Voluntary Provision of Public Goods: Experimental Results with Interior Nash Equilibria

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Introduction

The standard public goods experiment involves linear payoffs in which the unique Nash equilibrium is at the lower boundary, i.e. full free riding. Contributions in these experiments tend to decline toward the Nash equilibrium in most treatments, but contributions persist even after as many as 60 rounds. This observation raises the question of whether the persistence of contributions is merely a boundary condition, with residual noise keeping contributions from reaching the Nash equilibrium. In other experimental environments, behavior shows a tendency to differ from boundary equilibria, but is reasonably close to interior predictions (see Smith and Walker, 1993, for examples). One way to address this issue in public goods experiments is to modify the standard linear payoff structure so that the Nash equilibrium is located in the interior of the set of feasible contributions. This paper surveys the evidence from interior-Nash public goods experiments. In some papers, the internal equilibrium is a dominant strategy and in others it is not. The designs also differ in terms of where the equilibrium is located relative to the upper and lower boundaries of the decision space. These relatively new designs are important because they can be used to evaluate the effects of treatment variables (for example, endowments, group size, and information) when the data are not being pulled toward the boundary. In addition, moving the equilibria to the center of the set of feasible contributions tends to reduce or neutralize any bias due to decision errors.

Before considering the interior Nash designs, it is useful to review the standard linear structure that produces a boundary equilibrium. An individual i who contributes x_i to the public good out of an endowment of E units, thereby consumes $E - x_i$ units of the private good. If the marginal value of the private good is a constant, v, and the individual's marginal value of total contributions to the public good is also a constant, m, then the individual's earnings are given by:

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$$\pi_i = v(E - x_i) + mX \tag{1}$$

where X is the sum of all individuals' contributions to the public good. Notice that the constant marginal value of both the public and private goods produces a linear earnings function that is maximized at either the upper or lower boundary (unless v is exactly equal to m, in which case any level of contribution is optimal).[1] In a more realistic setting, it is natural to think that both the public and private goods are subject to diminishing marginal values after some point. This feature requires the earnings function to be nonlinear in private and/or public consumption.

The most common approach used to implement non-linearities is to think of (1) as additively separable with two possibly nonlinear components: $V(E - x_i)$ and $M(X)$, where the capitalized notation indicates that $V(\bullet)$ and $M(\bullet)$ are functions. In the linear public goods environment described by (1), both of these functions are linear in contributions to the public good, and a dominant strategy at one of the boundaries results. Non-linearities have typically been introduced in only one of the components of the earnings function. Either of these two quasi-linear approaches can result in interior Nash equilibria, but they have different implications for the nature of these equilibria, as described below.[2] We begin with the simpler (although less common) setup: a non-linear value of private consumption. This yields a unique Nash equilibrium in dominant strategies.

Dominant Strategy Designs

The simplest way to introduce a non-linearity in the value of the private good is by using a quadratic function:

$$\pi_i = a(E - x_i) - b(E - x_i)^2 + mX \tag{2}$$

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[1] These calculations are based on the assumption that individuals maximize their own monetary earnings; the presence of altruism and fairness considerations may lead to different equilibrium outcomes.

[2] The focus of this paper is on *continuous* public goods experiments, where the earnings function from the public good is a continuous function of contributions. Other studies, not surveyed here, have implemented an interior equilibrium in a provision point setting, where the public good yields no return (or a comparatively low return) until contributions reach a specified threshold level. For examples of this research, see Marwell and Ames (1979, 1980) and Bagnoli and McKee (1991).

where $a,b > 0$, mX is the value of the public good, and $(E - x_i)$ is the consumption of the private good. Thus private consumption has diminishing marginal value. Maximizing earnings results in a unique equilibrium level of individual contributions where the marginal cost of contributing $[a - 2b(E - x_i)]$ is equated with the marginal benefit (m). For an appropriately specified earnings function and endowment, this yields an internal equilibrium, given by:

$$x_i = \frac{m - a}{2b} + E \quad . \qquad (3)$$

Notice that the equilibrium contribution depends only on the parameters of the experiment and not on others' contributions. Thus the equilibrium contribution is a dominant strategy, as in the linear public goods experiments.

Keser (1996) implemented this design in an experiment that tested whether moving the equilibrium away from the boundary would result in more dominant-strategy behavior. The equilibrium contribution to the public good was 7 tokens (35 percent of each individual's endowment), while full contribution was the Pareto optimal outcome. Although the dominant-strategy Nash equilibrium was the modal contribution (overall, 25 percent of all observations were at this level), average contributions were greater than this level in each round. This may be seen in the left side of Figure 1. Average contributions declined over the 25-round sessions, from about 33 percent greater than the equilibrium in the first five rounds, to about 15 percent greater in the last five rounds. More dominant-strategy behavior was observed in the final rounds of the sessions.[3]

Additional studies using the dominant strategy earnings structure described by (2) have yielded contribution patterns similar to Keser's. In van Dijk, Sonnemans, and van Winden (1997), the equilibrium contribution for each subject was three of ten tokens, with full contribution again the Pareto optimal outcome. Average contributions were about 35 percent

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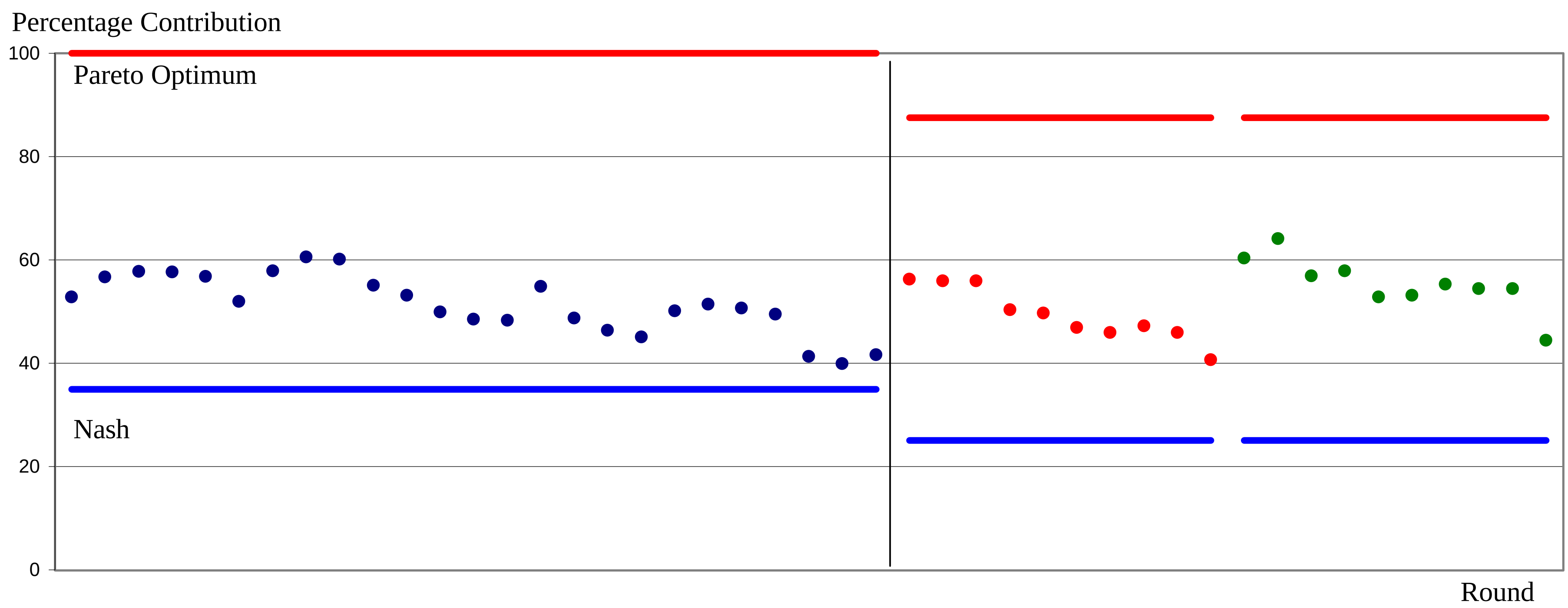
[3] Ortmann, Hansberry, and Fitzgerald (1997) found behavior more consistent with the dominant-strategy Nash equilibrium in a study intended to replicate Keser's. There were, however, several changes relative to her study: the introduction of a conversion factor resulted in three dominant strategy equilibria, subjects were reassigned between two groups after every round, the sessions lasted only eight rounds, greater controls for anonymity were implemented, and individual earnings were publicly posted after each round. It is not clear which of these changes may account for the difference in results.

Figure 1. Percentage of Endowment Contributed

Keser (1996), shown with blue dots, implemented an interior Nash equilibrium in which each subject had a dominant strategy to contribute a portion of their resource endowment.

Sefton and Steinberg (1996) compared a dominant strategy interior Nash equilibrium environment (red dots) with a non-dominant strategy environment (green dots).



Although contributions decline over the experiment, in every round average contributions are above the Nash equilibrium.

Average contributions are also above the Nash equilibrium in every round of the Sefton and Steinberg study.

Contributions are somewhat lower than the equilibrium when this is a dominant strategy, shown by the red dots, than when there is no dominant strategy, shown by the green dots.

higher than the equilibrium in early rounds, however there was less decay in contributions than observed in Keser's sessions, and a larger end-round drop in contributions. Sefton and Steinberg (1996) provided further evidence that equilibrium contributions are not, on average, attained by moving the dominant-strategy equilibrium into the interior of the decision space. As shown by the red dots on the right side of Figure 1, contributions in their study were about mid-way between the Nash equilibrium (set at 25 percent of the endowment) and the Pareto optimal outcome (set at 87.5 percent of the endowment).[4]

Considered together, these results indicate that moving the equilibrium away from the boundary is not sufficient in itself to induce Nash behavior in public goods experiments, even when the Nash equilibrium involves dominant strategies. Moreover, a systematic upward bias in contributions persists. The following section considers a number of other studies that have induced an interior equilibrium in a different way: by specifying a declining marginal value for the *public* good. They examine not only whether moving the equilibrium away from the boundary matters, but also how the location of the equilibrium in the set of feasible decisions affects contribution behavior.

Non-Dominant Strategy Designs

A second source of non-linearity is to introduce a diminishing marginal value of the public good, while maintaining the constant marginal cost of contributing (the marginal value of the private good). A simple quadratic specification of this quasi-linear environment is:

$$\pi_i = v(E - x_i) + aX - bX^2 \qquad (4)$$

where $a,b > 0$. Maximizing earnings results in an equilibrium contribution in which the marginal cost of contributing (v) is equated with the marginal benefit ($a - 2bX$). In equilibrium, the aggregate contribution is given by:

[4] Like Keser's design, the dominant strategy in Sefton and Steinberg's study was induced via a declining marginal value of the private good. However, the marginal value of the private good was not generated using an algebraic payoff function. This was done in order to hold constant the cost of a 1-unit deviation from equilibrium play between this treatment and a non-dominant-strategy treatment, described in the next section.

$$X = \frac{a - v}{2b} \quad . \tag{5}$$

Thus, for appropriately specified parameters, there is a unique equilibrium for *aggregate* contributions in the interior of the decision space.[5] However, there are multiple individual equilibria, each depending on the level of others' contributions, and there is no longer a dominant strategy. This adds a layer of complexity not present in the dominant-strategy design. In order to attain an equilibrium outcome, subjects must not only determine the optimal allocation (as in the dominant-strategy specification), but also correctly anticipate others' contribution decisions.

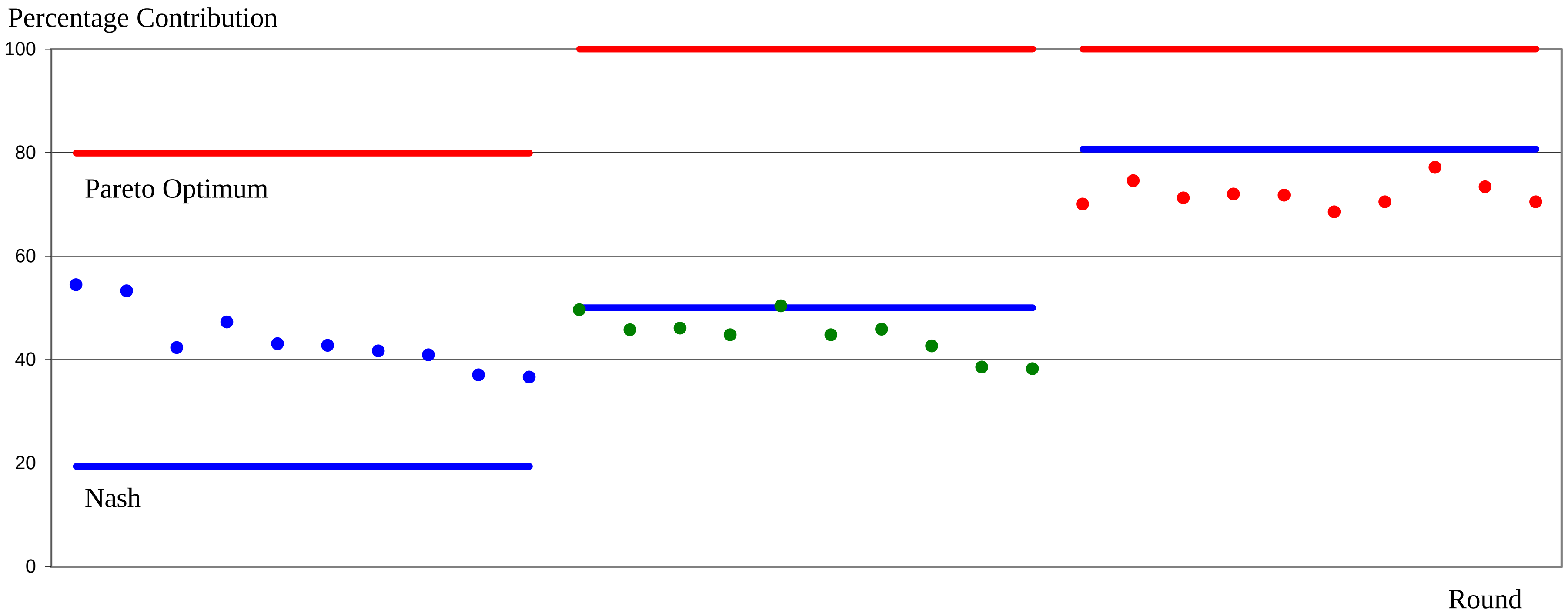
Given this additional coordination problem, it is perhaps not surprising that Sefton and Steinberg (1996) found that contributions had a higher variance in such a non-dominant-strategy environment than in a parallel, dominant-strategy treatment. Under both treatments, subjects were given the same endowment, faced the same aggregate Nash equilibrium and Pareto-optimal outcomes, and faced the same monetary loss from a one-unit deviation from equilibrium play. In addition to the higher variance in the non-dominant-strategy design, contributions were somewhat higher than in the dominant-strategy sessions (see the green dots on the right side of Figure 1). This difference, however, was not significant.

In each of the cases reviewed so far, the Nash equilibrium has been less than half of the feasible contribution amount, and average contributions remained above the Nash prediction, as can be seen in Figure 1. In a path-breaking paper, Isaac and Walker (1996) considered how the location of the aggregate Nash equilibrium, relative to the group's aggregate endowment, affected contributions to the public good. They considered treatments in which the aggregate Nash equilibrium was low relative to the group's endowment (19.4 percent), in the middle of the endowment space (50 percent) and high relative to the group's endowment (80.6 percent). As shown in Figure 2, the "low-Nash" and "high-Nash" treatments were mirror images of each other. Overall, only a small percentage of actual contributions were close to the aggregate Nash prediction. Like the studies discussed above, contributions in the low-Nash treatment were

[5] Two studies (Isaac, McCue and Plott, 1985, and Isaac and Walker, 1991) used the quadratic earnings structure in (4), but with the marginal cost set to be higher than the individual's marginal benefit at any level of contributions. Thus, zero contributions remained the dominant-strategy equilibrium.

Figure 2. Percentage of Endowment Contributed

Isaac and Walker (1996) studied how the location of the Nash equilibrium, within the range of feasible outcomes, affected the level of contributions to the public good in an environment without a dominant strategy.



Contributions below Nash are observed when the Nash equilibrium is high relative to total token endowment (see the red dots in the third treatment). The first and third treatments are mirror images of one another. However, in the "low Nash" treatment (blue dots, shown on the left) deviations from the Nash prediction are larger than in the "high Nash" treatment (red dots, shown on the right). The Nash equilibrium does relatively well when it is in the middle of the decision space, as in the second treatment (green dots, shown in the center).

significantly greater than the aggregate equilibrium. When the equilibrium was in the middle of the decision-space, contributions tracked the equilibrium initially, but declined below Nash by the final rounds of the sessions. Contributions in the high-Nash treatment were significantly below the Nash equilibrium.[6] However, the upward bias in low-Nash sessions was found to be significantly greater than the downward bias in high-Nash sessions, perhaps suggesting a greater tendency toward cooperative behavior.

A different approach toward exploring boundary effects is to hold the Nash equilibrium contribution constant, while moving the lower boundary closer to this level. Andreoni (1993) implemented such a design by imposing a minimum required contribution for each individual. Moving the boundary toward the equilibrium resulted in increased contributions, but by less than the amount of the "tax." Thus, some crowding out of voluntary contributions occurs, although the crowding out is incomplete. Chan, Godby, Mestelman, and Muller (1997) report a similar finding. They also examine how the size of the required transfer affects contributions. When the transfer is small relative to the equilibrium contribution, incomplete crowding out occurs. For a larger transfer, however, crowding out is complete.

It is also interesting to consider the level of contributions in the control treatments, i.e., in the absence of a required transfer. Average contributions in both studies were *less* than the Nash prediction, although this difference was not significant. While the equilibrium in Andreoni's study was close to half of the endowment (43 percent), the equilibrium in Chan et al. was considerably lower in the decision space (27 percent of the endowment). The clearest differences between these experiments and those reported earlier was in the payoff structure. Andreoni used an integer-approximation to a Cobb-Douglas payoff function, and Chan et al. added a Cobb-Douglas specification, $(E - x_i)X$, to the standard linear earnings function in (1). In the studies discussed so far, earnings from the private and public goods were additively separable, and hence were presented in *separate* tables that showed earnings from each good at various contribution levels. In contrast, subjects in the Andreoni and Chan et al. experiments were presented with a complete payoff matrix that showed total earnings as a function of both own and others'

[6] This downward bias is consistent with below-Nash contributions reported by Saijo and Nakamura (1995) in a linear public goods experiment where the dominant-strategy Nash equilibrium was located at the Pareto optimal upper boundary.

contributions to the public good.[7] We conjecture that subjects may have a hard time noticing and responding to the positive externality from the public good when the payoff structure is highly nonlinear, as in the Cobb-Douglas case, and earnings are presented in matrix form. This matrix presentation of earnings may make the outcome that maximizes individual earnings more apparent than that which maximizes group earnings, resulting in lower contributions than when earnings from the public and private goods are presented separately.

These observations suggest that the way in which subjects perceive the tradeoff between the private and public goods, and hence their contributions, could depend upon the way in which the earnings structure is presented. Laury, Walker, and Williams (1997) tested this by providing subjects with a table that explicitly showed the declining marginal benefit of the public good and the constant marginal benefit of the private good. In other sessions, without this information, there was evidence that subjects focused on the outcome that maximized earnings from the public good, ignoring the marginal cost of contributing. Providing subjects with additional earnings information was associated with a decrease in contributions to the public good. Although contributions remained above the Nash equilibrium level, complete free-riding was the modal contribution in these sessions.

While contributions in excess of the Nash equilibrium level are typical, these experiments have demonstrated that contributions may fall below the Nash level when this equilibrium is near the upper boundary or when the payoff structure is not separable and therefore must be presented in matrix form. Deviations from the predicted level are small when the equilibrium is near the middle of the set of feasible decisions. Providing subjects with detailed earnings information may also reduce the size of these deviations. The next session examines how two other treatments, individual endowment and group size, affect contributions in a non-dominant strategy interior Nash environment.

Treatment Effects

It is possible that contributions to the public good are a positive function of "wealth,"

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[7] Cason, Saijo, and Yamato (1997) also used a Cobb-Douglas earnings function and presented earnings to subjects in a payoff matrix. Contributions were not significantly different than the Nash equilibrium prediction for U.S. subjects, but were significantly less than this level for Japanese subjects.

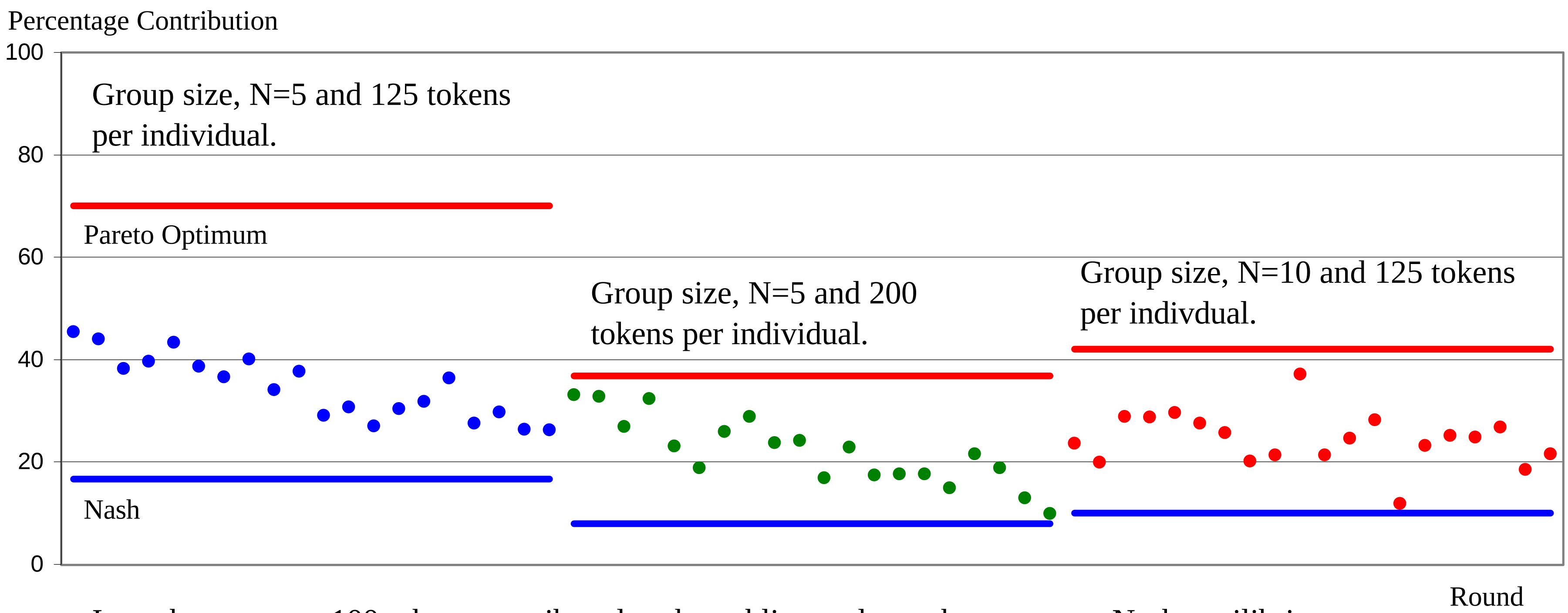
defined in the lab as subjects' resource endowment. As a special case of this, one can test whether subjects contribute a fixed percentage of their endowment. Laury, Walker, and Williams (1997) studied the effect of a change in individual endowments, from 125 to 200 tokens, holding constant the aggregate Nash prediction of 100 tokens. Contributions exceeded the Nash level in both treatments. When subjects were provided only with rough payoff tables, the *level* of contributions increased when the individual endowment was increased. However, the *percentage* of endowment contributed declined. When subjects were provided with additional earnings information, as described above, an increase in token endowment had almost no effect on the level of contributions. The results of these sessions (in terms of the percentage of endowment contributed to the public good) are shown on the left side of Figure 3.

Chan, Mestelman, Moir, and Muller (1996) instead examined the effect of changing the distribution of endowments among subjects, holding constant the three-person group's aggregate endowment. They found that small changes in the endowment distribution did not have a significant effect on contributions. However, a large inequality in endowment distribution led to a significant increase in contributions. On average, those subjects with relatively high endowments tended to under-contribute relative to the theoretical prediction, while subjects with relatively low endowments tended to over-contribute. A heterogeneous distribution of endowments was also implemented by van Dijk, Sonnemans, and van Winden (1997) in a dominant-strategy environment. In two-person groups, both the low-endowment and high-endowment subjects over-contributed relative to the Nash prediction. However, the extent of this over-contribution was greater for those with a low endowment, for whom the predicted contribution was lower (one of eight tokens for low-endowment subjects, compared with five of 12 tokens for high-endowment subjects).

The effect of changing group size, holding constant the aggregate Nash equilibrium level of contributions, has also been examined. Both Guttman (1986) and Laury, Walker, and Williams (1997) report similar group-size effects. Doubling group size (from three to six in Guttman's study and from five to ten in Laury, Walker, and Williams) results in an increase in aggregate contributions to the public good, but a decrease in individual contributions. Contributions are higher than the Nash equilibrium prediction in all of these sessions, however. Results from the Laury, Walker, and Williams experiments are shown in Figure 3.

Figure 3. Percentage of Endowment Contributed

Laury, Walker, and Williams (1997) examined the effect of group size and individual token endowment on contributions in a non-dominant strategy environment.



In each treatment, 100 tokens contributed to the public good was the aggregate Nash equilibrium. However, the Nash equilibrium varied as a percentage of total group resource endowment between treatments. For groups of size five, the percentage contribution is lower when each participant is given 200 tokens (green dots) than when each is given 125 tokens (blue dots), although the total number of tokens contributed is quite close between these two treatments. The percentage contribution is lower for groups of size ten (red dots) than for groups of size five (blue dots), although the total number of tokens contributed by groups of size ten is higher.

Final Observations

The majority of evidence from internal Nash public goods experiments shows that simply moving the equilibrium into the interior of the decision space is not sufficient to produce contributions that are, on average, close to the equilibrium prediction. When the Nash equilibrium falls between the lower boundary and the mid-point of the decision space, average contributions typically exceed the equilibrium level. This is true both in dominant-strategy and non-dominant-strategy environments. There is less variance in contribution decisions, however, when the interior Nash equilibrium is a dominant strategy. The most important determinant of the size and direction of these deviations appears to be the equilibrium's location relative to the group's aggregate endowment. For example, significant *under-contribution* is observed when the equilibrium is relatively close to the upper boundary. Other treatments have been shown to impact contributions as well. While a uniform change in individual endowment may not affect the *level* of contributions, a change in the distribution of endowments may. An increase in group size increases aggregate contributions, while individual contributions fall. There is also some evidence that the way in which the earnings function is presented to subjects may affect contributions, although a controlled study of this has not been conducted.

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