Electron scattering from nuclei in the quasielastic region and beyond

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Part 2

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Nuclear Response Function

\[ Q^2 = \vec{q}^2 - \nu^2 \]

\[ \nu = (E - E') \quad (\nu \equiv \omega) \]
Inclusive Electron Scattering from Nuclei

Two distinct processes

Quasielastic from the nucleons in the nucleus

Inelastic and DIS from the quark constituents of the nucleon.

Inclusive final state means no separation of two dominant processes

\[ x = \frac{Q^2}{2m_\nu} \]

\( \nu, \omega = \text{energy loss} \)
The two processes share the same initial state

**QES in IA**

\[
\frac{d^2\sigma}{dQd\nu} \propto \int dk \int dE \sigma_{ei} S_i(k, E) \delta() \]

**DIS**

\[
\frac{d^2\sigma}{dQd\nu} \propto \int dk \int dE W_{1,2}^{(p,n)} S_i(k, E) \]

However they have very different \(Q^2\) dependencies

\(\sigma_{ei} \propto \text{elastic (form factor)}^2\)

\(W_{1,2} \text{ scale with } \ln Q^2 \text{ dependence}\)

There is a rich, if complicated, blend of nuclear and fundamental QCD interactions available for study from these types of experiments.

The limits on the integrals are determined by the kinematics. Specific \((x, Q^2)\) select specific pieces of the spectral function.
Early 1970’s Quasielastic Data

→ getting the bulk features

$\vec{q} \approx 500 \text{MeV}/c$

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>$k_F$</th>
<th>$\bar{\epsilon}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^6\text{Li}$</td>
<td>169</td>
<td>17</td>
</tr>
<tr>
<td>$^{12}\text{C}$</td>
<td>221</td>
<td>25</td>
</tr>
<tr>
<td>$^{24}\text{Mg}$</td>
<td>235</td>
<td>32</td>
</tr>
<tr>
<td>$^{40}\text{Ca}$</td>
<td>251</td>
<td>28</td>
</tr>
<tr>
<td>nat $\text{Ni}$</td>
<td>260</td>
<td>36</td>
</tr>
<tr>
<td>$^{89}\text{Y}$</td>
<td>254</td>
<td>39</td>
</tr>
<tr>
<td>nat $\text{Sn}$</td>
<td>260</td>
<td>42</td>
</tr>
<tr>
<td>$^{181}\text{Ta}$</td>
<td>265</td>
<td>42</td>
</tr>
<tr>
<td>$^{208}\text{Pb}$</td>
<td>265</td>
<td>44</td>
</tr>
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</table>

compared to Fermi model: fit parameter $k_F$ and $\bar{\epsilon}$

• The shape of the low $\nu$ cross section is determined by the momentum distribution of the nucleons.
• As $Q^2 \gg$ inelastic scattering from the nucleons begins to dominate
• We can use $x$ and $Q^2$ as knobs to dial the relative contribution of QES and DIS.

The quasielastic peak (QE) is broadened by the Fermi-motion of the struck nucleon.

The quasielastic contribution dominates the cross section at low energy loss ($\nu$) even at moderate to high $Q^2$. 

$^3$He SLAC (1979)
A dependence: higher internal momenta broadens the peak.
Scaling

- Scaling refers to the dependence of a cross section, in certain kinematic regions, on a single variable. If the data scales in the single variable then it validates the assumptions about the underlying physics and scale-breaking provides information about conditions that go beyond the assumptions.

- At moderate $Q^2$ inclusive data from nuclei has been well described in terms $y$-scaling, one that arises from the assumption that the electron scatters from quasi-free nucleons.

- We expect that as $Q^2$ increases we should see for evidence ($x$-scaling) that we are scattering from a quark that has obtained its momenta from interactions with partons in other nucleons. These are super-fast quarks.
Galileo realized that if one simply scaled up an animal's size, its weight would increase significantly faster than its strength, "...you can plainly see the impossibility of increasing the size of structures to vast dimensions...if his height be increased inordinately, he will fall and be crushed under his own weight"

\[
\frac{\text{Strength}}{\text{Weight}} \propto \frac{A}{V} \propto \frac{1}{l} \propto \frac{1}{W^{1/3}}
\]

Smaller animals appear stronger

Explains why small animals can leap as high as large one ...

G. West, LANL report
Metabolism

- How does the metabolic rate ($B$) vary from animal to animal?
- $B = \text{heat lost by a body in steady inactive state}$
- Should be dominated by the surface affects of sweating and radiation

$$B \propto W^{2/3}$$

Note that best fit slope is $\approx 3/4$
Something other than pure geometry is playing a role

Deviations from the geometrical or kinematic analysis reflects the dynamics of the system.

One can view deviations from naive scaling as a probe of the dynamics
Respiration

- Can we understand this?
- Pore length = thickness of shell suggests its strength $\propto W^{1/3}$.
- Conductance $\propto$ total pore area and $\propto 1/$pore length

Assume pore spacing the same from bird to bird, then the two factors go as Surface area ($W^{2/3}$) and $1/l$ ($1/W^{1/3}$)

$$K \propto \frac{W^{2/3}}{W^{1/3}} = W$$
Selecting the relevant variables

The Dace, a fresh water fish

Scaling and scaling violations reveal information about the dynamics of the system

Knut Schmidt-Nielsen, from Scaling: Why is Animal Size So Important?
Scaling in DIS

Existence of partons (quarks) revealed by DIS at SLAC in 1960's

Ratio of measured cross-section to pointlike prediction for the proton = form factor!

Invariant mass of the final hadronic state

“Scaling” – in this regime, the form factors are approximately equal and are almost independent of momentum transfer...

$Q^2 = -q^2 \gg M^2$ "deep" $\left( M \equiv M_p \right)$

$W^2 = (p+q)^2 \gg M^2$ "inelastic"
Scaling

Scaling Violations

Quarks AND Gluons
$F_2$ dominates cross-section

Range in $x$: 0.00001 – 1

Range in $Q^2$: $\sim 1$ – 30000 GeV$^2$

Measured with $\sim 2$–3% precision

Directly sensitive to sum of all quarks and anti-quarks

Indirectly sensitive to gluons via QCD radiation – scaling violations
\begin{equation}
F(y) = \frac{\sigma^{\text{exp}}}{(Z\tilde{\sigma}_p + N\tilde{\sigma}_n)} \cdot K
\end{equation}

\begin{equation}
\eta(k) = -\frac{1}{2\pi y} \frac{dF(y)}{dy}
\end{equation}

Assumption: scattering takes place from a quasi-free proton or neutron in the nucleus.

\( y \) is the momentum of the struck nucleon parallel to the momentum transfer:
\( y \approx -q/2 + mv/q \)
y-scaling in PWIA

\[
\frac{d^2\sigma}{dEdQ_{e'}} = \sum_{i=1}^{A} \int dk \int dE_s \sigma_{ei} S_i(E_s, k) \times \delta(\omega - E_s + M_A - (M_A^2 + \vec{k}^2)^{1/2} - (M_{A-1}^2 + \vec{k}^2)^{1/2}),
\]

\[
\frac{d^2\sigma}{dEdQ_{e'}} = 2\pi \sum_{i=1}^{A} \int_{E_{min}}^{E_{max}} dE_s \int_{k_{min}}^{k_{max}} dk k \sigma_{ei} S_i(E_s, k) \left| k \left( \frac{\partial \omega}{\partial \cos \theta_{kq}} \right) \right|^{-1}
\]

\[
\sigma_{ei} = f(q, \omega, \vec{k}, E_s)
\]

\[
E_{min} = M_{A-1} + M - M_A, \quad E_{max} = M^*_A - M_A \quad K = q/(M^2 + (\vec{k} + \vec{q})^2)^{1/2}
\]

\[
M^*_A = [(\omega + M_A)^2 - q^2]^{1/2}
\]

\[
k_{min} \text{ and } k_{max} \text{ are determined from } \cos \theta = \pm 1
\]

\[
\omega - E_s + M_A = (M^2 + q^2 + k^2 \pm 2kq)^{1/2} + (M_{A-1}^2 + k^2)^{1/2}
\]
y-scaling in PWIA

- lower limit becomes $y = y(q, \omega)$
- upper limits grows with $q$ and because momentum distributions are steeply peaked, can be replaced with $\infty$
- Assume $S(E_s, k)$ is isospin independent and neglect $E_s$ dependence of $\sigma_{ei}$ and kinematic factor $K$ and pull outside
- At very large $q$ and $\omega$, we can let $E_{max} = \infty$, and integral over $E_s$ can be done

$$n(k) = \int S(E_s, k) \, dE_s$$

Now we can write

$$\frac{d^2 \sigma}{dEd\Omega_{e'}} = (Z \bar{\sigma}'_{ep} + N \bar{\sigma}'_{en}) K' F(y)$$

where

$$F(y) = 2\pi \int_{|y|}^{\infty} n(k)k \, dk$$

Scaling (independent of $Q^2$) of QES provides direct access to momentum distribution
Assumptions & Potential Scale Breaking Mechanisms

- No FSI
- No internal excitation of (A-1)
- Full strength of Spectral function can be integrated over at finite q
- No inelastic processes
- No medium modifications
Assumption: scattering takes place from a quasi-free proton or neutron in the nucleus.

$y$ is the momentum of the struck nucleon parallel to the momentum transfer:

$$y \approx \frac{-q}{2} + \frac{m v}{q}$$
In nuclei the distribution of the strength in energy complicates the relationship between the scaling function and \( n(k) \).

The spectral function \( S(k,E) \) for \(^3\text{He}\).
As q increases, more and more of the spectral function $S(k,E)$ is integrated.

Is the energy distribution as calculated (scaling occurs at much lower q)?

Do other processes play a role?

FSI or/and DIS
Scaling of the response function shows up in a variety of disciplines. Scaling in inclusive neutron scattering from atoms provides access to the momentum distributions.

Momentum distributions are “distorted” by the presence of FSI. 

$y$-scaling as a test for presence of FSI

FSI have a $1/q$ dependence

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Convergence of \( F(y,q) \)

\[ 3^3\text{He} \]

\[ \text{He}^3, \ y = -0.2 \]
\[ \text{slope} = 0.12 \]

\[ 3^3\text{He} \]

\[ \text{He}^3, \ y = -0.4 \]
\[ \text{slope} = 5(-3) \]

\[ \text{Fe} \]

\[ \text{Fe}, \ y = -0.2 \]
\[ \text{slope} = 0.28 \]

\[ \text{Fe} \]

\[ \text{Fe}, \ y = -0.4 \]
\[ \text{slope} = 35(-3) \]
In (e,e') flux of outgoing protons strongly suppressed: 20–40% in C, 50–70% in Au

In (e,e') the failure of IA calculations to explain dσ at small energy loss

FSI has two effects: energy shift and a redistribution of strength

Benhar et al proposed approach based on NMBT and Correlated Glauber Approximation
Final State Interactions in CGA

Benhar et al. PRC 44, 2328
Benhar, Pandharipande, PRC 47, 2218
Benhar et al. PLB 3443, 47
Sensitivity to SRC increase with $Q^2$ and $x$

We want to be able to isolate and probe two-nucleon and multi-nucleon SRCs

Dotted = mean field approx.  
Solid = +2N SRCs.  
Dashed = +multi-nucleon.

11 GeV can reach $Q^2 = 20(13)$ GeV$^2$ at $x = 1.3(1.5)$  
- very sensitive, especially at higher $x$ values
CS Ratios and SRC

In the region where correlations should dominate, large $x$, $a_j(A)$ are proportional to finding a nucleon in a $j$-nucleon correlation. It should fall rapidly with $j$ as nuclei are dilute.

$$
\sigma_2(x, Q^2) = \sigma_{eD}(x, Q^2) \quad \text{and} \quad \sigma_j(x, Q^2) = 0 \quad \text{for} \quad x > j.
$$

$$
\Rightarrow \quad \frac{2 \sigma_A(x, Q^2)}{A \sigma_D(x, Q^2)} = a_2(A) \bigg|_{1 < x \leq 2}
$$

$$
\frac{3 \sigma_A(x, Q^2)}{A \sigma_{A=3}(x, Q^2)} = a_3(A) \bigg|_{2 < x \leq 3}
$$

In the ratios, off-shell effects and FSI largely cancel. $a_j(A)$ is proportional to probability of finding a $j$-nucleon correlation.

$$
\sigma(x, Q^2) = \sum_{j=1}^{A} A \frac{1}{j} a_j(A) \sigma_j(x, Q^2)
$$

$$
= \frac{A}{2} a_2(A) \sigma_2(x, Q^2) + \frac{A}{3} a_3(A) \sigma_3(x, Q^2) + \ldots
$$
Ratios and SRC

\[ \frac{2 \sigma_A}{A \sigma_D} = a_2(A); \quad (1.4 < x < 2.0) \]

A(e,e'), 1.4<Q^2<2.6


\[ \alpha_{2N} \approx 20\% \]
\[ \alpha_{3N} \approx 1\% \]

CLAS data
Egiyan et al., PRL 96, 082501, 2006
Arguments about role of FSI

Benhar et al.: FSI includes a piece that has a weak $Q^2$ dependence, Benhar et al. PLB 3443, 47

There is the cancellation of two large factors ($\approx 3$) that bring the theory to describe the data. These factors are $Q^2$ and $A$ dependent

The solution

- Direct ratios to $^{2}\text{H}$, $^{3}\text{He}$, $^{4}\text{He}$ out to large $x$ and over wide range of $Q^2$

- Study $Q^2$, $A$ dependence (FSI)

- Absolute Cross section to test exact calculations and FSI

- Extrapolation to NM
Sensitivity to SRC increase with $Q^2$ and $x$

We want to be able to isolate and probe two-nucleon and multi-nucleon SRCs

Dotted = mean field approx.
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Dashed = +multi-nucleon.

$11$ GeV can reach $Q^2 = 20(13)$ GeV$^2$ at $x = 1.3(1.5)$
- very sensitive, especially at higher $x$ values
Duality= resonances average to DIS

FIG. 1 (color). Extracted $F_2$ data in the nucleon resonance region for hydrogen (a) and deuterium (b) targets, as functions of the Nachtmann scaling variable $\xi$. For clarity, only a selection of the data is shown here. The solid curves indicate the result of the NMC fit to deep inelastic data for a fixed $Q^2 = 10 \text{ (GeV/c)}^2$ [16].

JLAB data, Niculescu et al.
x and $\xi$ scaling

An alternative view is suggested when the data (deuteron) is presented in terms of scattering from individual quarks.

$\nu W_2^A$ versus $x$

$\nu W_2^A$ versus $\xi$

$$\nu W_2^A = \nu \cdot \frac{\sigma^{\text{exp}}}{\sigma_M} \left[ 1 + 2 \tan^2(\theta/2) \cdot \left( \frac{1 + \nu^2/Q^2}{1 + R} \right) \right]^{-1}$$
The Nachtmann variable (fraction $\xi$ of nucleon light cone momentum $p^+$) has been shown to be the variable in which logarithmic violations of scaling in DIS should be studied.

Local duality (averaging over finite range in $x$) should also be valid for elastic peak at $x = 1$ if analyzed in $\xi$.

Evidently the inelastic and quasielastic contributions cooperate to produce $\xi$ scaling. Is this duality?
Medium Modifications generated by high density configurations

A single nucleon, $r = 1$ fm, has a volume of 4.2 fm$^3$
197 times 4.2 fm$^3 \approx 830$ fm$^3$
60% of the volume is occupied - very closely packed!

$V(r) \approx 0$ for $r \approx 1$ fm

Potential between two nucleons

Nucleon separation is limited by the short range repulsive core

Even for a 1 fm separation, the central density is about 4x nuclear matter

Comparable to neutron star densities!

High enough to modify nucleon structure?

To which nucleon does the quark belong?
Sensitivity to non-hadronic components

Ratio: With/Without

Ratio: With/Without

Mulders & Thomas
Quark distributions at $x > 1$

Two measurements (very high $Q^2$) exist so far:

CCFR ($\nu$-C): $F_2(x) \propto e^{-sx}$ \quad s = 8

BCDMS ($\mu$-Fe): $F_2(x) \propto e^{-sx}$ \quad s = 16

Limited $x$ range, poor resolution
Limited $x$ range, low statistics

With 11 GeV beam, we should be in the scaling region up to $x \approx 1.4$
Welcome to Quasielastic Electron Nucleus Scattering Archive

In connection with a review article (Quasielastic Electron-Nucleus Scattering, by O. Benhar, D. Day and I. Sick) to be submitted to Reviews of Modern Physics, we have collected here an extensive set of quasielastic electron scattering data in order to preserve and make available these data to the nuclear physics community.

We have chosen to provide the cross section only and not the separated response functions. Unless explicitly indicated the data do not include Coulomb corrections.

Our criteria for inclusion into the data base is the following:

2. Radiative corrections applied to data.
3. No known or acknowledged pathologies

At present there are about 600 different combinations of targets, energies and angles consisting of some 19,000 data points.

In the infrequent event that corrections were made to the data after the original publications, we included the latest data set, adding an additional reference, usually a private communication.

As additional data become known to us, we will add to the data sets.

If you wish to be alerted to changes in the archive or to the inclusion of new data, send an email to me. Send any comments or corrections you might have as well.