymmetry  $\sigma_{HF}/\sigma_{HH}$  (depicted in Figure 3) and the volatility of noise shocks  $\theta$  (depicted in Figure 4). We vary each of these over a very wide range: information dispersion  $(\sigma_{HH} + \sigma_{HF})/2$  from 0 to 2, the information asymmetry  $\sigma_{HF}/\sigma_{HH}$  from 1 to 2, and the volatility of noise shocks  $\theta$  from 1 to 1000. Each figure displays the standard deviation of gross capital flows (panel A), the standard deviation of net capital flows (panel B), the correlation between capital inflows and outflows (panel C), the extent of disconnect in capital flows, measured by the share of the variance of flows that is explained by unobserved fundamentals (panel D), and the predictive content of capital flows, measured by the  $R^2$  in a regression of  $\varepsilon_{t+1}^D$  and  $\varepsilon_{t+1}^A$  on net and gross capital flows, respectively (panel E).

#### Information dispersion

Private signals carry little information when the variance of the signal errors becomes very high. This significantly reduces the volatility of both gross and net capital flows (Figure 2, panels A and B). The correlation between capital inflows and outflows goes down as the private information becomes weaker (panel C). But even when  $(\sigma_{HH} + \sigma_{HF})/2$  is equal to 2, which is ten times as high as estimated, the correlation remains positive at 0.46.

Not surprisingly, as private signals become weaker the disconnect of capital flows from observed fundamentals (panel D) and predictive power for future fundamentals (panel E) become weaker. This decline is more acute for net capital flows than for gross capital flows. Gross capital flows depend on  $\varepsilon_t^A - \varepsilon_{t+1}^A$  through an information asymmetry across the countries. As private signals become weaker, it weakens the impact of both the observed fundamental  $\varepsilon_t^A$  and the future fundamental  $\varepsilon_{t+1}^A$ . As the weight of the unobserved future fundamental  $\varepsilon_{t+1}^A$  in capital flows declines, therefore so does the weight of the observed fundamental  $\varepsilon_t^A$ .

#### International Information Asymmetry

The extent of asymmetric information across countries  $\sigma_{HF}/\sigma_{HH}$  has little impact on net capital flows (Figure 3). We therefore focus our discussion on gross capital flows. It is through this information asymmetry that  $\varepsilon_{t+1}^A$  affects gross



capital flows in our model. Not surprising therefore, the volatility of gross capital flows depends critically on the extent of this information asymmetry (panel A). Information asymmetry also matters for the correlation between capital inflows and outflows (panel C). As a result of the information asymmetry capital inflows and outflows both drop when  $\varepsilon_{t+1}^A$  rises, as agents retrench towards domestic assets about which they have more information and are therefore more optimistic. But we only need a small amount of information asymmetry to generate a high correlation between capital inflows and outflows, with the correlation reaching 0.8 even if we set  $\sigma_{HF}/\sigma_{HH}$  as low as 1.1 (rather than the 1.5 in the benchmark).

The extent of information asymmetry has very little effect on the extent of disconnect (panel D) and the predictive content of gross capital flows (panel E). A lower value of  $\sigma_{HF}/\sigma_{HH}$ , implying less information asymmetry across countries, reduces the impact of both  $\varepsilon_{t+1}^A$  and  $\varepsilon_t^A$  on gross capital flows. It proportionally reduces the impact of both observed and unobserved state variables.

### Noise

A lower volatility of noise shocks  $\theta$  reduces the information asymmetry across countries as the relative asset price then contains more information about relative future productivity. This in turn reduces the impact of  $\varepsilon_{t+1}^A$  on gross capital flows and therefore its volatility (Figure 4, panel A). The volatility of net flows by contrast is not much affected by the noise (panel B). There are two offsetting effects. On the one hand more noise implies that the relative asset price is more affected by the noise. On the other hand, the noise reduces the information content of the relative price, which limits the extent to which private information about  $\varepsilon_{t+1}^D$  is aggregated into the relative price. So, while one unobserved fundamental ( $\varepsilon_t^\tau$ ) generates more volatility of the relative asset price and net capital flows when  $\theta$  rises, another unobserved fundamental ( $\varepsilon_{t+1}^D$ ) does the opposite.

The correlation between inflows and outflows is insensitive to the noise (panel C). The noise also has little impact on the extent of disconnect in gross and net capital flows (panel D). While the noise has little impact on the predictive content of gross capital flows (panel E), it affects the predictive content of net flows. More noise implies a larger role for  $\varepsilon_t^\tau$  relative to  $\varepsilon_{t+1}^D$ , which lowers the predictive content of net capital flows.

## 7 Conclusion

We investigate the impact of dispersed information on international capital flows within the context of a DSGE two-country model. We show that it increases the volatility of both gross and net capital flows, generates a large positive correlation between capital inflows and outflows, leads to a disconnect between capital flows and observed macro fundamentals, and makes capital flows a relevant source of information about future macro fundamentals. Our calibration exercise shows that all of these effects can be quantitatively large. In addition, we show that dispersed private information can also increase the impact of observed macro fundamentals on capital flows.

While we illustrate that the impact of dispersed information on international capital flows can be potentially large, we do not empirically estimate this aspect. Assessing the magnitude of this impact will be an important topic for future work. We see a number of possible ways to approach this. A first approach is to evaluate the extent of disconnect by regressing capital flows on observed (current and past) macro fundamentals. Similarly, one can conduct Granger causality tests to evaluate the predictive content of capital flows, controlling for current and past macro fundamentals.<sup>25</sup>

An important drawback of this approach is that it is hard to distinguish between private signals and public signals that are omitted by the econometrician. An alternative approach is to directly measure the impact of private information on capital flows by using data on order flow, which is known to aggregate all private information in the market. This has been the approach followed by Evans and Lyons (2002) for exchange rates. One can for example use high frequency order flow data from equity and FX markets and consider the explanatory power for high frequency capital flow data, such as weekly data from State Street Corporation of institutional agent flows.

A final approach is to consider a more extensive calibration exercise in order to shed light on a wide range of reported stylized facts associated with international capital flows. We have made many simplifying assumptions for the sake of tractability and transparency. Natural extensions include relaxing the OLG

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<sup>25</sup>In a previous draft of this paper we took a very preliminary shot at this approach, finding evidence of substantial disconnect for both gross and net capital flows and predictive content of particularly gross capital flows for the future world profit rate.

assumption, introducing other assets, different types of private information (including private information about future labor income) and non-log utility. These lead to additional channels through which dispersed private information affects capital flows. Ultimately this should lead to better insight into the quantitative role that private information plays in accounting for various aspects of gross and net capital flow data.

## Appendix A: Signal Extraction

This Appendix discusses signal extraction in the private information model. Home agent  $j$  forms her expectation of innovations by relying on three sources of information: the asset price (more precisely its component  $h_t$ , which only has a first-order component), her private signals ( $v_{j,t}^{H,H}$  and  $v_{j,t}^{H,F}$ , which have zero- and first-order components), and the unconditional distribution of innovations. We denote the vectors of productivity innovations by  $\xi_{t+1} = (\varepsilon_{H,t+1}, \varepsilon_{F,t+1})'$ , the vector of Home agent  $j$ 's signals by  $Y_t^{j,H} = (h_t, v_{j,t}^{H,H}, v_{j,t}^{H,F}, 0, 0)'$ , and the vector of shocks on signals by  $v_t^{j,H} = (\lambda\tau_t^D/\tau, \epsilon_{j,t}^{H,H}, \epsilon_{j,t}^{H,F}, -\varepsilon_{t+1}^H, -\varepsilon_{t+1}^F)'$ . Signals are linear functions of productivity innovations and shocks:

$$Y_t^{j,H} = X^{j,H}\xi_{t+1} + v_t^{j,H}$$

where  $X^{j,H}$  is a matrix with zeros and ones.<sup>26</sup>  $R^{j,H}$  is diagonal variance-covariance matrix of the shocks  $v_t^{j,H}$  with  $\text{diag}(R^{j,H}) = (4\lambda^2\theta\sigma_a^2, \sigma_{H,H}^2, \sigma_{H,F}^2, \sigma_a^2, \sigma_a^2)'$ . Home agent  $j$ 's expectation of innovations are given by:

$$\begin{aligned} E_t^{j,H}\xi_{t+1} &= \left[ (X^{j,H})' (R^{j,H})^{-1} (X^{j,H}) \right]^{-1} (X^{j,H})' (R^{j,H})^{-1} Y_t^{j,H} \\ &= \begin{pmatrix} \alpha_{\varepsilon H,h}^{j,H} & \alpha_{\varepsilon H,vH}^{j,H} & \alpha_{\varepsilon H,vF}^{j,H} \\ \alpha_{\varepsilon F,h}^{j,H} & \alpha_{\varepsilon F,vH}^{j,H} & \alpha_{\varepsilon F,vF}^{j,H} \end{pmatrix} \begin{pmatrix} h_t \\ v_{j,t}^{H,H} \\ v_{j,t}^{H,F} \end{pmatrix} \end{aligned}$$

where the coefficients  $\alpha$  are functions of the variance of innovations  $\sigma_a^2$  and the dispersion of private signals  $\sigma_{H,H}^2$  and  $\sigma_{H,F}^2$ . These coefficients therefore do not merely consist of zero-order terms, but include terms of higher order that are computed from a Taylor expansion with respect to  $\sigma_a$  as in equation (31). The coefficients on  $h_t$  have zero- and second-order components, while the coefficients on the private signals only have second-order components.<sup>27</sup>

The expectations of a Foreign agent are constructed following similar steps, and the coefficients on the various signals are similar to the ones for a Home agent:

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<sup>26</sup>Specifically:  $X^{j,H} = \begin{pmatrix} 1 & 1 & 0 & 1 & 0 \\ -1 & 0 & 1 & 0 & 1 \end{pmatrix}'$ .

<sup>27</sup>They also have fourth- and higher order components, but we are not interested in these.

$$\alpha_{\varepsilon_{H,h}}^{j,F} = -\alpha_{\varepsilon_{F,h}}^{j,H}, \quad \alpha_{\varepsilon_{F,h}}^{j,F} = -\alpha_{\varepsilon_{H,h}}^{j,H}, \quad \alpha_{\varepsilon_{H,vH}}^{j,F} = \alpha_{\varepsilon_{F,vF}}^{j,H}, \quad \alpha_{\varepsilon_{H,vF}}^{j,F} = \alpha_{\varepsilon_{F,vH}}^{j,H}, \quad \alpha_{\varepsilon_{F,vH}}^{j,F} = \alpha_{\varepsilon_{H,vF}}^{j,H}, \quad \text{and } \alpha_{\varepsilon_{F,vF}}^{j,H} = \alpha_{\varepsilon_{H,vH}}^{j,F}.$$

We combine the order components of signals and coefficients to compute the various order components of the expected innovations. The first-order component of expectations depends on  $h_t$  times the zero-order components of its coefficients. As it is associated with information from the publicly observed asset price, it is the same for all agents:  $\left[ E_t^{j,H} \varepsilon_{t+1}^A \right] (1) = \left[ E_t^{j,F} \varepsilon_{t+1}^A \right] (1) = 0$  and  $\left[ E_t^{j,H} \varepsilon_{t+1}^D \right] (1) = \left[ E_t^{j,F} \varepsilon_{t+1}^D \right] (1) = (1 + 2\lambda^2\theta)^{-1} h_t$ .

The second-order component of expectations reflects the zero-order components of private signals (the idiosyncratic elements of signals) and the second-order components of the coefficients on private signals. They differ across agents, but average to zero in each country:

$$\begin{aligned} \left[ E_t^{j,H} \varepsilon_{t+1}^A \right] (2) &= \left( \frac{\varepsilon_{j,t}^{H,H}}{\sigma_{H,H}^2} + \frac{\varepsilon_{j,t}^{H,F}}{\sigma_{H,F}^2} \right) \frac{\sigma_a^2}{2} \quad ; \quad \left[ E_t^{j,F} \varepsilon_{t+1}^A \right] (2) = \left( \frac{\varepsilon_{j,t}^{F,H}}{\sigma_{H,F}^2} + \frac{\varepsilon_{j,t}^{F,F}}{\sigma_{H,H}^2} \right) \frac{\sigma_a^2}{2} \\ \left[ E_t^{j,H} \varepsilon_{t+1}^D \right] (2) &= \left( \frac{\varepsilon_{j,t}^{H,H}}{\sigma_{H,H}^2} - \frac{\varepsilon_{j,t}^{H,F}}{\sigma_{H,F}^2} \right) \frac{2\lambda^2\theta}{1 + 2\lambda^2\theta} \sigma_a^2 \\ \left[ E_t^{j,F} \varepsilon_{t+1}^D \right] (2) &= \left( \frac{\varepsilon_{j,t}^{F,H}}{\sigma_{H,F}^2} - \frac{\varepsilon_{j,t}^{F,F}}{\sigma_{H,H}^2} \right) \frac{2\lambda^2\theta}{1 + 2\lambda^2\theta} \sigma_a^2 \end{aligned}$$

Finally, the third-order component of expectations reflects the first-order components of private signals (the true productivity innovations) and the second-order components of the coefficients on private signals. They are the same for all agents in a given country:

$$\begin{aligned} \left[ E_t^{j,H} \varepsilon_{t+1}^D \right] (3) &= \left[ \frac{\varepsilon_{H,t+1}}{\sigma_{H,H}^2} - \frac{\varepsilon_{F,t+1}}{\sigma_{H,F}^2} - \frac{\sigma_{H,H}^2 + \sigma_{H,F}^2}{\sigma_{H,H}^2 \sigma_{H,F}^2} \frac{1}{2(1 + 2\lambda^2\theta)} h_t \right] \frac{2\lambda^2\theta}{1 + 2\lambda^2\theta} \sigma_a^2 \\ \left[ E_t^{j,H} \varepsilon_{t+1}^A \right] (3) &= \left[ \frac{\varepsilon_{H,t+1}}{\sigma_{H,H}^2} + \frac{\varepsilon_{F,t+1}}{\sigma_{H,F}^2} - \frac{\sigma_{H,F}^2 - \sigma_{H,H}^2}{\sigma_{H,H}^2 \sigma_{H,F}^2} \frac{1}{2(1 + 2\lambda^2\theta)} h_t \right] \frac{\sigma_a^2}{2} \\ \left[ E_t^{j,F} \varepsilon_{t+1}^D \right] (3) &= \left[ \frac{\varepsilon_{H,t+1}}{\sigma_{H,F}^2} - \frac{\varepsilon_{F,t+1}}{\sigma_{H,H}^2} - \frac{\sigma_{H,H}^2 + \sigma_{H,F}^2}{\sigma_{H,H}^2 \sigma_{H,F}^2} \frac{1}{2(1 + 2\lambda^2\theta)} h_t \right] \frac{2\lambda^2\theta}{1 + 2\lambda^2\theta} \sigma_a^2 \\ \left[ E_t^{j,F} \varepsilon_{t+1}^A \right] (3) &= \left[ \frac{\varepsilon_{H,t+1}}{\sigma_{H,F}^2} + \frac{\varepsilon_{F,t+1}}{\sigma_{H,H}^2} + \frac{\sigma_{H,F}^2 - \sigma_{H,H}^2}{\sigma_{H,H}^2 \sigma_{H,F}^2} \frac{1}{2(1 + 2\lambda^2\theta)} h_t \right] \frac{\sigma_a^2}{2} \end{aligned}$$

Equation (46) in the text follows immediately.

## Appendix B: Saving, Investment and Capital flows

In this Appendix we compute the equation (50) for outflows. The steps leading to equation (51) for inflows are similar. In period  $t$  the old Home agents enter the period with the following quantities of Home and Foreign assets (we abstract from  $j$  indexes as we focus on first-order aggregates):

$$G_{H,t-1}^H = \frac{z_{H,t-1} (W_{H,t-1} - C_{y,t-1}^H)}{Q_{H,t-1}} \quad ; \quad G_{F,t-1}^H = \frac{(1 - z_{H,t-1}) (W_{H,t-1} - C_{y,t-1}^H)}{Q_{F,t-1}}$$

The savings by young agents in period  $t$  is:  $S_{y,t}^H = W_{H,t} - C_{y,t}^H$ . The consumption of old agents is the total return on their portfolio. Their income is the dividend stream they receive (capital gains are not counted as income streams in national accounts). We consider income is net of depreciation. The savings of the old Home agents are then (we ignore the iceberg cost as it is second order and we focus on first-order flows):

$$\begin{aligned} S_{o,t}^H &= [(1 - \omega) A_{Ht} (K_{H,t})^{-\omega} - \delta Q_{H,t}] G_{H,t-1}^H + [(1 - \omega) A_{Ft} (K_{F,t})^{-\omega} - \delta Q_{F,t}] G_{F,t-1}^H \\ &\quad - (R_{H,t} Q_{H,t-1} G_{H,t-1}^H + R_{F,t} Q_{F,t-1} G_{F,t-1}^H) \\ &= - \left[ z_{H,t-1} \frac{Q_{H,t}}{Q_{H,t-1}} + (1 - z_{H,t-1}) \frac{Q_{F,t}}{Q_{F,t-1}} \right] (W_{H,t-1} - C_{y,t-1}^H) \end{aligned}$$

The dissaving by old agents reflects the liquidation value of their portfolio. National savings in the Home country  $S_t^H = S_{y,t}^H + S_{o,t}^H$  are:

$$S_t^H = W_{H,t} - C_{y,t}^H - \left[ z_{H,t-1} \frac{Q_{H,t}}{Q_{H,t-1}} + (1 - z_{H,t-1}) \frac{Q_{F,t}}{Q_{F,t-1}} \right] (W_{H,t-1} - C_{y,t-1}^H)$$

Taking a linear Taylor expansion, the first-order component of national Home savings is:

$$s_t^H(1) = \Delta a_{H,t}(1) + (1 - \omega) \Delta k_{H,t}(1) - z_H(0) \Delta q_{H,t}(1) - (1 - z_H(0)) \Delta q_{F,t}(1)$$

where  $s_t^H(1) = S_t^H(1) / W^F(0)$ ,  $W^F(0) = \beta(1 + \beta)^{-1} W_H(0)$  is the zero-order component of financial wealth,  $w_{H,t}(1) = W_{H,t}(1) / W^F(0)$ . Following similar steps, Foreign saving is:

$$s_t^F(1) = \Delta a_{F,t}(1) + (1 - \omega) \Delta k_{F,t}(1) - (1 - z_H(0)) \Delta q_{H,t}(1) - z_H(0) \Delta q_{F,t}(1)$$

Taking the average and cross-country difference of these Home and Foreign saving expressions gives equations (60) and (61).

Investment is defined as net of depreciation, so that Home investment is:

$$I_{H,t}^{net} = I_{H,t} - \delta K_{H,t} = K_{H,t+1} - K_{H,t}$$

which in terms of a linear approximation is  $i_{H,t}^{net}(1) = \Delta k_{H,t+1}(1)$ , from which equation (62) follows immediately using the Tobin Q relation.

Gross capital outflows from the Home country are the change in the value of cross-border asset holdings, evaluated at current asset prices:

$$\begin{aligned} OUTFLOWS_t &= Q_{F,t} (G_{F,t}^H - G_{F,t-1}^H) \\ &= (1 - z_{H,t}) (W_{H,t} - C_{y,t}^H) - \frac{Q_{F,t}}{Q_{F,t-1}} (1 - z_{H,t-1}) (W_{H,t-1} - C_{y,t-1}^H) \end{aligned}$$

The first-order component of gross outflows is obtained from a linear Taylor approximation:

$$\begin{aligned} outflows_t(1) &= -\Delta z_{H,t}(1) + (1 - z_H(0)) [\Delta a_{H,t}(1) + (1 - \omega) \Delta k_{H,t}(1)] \\ &\quad - (1 - z_H(0)) \Delta q_{F,t}(1) \end{aligned}$$

where:  $outflows_t(1) = OUTFLOWS_t(1) / W^F(0)$ . Using the expression for savings and equation (52) we write:

$$outflows_t(1) = (1 - z_H(0)) s_t^H(1) - [\Delta z_{H,t}(1) - \Delta \tilde{z}_t(1)]$$

## Appendix C: Calibrating Information Dispersion

In this Appendix we provide some details regarding the calibration of dispersed information about future fundamentals. We set the average dispersion of private signals,  $(\sigma_{HH} + \sigma_{HF})/2$ , to generate a cross-sectional dispersion of expected asset price changes that matches the evidence from surveys of forecasters.

For this purpose we use a survey from the International Center for Finance at the Yale School of Management that reports expected stock price changes by

a large number of financial institutions.<sup>28</sup> The survey has data for two countries, the United States and Japan. For both countries the survey asks about expected percentage change in the stock price (respectively Dow Jones Industrial Index and Nikkei Dow) over the next 1, 3, and 12 months, with our parameterization focusing on the 1-year ahead forecasts.

For each country the survey is based on about 400 financial institutions. For Japan the survey is mailed to most of the major financial institutions, including 165 banks, 46 insurance companies, 113 security companies and 45 investment trust companies. For the U.S. about 400 randomly drawn institutions are selected from “Investment Managers” in the “Money Market Directory of Pension Funds and their Investment Managers”.

The survey starts in 1989 with six-month interval surveys until 1998, after which monthly surveys are conducted.<sup>29</sup> We have collected the data through October 2004.

Since it is important to compare expectations at the same point in time, and financial institutions do not all respond to the survey on the same day, we only consider the cross sectional distribution of responses that take place on the same day. Moreover, we eliminate days where there were fewer than 5 responses.

The average cross-sectional standard deviation of the expected one-year percentage stock price change across respondents is 0.1278 for the U.S. and 0.1341 for Japan. This is scaled by the variance of stock price changes. Here we use historical numbers of the standard deviation of stock price changes from Jorion and Goetzmann (1999), which are respectively 0.1584 and 0.1579 for the U.S. and Japan. Our scaled measure of dispersion of expected stock price changes is then 4.99 for the U.S. and 5.23 for Japan.

In the model this scaled measure of dispersion of expected stock price changes is the standard deviation of  $E_t^{Hj} q_{t+1}^H$  across agents, divided by the unconditional variance of  $\Delta q_t^H$ . We set  $(\sigma_{HH} + \sigma_{HF})/2 = 0.21$ , which leads to a scaled measure of dispersion of expected stock price changes of 5.0, close to that for both the U.S. and Japan.

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<sup>28</sup>We would like to thank the International Center for Finance for making these data available to us.

<sup>29</sup>See Shiller et al. (1996) and <http://icf.som.yale.edu/confidence.index/explanations.html> for more details.



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Table 1: Capital Flow Moments

	Data	Model		
		Private Signals	Standard	Public Signals
	Standard Deviations			
capital outflows	2.89	2.99	0.05	0.15
capital inflows	2.99	2.99	0.05	0.15
gross capital flows	5.78	5.96	0.02	0.07
net capital flows	0.96	0.46	0.09	0.29
current account	0.70	0.46	0.09	0.29
	Correlation			
capital inflows and outflows	0.89	0.99	-0.90	-0.90
	Share of Variance Explained by Dispersed Information			
capital outflows	-	0.50	0	0
capital inflows	-	0.50	0	0
gross capital flows	-	0.50	0	0
net capital flows	-	0.40	0	0
	Information Content Capital Flows			
$R^2$ of regression of $\varepsilon_{t+1}^A$ on gross flows	-	0.50	0	0
$R^2$ of regression of $\varepsilon_{t+1}^D$ on net flows	-	0.34	0	0

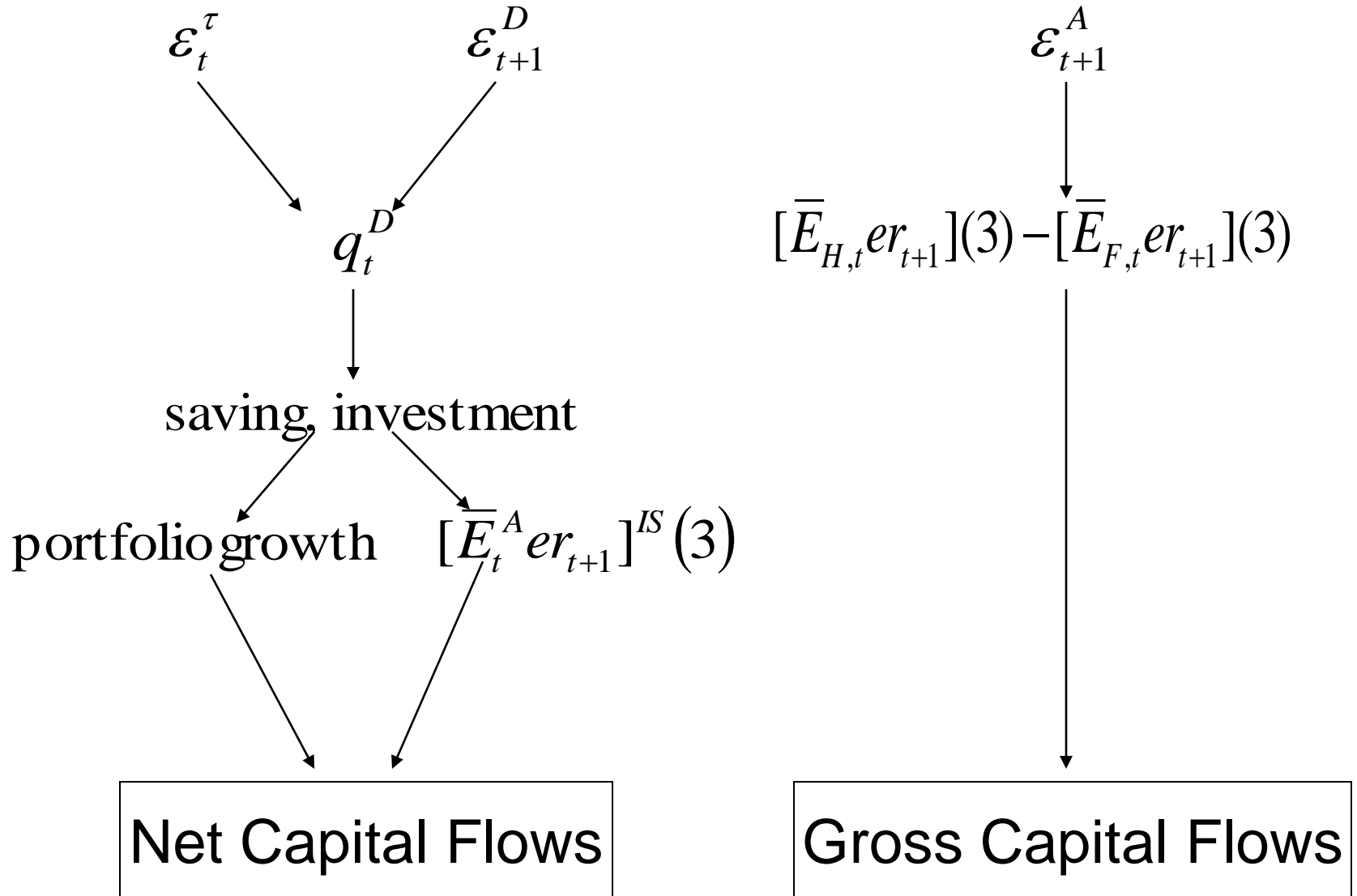
Note: The top half shows the standard deviation of various measures of capital flows, as well as the correlation between gross outflows and inflows, in the data (column 1) and in the three variants of the model (columns 2-4). Gross capital flows are defined as capital outflows plus capital inflows. Net capital flows are capital outflows minus capital inflows. All capital flow data are from the IMF International Financial Statistics (IFS). They are multiplied by the exchange rate to convert to the local currency and then divided by GDP (also from the IFS). These scaled capital flows are HP(10) filtered. The corresponding moments in the three versions of the model are based on the first-order component of capital flows divided by GDP, which is HP(10) filtered. The bottom half presents moments for the three variants of the model that are based on the unfiltered first-order components of capital flows. The  $R^2$  reported in the bottom two rows refers to the increase in the  $R^2$  when adding capital flows to regressions of  $\varepsilon_{t+1}^D$  and  $\varepsilon_{t+1}^A$  on variables in the public information sets of the various models.

Table 2: Decomposition of Capital Flow Volatility

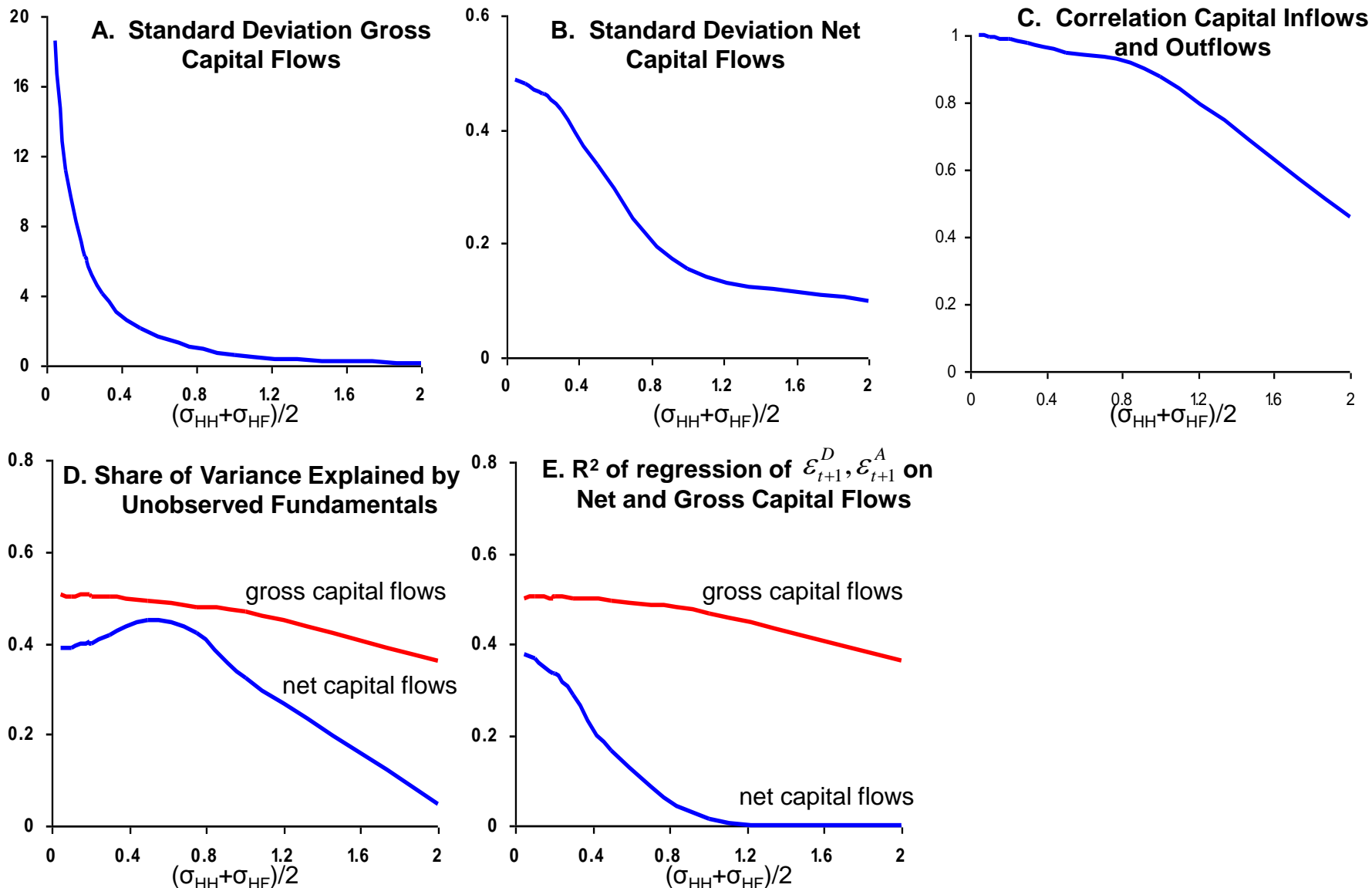
	portfolio growth	time-varying risk	average expected return	difference expected return across countries
Private Signals Model				
capital outflows	0.06	0.02	0.17	3.00
gross capital flows	0.02	0.04	0	6.01
net capital flows	0.12	0	0.34	0
Standard Model				
capital outflows	0.04	0.04	0.01	0
gross capital flows	0.02	0.08	0	0
net capital flows	0.07	0	0.24	0
Public Signals Model				
capital outflows	0.02	0	0.03	0
gross capital flows	0.02	0	0	0
net capital flows	0.04	0	0.06	0

Note: The table shows the contributions of the four drivers of capital flows to capital flow volatility, measured as the standard deviation after applying an HP(10) filter. The first component is due to portfolio growth. The second component is due to time-varying risk. The third is associated with changes in the averaged expected return related to changes in saving and investment. The last component is due to the cross-country difference in the average expected excess return. The results are shown for the private signals model, the standard model and the public signals model.

# Figure 1 Role of Information Dispersion



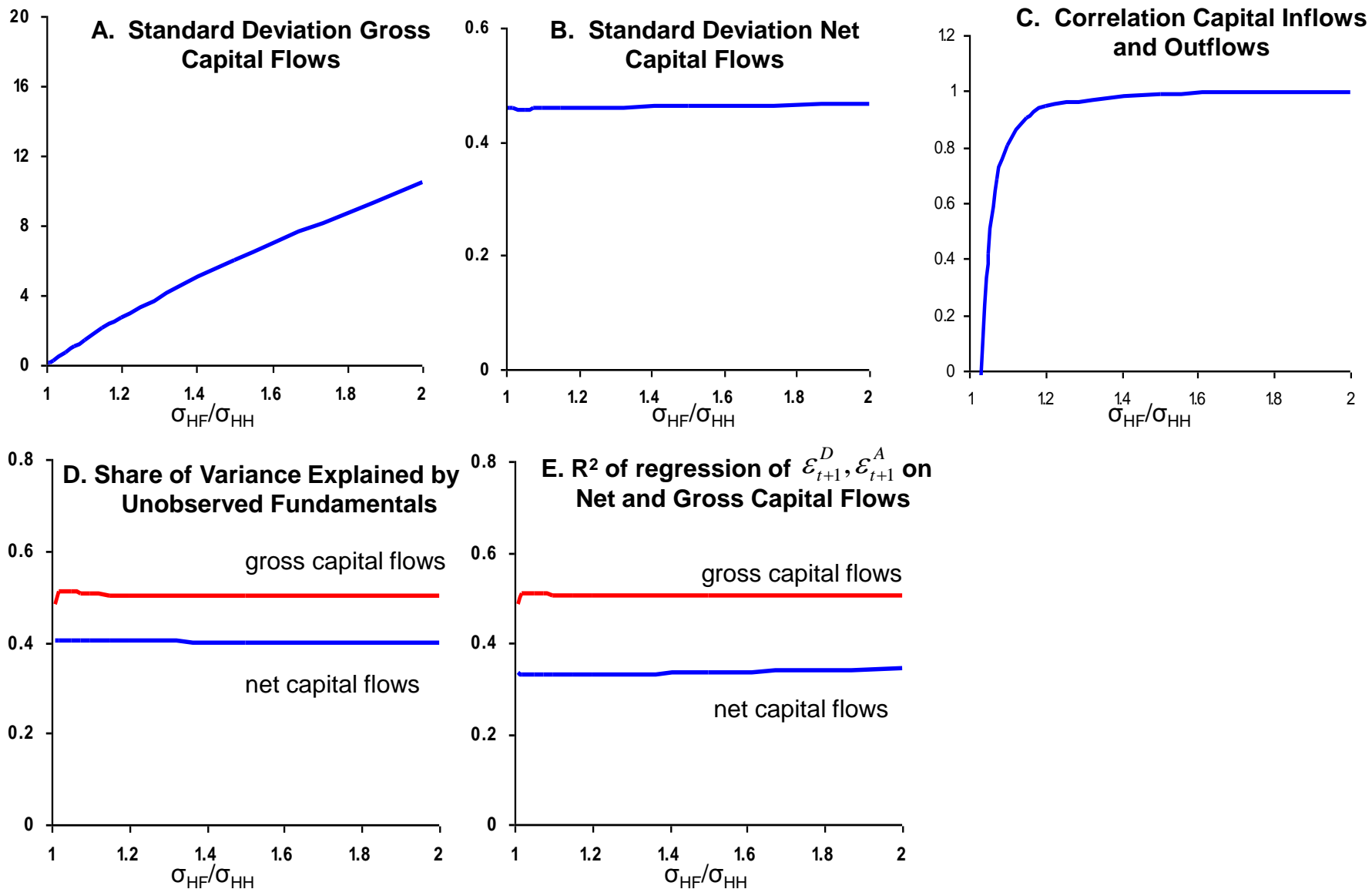
# Figure 2 Model Simulation: Information Dispersion



Gross Capital flows=capital outflows+capital inflows; Net capital flows=capital outflows-capital inflows. Results are based on a simulation of the model over 100,000 periods.

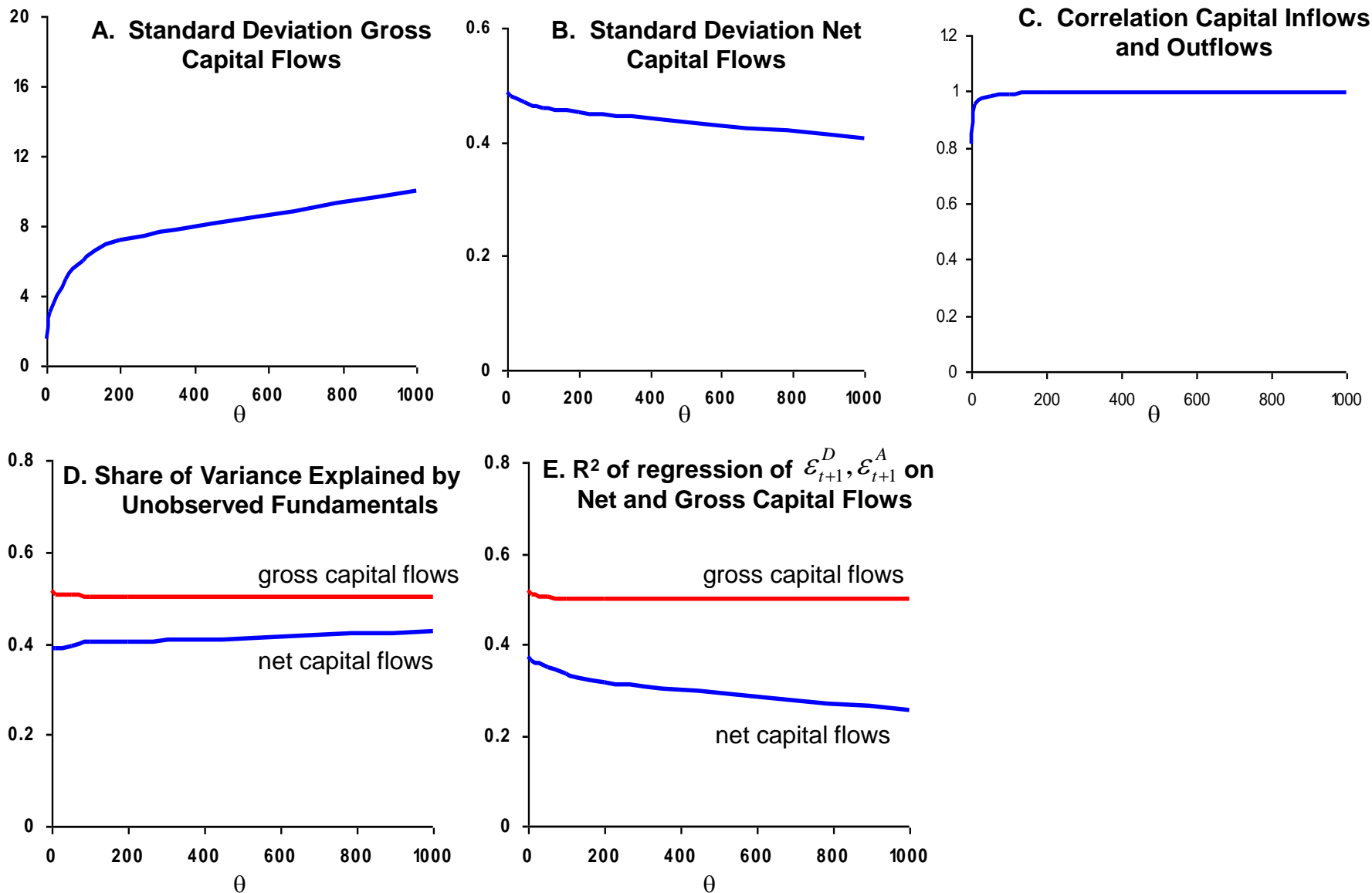


# Figure 3 Model Simulation: Information Asymmetry



Gross Capital flows=capital outflows+capital infows; Net capital flows=capital outflows-capital infows. Results are based on a simulation of the model over 100,000 periods.

# Figure 4 Model Simulation: Noise



Gross Capital flows=capital outflows+capital inflows; Net capital flows=capital outflows-capital inflows. Results are based on a simulation of the model over 100,000 periods.