Electron scattering from nuclei in the quasielastic region and beyond

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Part 1

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Nuclear Response Function

\[ Q^2 = \bar{q}^2 - \nu^2 \]

\[ \nu = (E - E') \quad (\nu \equiv \omega) \]

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**Figure 1**

Schematic of nuclear response function with various regions:

- **I. Elastic Scattering**
- **II. Quasi-Free Scattering**
- **III. Deep Inelastic "EMC"**
- **IV. Deep Inelastic "Quarks"**

**Axes:**
- \( Q^2 \) vs. \( \nu \)
- \( Q^2 = 2 \Delta M^2 \)

**Proton and Nucleus:**
- \( x = 1 \)
- \( x = \frac{Q^2}{2M\nu} \)

**Regions:**
- Giant Resonance
- Photon absorption
- Lepton scattering
Structure of the nucleus

• nucleons are bound
• energy (E) distribution
• shell structure
• nucleons are not static
• momentum (k) distribution

on average:
- binding energy: ~ 8 MeV
- distance: ~ 2 fm

determined by N-N potential

repulsive core

long-range

attractive part

short-range

\sigma, \omega, \rho

V(\text{r}) [\text{MeV}]

0 100

0 50

1 2 3 4 5 d [\text{fm}]

d
How well do we understand nuclear structure?

- **The shell model**
  - Basis upon which most model calculations of nuclear structure rely.
- **The underlying physical picture**
  - Dense system of fermions whose motions to first order can be treated as independent particles moving in a mean field.
- **Electromagnetic interactions**
  - Best probe for investigating the validity of the independent particle picture because they are sensitive to a much larger fraction of the nuclear volume.
Early hint on shell structure in the nucleus

particular stable nuclei with $Z,N = 2, 8, 20, 28, 50, 82, 126$ (magic numbers)

large separation energy $E_s$

Weizsäcker Formula == Semi Empirical Mass Formula
Large deviations from the SEMF curve at small mass number, e.g. A = 4.

Systematic pattern of deviations occurs, with maxima in B occurring for certain "magic" values of N and Z, given by:
N/Z = 2, 8, 20, 28, 50, 82, 126.

These values of neutron and proton number are anomalously stable with respect to the average –the pattern must therefore reflect something important about the average nuclear potential V(r) that the neutrons and protons are bound in....
Shell structure (Maria Goeppert-Mayer, Jensen, 1949)

But: there is experimental evidence for shell structure

Pauli Exclusion Principle:

nucleons can not scatter into occupied levels:
Suppression of collisions between nucleons

nuclear density $10^{18}$ kg/m$^3$

With the enormous strong force acting between them and with so many nucleons to collide with, how can nucleons possibly complete whole orbits without interacting?
Independent Particle Shell model (IPSM)

- **single particle approximation:**
  nucleons move independently from each other
  in an average potential created by the surrounded nucleons (mean field)

  spectral function \( S(E, k) \):
  probability of finding a proton with initial momentum \( k \) and energy \( E \) in the nucleus

- **factorizes into energy & momentum part**

nuclear matter:

\[
Z(E) = \begin{cases} \text{occupied} & \text{if } E < E_F \\ \text{empty} & \text{if } E \geq E_F \end{cases}
\]

\[
Z(k) = \begin{cases} \text{occupied} & \text{if } k < k_F \\ \text{empty} & \text{if } k \geq k_F \end{cases}
\]

\[
E_F = \frac{k_F^2}{2M}
\]
Simple model yet excellent first approximation to structure of the nucleus
The single-particle energies $\xi_\alpha$ and wave function $\Phi_\alpha$ are the basic quantities in IPSM
In high energy knockout reaction we can directly measure $\xi_\alpha$ and $\Phi_\alpha$
Observed first in Uppsala in 1957 in $(p,2p)$ reactions on $^{12}\text{C}(p,2p)^{11}\text{B}$

$$S(\vec{p},E) = \sum_i |\Phi_\alpha(p)|^2 \delta(E + \epsilon_\alpha)$$

The spectral function should exhibit a structure at fixed energies with momentum distributions characteristic of the shell (orbit).
Quasi free Knockout Reactions

\[ \vec{k}_{A-1} = \vec{k}_0 - \vec{k}_1 - \vec{k}_2 = -\vec{k} \]

Momentum of particle in target nucleus that is knocked out

\[ E_s = T_1 + T_2 + T_{A-1} - T_0 \]

Separation energy (missing energy)

Energy required for separation of the nucleon from the target nucleus [Includes possible excitation of residual nucleus]
Fig. 4 and Fig. 5. Absolute cross sections for the (p, 2p) reaction at 185 MeV versus binding energy of the removed proton. Separate energy scales for each target also show the corresponding excitation energy of the residual nuclei. Relative errors should not be much larger than the statistical errors shown on each point; absolute errors should be less than 40%, and the error in comparing two spectra somewhat less.

(p,2p) experiments provided information on the binding energies of the inner shells of nuclei and their momentum. These experiments suffered from distortion of the proton (strongly interacting): Jacob and Maris (1966) suggested using high energy electrons - nucleus is almost transparent to them.
(e,e′p)-reaction: coincidence experiment
measured values: momentum, angles

electron energy: \( E_e \)
proton: \( \vec{p}_p' \)

electron: \( \vec{k}_e' \), \( E_{e'} = |\vec{k}_e'| \)

reconstructed quantities:
missing energy:
\( E_m = E_e - E_{e'} - T_{p'} - T_{A-1} \)
missing momentum:
\( \vec{p}_m = \vec{q} - \vec{p}_p' \)

in PWIA:
direct relation between measured quantities and theory:
\[ |E| \equiv E_m \quad k \equiv -\vec{p}_m \]
IA and IPSM

The QFS reaction cross section

\[
\frac{d\sigma_{fi}}{dE_1 d\Omega_1 dE_2 d\Omega_2} = K_S(\vec{k}, E) \frac{d\sigma_{\text{free}}}{d\Omega}
\]

factorized

Other reaction proportional \( S(p,E) \) are single nucleon pickup \([(p,d), (d, ^3\text{He}), (\gamma,p)]\)

Provides complimentary information but... strong absorption in nucleus hinders mapping out the spectral function.

**IA and IPSM is a considerable simplification**

- Assumption that asymptotic \( p_m \) and \( E_m \) are equal to values just before knockout
- Elementary reaction = free
- No FSI

Factorized form is preserved when strong interaction effects are considered – DWIA
The first (e,e'p) measurement: identification of different orbits

Frascati Synchrotron, Italy

$^{12}$C(e,e'p)

$^{27}$Al(e,e'p)


Moderate resolution: FWHM: 20 MeV
Characteristic momentum behavior of the s and p shells can be clearly seen. J. Mougey "The (e,e'p) reaction" Nuclear Physics A Volume 335, (1980) 35-53

\[ S(\vec{p}, E) = \sum_i |\Phi_a(p)|^2 \delta(E + \epsilon_a) \]
shape described by Lorentz function with central energy $E_\alpha$ + width $\Gamma_\alpha$

Steenhoven et al., PRC 32, 1787 (1985)

$\Rightarrow$ electrons are a suitable probe to examine the nucleus
Shell Model: describes basic properties like spin, parity, magic numbers ...

Momentum distribution:
- characteristic for shell \((l, j)\)
- Fourier transformation of \(\Psi_{lj}(r)\) ==> info about radial shape

![Graphs showing momentum distribution](image)

NIKHEF results

Momentum distribution:
- characteristic for shell \((l, j)\)
- Fourier transformation of \(\Psi_{lj}(r)\) ==> info about radial shape
Theory on previous slide (solid line):

Distorted wave impulse approximation (DWIA) solves the Schrödinger equation using an optical potential (fixed by $p^{12}\text{C}$) (Hartree-Fock, self-consistent)

- real part: Wood-Saxon potential
- imaginary part: accounts for absorption in the nucleus

Correction for Coulomb distortion

--- well reproduced shape

strength of the transition smaller!

Number of nucleons in each shell (IPSM): $2j + 1$

Spectroscopic factor $Z_\alpha$

$$Z_\alpha = 4\pi \int k_f dE dk k^2 S(k,E)$$

- single particle state $\alpha$:
- = number of nucleons in shell

NIKHEF

IPSM

Number of nucleons in each shell (IPSM): = 2j + 1

- 65%
k < k_F: single-particle contribution dominates

k ≈ k_F: SRC already dominates for E > 50 MeV

k > k_F: single-particle negligible

consequence: search for SRC at large E, k

method: (e,e'p)-experiment
Modern many-body theories:

- Correlated Basis Function theory (CBF)

- Green’s function approach (2nd order)

- Self-consistent Green’s function (T = 2 MeV)
Missing strength already at moderate $p_m$ compared to IPSM

$200 \text{ MeV/c} < p_m < 300 \text{ MeV/c}$

Spectral function containing SRC: good agreement with data

Data on $^{12}\text{C}$

IPSM*0.85

CBF

Radiation tail
Setup in Hall C:

- **HMS**
  - 0.5–7.4 GeV/c
- **SOS**
  - 0.1–1.75 GeV/c

Iron magnets

**Superconducting magnets, quadrupole: focusing/defocusing**

**Performance**

<table>
<thead>
<tr>
<th>Performance</th>
<th>HMS</th>
<th>SOS</th>
</tr>
</thead>
<tbody>
<tr>
<td>momentum range</td>
<td>0.5–7.4</td>
<td>0.1–1.75</td>
</tr>
<tr>
<td>acceptance $\delta$ (%)</td>
<td>$\pm 10$</td>
<td>$\pm 15$</td>
</tr>
<tr>
<td>solid angle (msr)</td>
<td>6.7</td>
<td>7.5</td>
</tr>
<tr>
<td>target acceptance (cm)</td>
<td>$\pm 7$</td>
<td>$\pm 1.5$</td>
</tr>
</tbody>
</table>
Data at high $p_m$, $E_m$ measured in Hall C at Jlab:

- targets: C, Al, Fe, Au
- kinematics: 3 parallel $p \parallel q$

To map out $S(E_m, p_m)$ vary $q$ keeping $p'$ ($T_p$) constant so that FSI are constant.

- kinematics: 2 perpendicular $p \perp q$

- Fix $e$, $\theta_e$, $p'$
- Vary $E_m$ thru $e'$
- Vary $p_m$ with proton angle $\theta_p$
Data at high $p_m$, $E_m$ measured in Hall C at Jlab:
- targets: C, Al, Fe, Au

Covered $E_m$-$p_m$ range:

- perpendicular kinematics
- parallel kinematics

high $E_m$ - region: dominated by $\Delta$ resonance
Extraction of the spectral function:
only in PWIA possible, care for corrections later

\[
\frac{d\sigma^{fi}}{dE_e d\Omega_e dE_p d\Omega_p} = K \sigma^{\text{free}} S(p_m, E_m) T_A
\]

exp. c.s.:

\[
\exp. \text{c.s.:} \quad \frac{d\sigma^{fi}}{dE_e d\Omega_e dE_p d\Omega_p} = K \sigma^{\text{free}} S(p_m, E_m) T_A
\]

Binning of the data \((E_m, p_m)_{ij} \): \(\Delta E_m = 10-50\) MeV, \(\Delta p_m = 40\) MeV/c

\[
\left( \frac{d\sigma}{dE_e d\Omega_e dE_p d\Omega_p} \right)_{ij} = \tilde{K} \tilde{\sigma}^{\text{free}} S(\tilde{p}_m, \tilde{E}_m)_{ij} T_p
\]

\[
= \frac{(N_{ij} - N_{ij}^{bg}) / \epsilon}{L P_{i,j}}
\]

Luminosity
Efficiency, dead time ...
phase space from M.C.

FWHM: 0.5ns
N_{ij}
N_{ij}^{bg}

N_{ij}^{bg}
2ns

0 -10 -5 0 5 10
coincidence time (ns)
Spectral function for $^{12}$C using cc
parallel: Kin3, Kin4, Kin5

$E_m = \frac{P_m^2}{2 M_p}$
Integrated strength in the covered $E_m-p_m$ region:

$$Z_C = 4\pi \int dp_m \, p_m^2 \int dE_m \, S(E_m,p_m)$$

230 MeV/c

Strength distribution in % (from CBF)

The region used for integral contains $\approx$ half of the total strength.
“correlated strength” in the chosen $E_m-p_m$ region:

<table>
<thead>
<tr>
<th></th>
<th>$^{12}\text{C}$</th>
<th>exp.</th>
<th>CBF theory</th>
<th>G.F. 2.order</th>
<th>selfconsistent G.F.</th>
</tr>
</thead>
<tbody>
<tr>
<td>experimental area</td>
<td>0.61</td>
<td>0.64 $\approx$ 10 %</td>
<td>0.46</td>
<td>0.61</td>
<td></td>
</tr>
<tr>
<td>in total (correlated part)</td>
<td></td>
<td>22 %</td>
<td>12 %</td>
<td>$\approx$ 20 %</td>
<td></td>
</tr>
</tbody>
</table>

contribution from FSI: -4 %

- $\approx$ 10% of the protons in $^{12}\text{C}$ at high $p_m$, $E_m$ found
- first time directly measured

comparing to theory leads to conclusion that
$\approx$ 20% of the protons in Carbon are beyond the IPSM region

Rohe et al.,
R(£, ¥)
NUCLEAR RESPONSE FUNCTION

Photon absorption

GIAN\ T RESONANCE

Elastic

Quasi-Free Scattering

Deep Inelastic "EMC"

Deep Inelastic "Quarks"

Lepton scattering

Q²

Q² = 0

50 MeV 300 MeV

Lepton scattering

x = 1

x = Q² / 2Mv

PROTON

NUCLEUS

Elastic

Q1  Q2  Q3

Dipole

27m

e

k

q

e'

Roth

show clearly this scaling behavior. At present none of the three-body

... of validity £ >> kF is satisfied.
Inclusive Electron Scattering from Nuclei

Two distinct processes

1. Quasielastic from the nucleons in the nucleus
   \[ M_A M^* A^{-1}, -\mathbf{k} \]
   \[ W^2 = M^2 \]

2. Inelastic and DIS from the quark constituents of the nucleon.
   \[ W^2 \geq (M_n + m_\pi)^2 \]

Inclusive final state means no separation of two dominant processes

\[ x = Q^2/(2\mu \omega) \]

\( \mu, \omega = \)energy loss

\[ x > 1 \quad x < 1 \]
Formalism

\[ \frac{d\sigma^2}{dQ_e' dE_e'} = \frac{d^2 E_e'}{Q^4 E_e} L_{\mu\nu} W^{\mu\nu} \]

\[ L_{\mu\nu} = 2 \left[ k_{e}^\mu k_{e'}^\nu + k_{e}^\nu k_{e'}^\mu - g^{\mu\nu}(k_{e} k_{e'}) \right] \quad W^{\mu\nu} = \sum_X \langle 0|J^\mu|X \rangle \langle X|J^\nu|0 \rangle \delta^{(4)}(p_0 + q - p_X) \]

Currents can be written as sum of one-body currents which (eventually) allows (See O. Benhar)

\[ W^{\mu\nu}(q, \omega) = \int d^3k \ dE \left( \frac{m}{E_k} \right) \left[ ZS_p(k, E)w^{\mu\nu}_p(\tilde{q}) + (A - Z)S_n(k, E)w^{\mu\nu}_n(\tilde{q}) \right] \]

where \( w^{\mu\nu} \) describes the e/m response of a bound nucleon with momentum \( k \) which consists of an elastic and inelastic component.

QES in IA

\[ \frac{d^2\sigma}{dQ d\nu} \propto \int d\vec{k} \int dE \sigma_{ei} S_i(k, E) \delta() \]

Spectral function

\[ G_{E}^{p,n}(Q^2) \text{ and } G_{M}^{p,n}(Q^2) \]

DIS

\[ \frac{d^2\sigma}{dQ d\nu} \propto \int d\vec{k} \int dE \ W_{1,2}^{(p,n)}(k, E) \]

Spectral function

\[ W_{1,2}^{p,n}(Q^2, \nu) \rightarrow W_{1,2}^{p,n}(x) + \log(Q^2) \text{ corrections} \]
The two processes share the same initial state

**QES in IA**

\[
\frac{d^2 \sigma}{dQ d\nu} \propto \int dk \int dE \sigma_{ei} S_i(k, E) \delta() \]

**DIS**

\[
\frac{d^2 \sigma}{dQ d\nu} \propto \int dk \int dE W_{1,2}^{(p,n)} S_i(k, E) \]

However they have very different \(Q^2\) dependencies

\(\sigma_{ei} \propto \text{elastic (form factor)}^2\)

\(W_{1,2}\) scale with \(\ln Q^2\) dependence

Exploit this dissimilar \(Q^2\) dependence
Relation to charged current neutrino-nucleus scattering

\[ e + A \rightarrow e' + X \]
\[ \nu_l + A \rightarrow l^- + X \]

\( \frac{d\sigma^2}{d\Omega_{e'} dE_{e'}} = \frac{a^2 E'_e}{Q^4 E_e} L_{\mu\nu} W_{\mu\nu} \)

\( \frac{d\sigma^2}{d\Omega_l dE_l} = \frac{G^2}{32\pi^2} \frac{|k'|}{|k|} L_{\mu\nu} W_{\mu\nu} \)

Both can be cast in the same form

\[ \frac{d^2\sigma}{d\Omega d\nu} \propto \int dk \int dE \sigma_{ei} S_i(k, E) \delta() \]

Spectral function

\[ \sigma_{ei} \rightarrow \sigma_{\nu i} \]

weak charged current interaction with a nucleon
Early 1970’s Quasielastic Data

→ getting the bulk features

\[ \vec{q} \approx 500 \text{MeV/c} \]

500 MeV, 60 degrees

compared to Fermi model: fit parameter \( k_F \) and \( \epsilon \)

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>( k_F )</th>
<th>( \epsilon )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(^6\text{Li})</td>
<td>169</td>
<td>17</td>
</tr>
<tr>
<td>(^{12}\text{C})</td>
<td>221</td>
<td>25</td>
</tr>
<tr>
<td>(^{24}\text{Mg})</td>
<td>235</td>
<td>32</td>
</tr>
<tr>
<td>(^{40}\text{Ca})</td>
<td>251</td>
<td>28</td>
</tr>
<tr>
<td>nat (^{56}\text{Ni})</td>
<td>260</td>
<td>36</td>
</tr>
<tr>
<td>(^{89}\text{Y})</td>
<td>254</td>
<td>39</td>
</tr>
<tr>
<td>nat (^{116}\text{Sn})</td>
<td>260</td>
<td>42</td>
</tr>
<tr>
<td>(^{181}\text{Ta})</td>
<td>265</td>
<td>42</td>
</tr>
<tr>
<td>(^{208}\text{Pb})</td>
<td>265</td>
<td>44</td>
</tr>
</tbody>
</table>

The shape of the low $\nu$ cross section is determined by the momentum distribution of the nucleons.

As $Q^2 \gg$ inelastic scattering from the nucleons begins to dominate

We can use $x$ and $Q^2$ as knobs to dial the relative contribution of QES and DIS.

The quasielastic peak (QE) is broadened by the Fermi-motion of the struck nucleon.

The quasielastic contribution dominates the cross section at low energy loss ($\nu$) even at moderate to high $Q^2$. 

• The shape of the low $\nu$ cross section is determined by the momentum distribution of the nucleons.
• As $Q^2 \gg$ inelastic scattering from the nucleons begins to dominate
• We can use $x$ and $Q^2$ as knobs to dial the relative contribution of QES and DIS.
A dependence: higher internal momenta broadens the peak
Shaded domain where scattering is restricted solely to correlations. Czyz and Gottfried (1963)

\[
\omega_c = \frac{(k + q)^2}{2m} + \frac{q^2}{2m}
\]

\[
\omega'_c = \frac{q^2}{2m} - \frac{q k_f}{2m}
\]

Czyz and Gottfried proposed to replace the Fermi n(k) with that of an actual nucleus. (a) hard core gas; (b) finite system of noninteracting fermions; (c) actual large nucleus.
Studying Superfast Quarks

- In the nucleus we can have $0 < x < A$
- In the Bjorken limit, $x > 1$ DIS tells us the virtual photon scatters incoherently from quarks
- **Quarks can obtain** momenta $x > 1$ by abandoning confines of the nucleon
  - deconfinement, color conductivity, parton recombination multiquark configurations
  - correlations with a nucleon of high momentum (short range interaction)
- DIS at $x > 1$ is a filter that selects out those nuclear configurations in which the nucleon wave functions overlap. We are studying the dynamics of partons that have abandoned the confines of the nucleon.

\[ < r_{NN} > \approx 1.7 \text{ fm} \approx 2 \times r_n = 1.6 \text{ fm} \]

The probability that nucleons overlap is large and at $x > 1$ we are kinematically selecting those configurations.
Short Range Correlations (SRCs)

Mean field contributions: \( k < k_F \)
Well understood, Spectroscopic Factors \( \approx 0.65 \)

High momentum tails: \( k > k_F \)
Calculable for few-body nuclei, nuclear matter.
Dominated by two-nucleon short range correlations.
Isolate short range interactions (and SRC's) by probing at high \( p_m \): \((e,e'p)\) and \((e,e')\)

Poorly understood part of nuclear structure.
Sign. fraction have \( k > k_F \)

Uncertainty in SR interaction leads to uncertainty at \( k>> \), even for simplest systems.
Fig. 2. Momentum distributions for $^4$He, HJ: Hamada–Johnston potential, RSC: Reid soft core potential, SSCB: de Tourreil–Sprung super soft core potential B, UNC: uncorrelated, for the RSC potential. The other uncorrelated distributions do not differ appreciably for $q > 2$ fm$^{-1}$.

Calculated by Zabolitzky and Ey, PLB 76, 527.

Fig. 3. Same as fig. 2, for $^{16}$O.

Calculated by Van Orden et al., PRC 21, 2628.

Calculations of SRC show up at large momentum.
Correlations are accessible in QES and DIS at large $x$ (small energy loss)